VALLIAMMAI ENGINEERING COLLEGE

S.R.M. Nagar, Kattankulathur - 603203

DEPARTMENT OF MATHEMATICS

QUESTION BANK

(COMMON TO ALL BRANCHES)

MA 8251- MATHEMATICS –II

Regulation – 2017

Academic Year – 2018 - 19

Prepared by

Dr.T.Isaiyarasi
Mr.V.Dhanasekaran
Ms.A.Karpagam
Ms.C.V.Dhanya

VALLIAMMAI ENGINEERING COLLEGE
SRM Nagar, Kattankulathur – 603 203.

DEPARTMENT OF MATHEMATICS

QUESTION BANK

SUBJECT : MA8251- ENGINEERING MATHEMATICS-II
SEM / YEAR : II SEMESTER / I YEAR (COMMON TO ALL BRANCHES)

UNIT I MATRICES			
Eigen values and Eigenvectors of a real matrix – Characteristic equation – Properties of Eigen values and Eigenvectors – Cayley-Hamilton theorem – Diagonalization of matrices – Reduction of a quadratic form to canonical form by orthogonal transformation – Nature of quadratic forms.			
Q.No.	**Question**	**BT Level**	**Competence**
PART-A			
1	Obtain the eigen values of A^3 where A=$\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	BTL-1	Remembering
2	Find the eigen values of $2A^2$ if A=$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$	BTL-1	Remembering
3	Find the sum and product of the eigen values of A= $\begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{pmatrix}$	BTL-1	Remembering
4	Find the sum and squares of the eigen values of A= $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$	BTL-1	Remembering
5	The product of the 2 eigen values of A= $\begin{pmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 14. Find the 3rd eigen value.	BTL-1	Remembering
6	Show that the eigen values of a null matrix are zero.	BTL-1	Remembering
7	State Cayley-Hamilton theorem.	BTL-2	Understanding
8	Use Cayley Hamilton theorem to find A^{-1} if A=$\begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}$	BTL-2	Understanding
9	If A=$\begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$ find A^3 using Cayley Hamilton theorem	BTL-2	Understanding
10	Write any 2 applications of Cayley Hamilton theorem	BTL-2	Understanding
11	Prove that the eigen values of A^{-1} are $\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3}$..........$\frac{1}{\lambda_n}$	BTL-3	Applying
12	Prove that sum of eigen values of a matrix is equal to its trace.	BTL-3	Applying
13	If 3 and 5 are two eigen values find the third eigen value of A= $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	BTL-3	Applying
14	Find the constants a and b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3,-2 be the eigen values of A	BTL-4	Analyzing
15	For what values of c, the eigen values of the matrix $\begin{pmatrix} 1 & 2 \\ c & 4 \end{pmatrix}$are real and unequal , real and equal, complex conjugates.	BTL-4	Analyzing
16	Find the matrix corresponding to the quadratic form 2xy+2yz+2zx.	BTL-4	Analyzing
17	Find the quadratic form corresponding to the matrix A= $\begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 6 \end{pmatrix}$	BTL-5	Evaluating
18	What is the nature of the quadratic form $x^2+y^2+z^2$ in 4 variables?	BTL-5	Evaluating
19	If 2,-1,-3 are the eigen values of the matrix A, then find the eigen values of A^2-2I	BTL-6	Creating

20	Find the symmetric matrix A, whose eigen values are 1 and 3 with corresponding eigen vectors $\begin{pmatrix}1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\1\end{pmatrix}$	BTL-6	Creating
	PART-B		
1(a)	Find the eigen values and eigen vectors of the matrix A= $\begin{pmatrix}11 & -4 & -7\\7 & -2 & -5\\10 & -4 & -6\end{pmatrix}$	BTL-2	Understanding
1(b)	Verify Cayley-Hamilton theorem and hence find A^{-1} of A= $\begin{pmatrix}1 & 2 & 1\\2 & 2 & 1\\1 & 1 & 3\end{pmatrix}$	BTL-1	Remembering
2(a)	Find the eigen values and eigen vectors of the matrix A= $\begin{pmatrix}2 & -2 & 2\\1 & 1 & 1\\1 & 3 & -1\end{pmatrix}$	BTL-4	Analyzing
2(b)	Diagonalise the matrix A= $\begin{pmatrix}2 & 1 & -1\\1 & 1 & -2\\-1 & -2 & 1\end{pmatrix}$ by means of an orthogonal transformation.	BTL-5	Evaluating
3(a)	Find the eigen values and eigen vectors of the matrix A= $\begin{pmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{pmatrix}$	BTL-2	Understanding
3(b)	Diagonalise the matrix A= $\begin{pmatrix}2 & 0 & 4\\0 & 6 & 0\\4 & 0 & 2\end{pmatrix}$ by orthogonal transformation	BTL-5	Evaluating
4(a)	Obtain the eigen values and eigen values and eigen vectors of the matrix A= $\begin{pmatrix}2 & 0 & 1\\0 & 2 & 0\\1 & 0 & 2\end{pmatrix}$ and verify that the eigen vectors are orthogonal in pairs.	BTL-4	Analyzing
4(b)	If $\lambda_1, \lambda_2, \dots \dots \dots \dots \dots \dots \dots \lambda_n$ are the eigen values of a matrix A, then (i) If k$\lambda_1, k\lambda_2, \dots \dots \dots \dots \dots \dots \dots k\lambda_n$ are the eigen values of the matrix kA, where k is a non-zero scalar. (ii)${\lambda_1}^p, {\lambda_2}^p, \dots \dots \dots \dots \dots \dots \dots {\lambda_n}^p$are the eigen values of the matrix A^P, where P is a positive integer. (iii)$\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3} \dots\dots\dots \frac{1}{\lambda_n}$ are the eigen values of A^{-1}	BTL-5	Evaluating
5(a)	Verify Cayley-Hamilton theorem for the matrix A= $\begin{pmatrix}1 & 3 & 7\\4 & 2 & 3\\1 & 2 & 1\end{pmatrix}$ and also find A^{-1}.	BTL-4	Analyzing
5(b)	Obtain the eigen values and eigen vectors of the matrix A= $\begin{pmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{pmatrix}$	BTL-5	Evaluating
6(a)	Verify that the matrix A= $\begin{pmatrix}2 & -1 & 2\\-1 & 2 & -1\\1 & -1 & 2\end{pmatrix}$ satisfies the characteristic equation and hence find $A^{4.}$	BTL-4	Analyzing
6(b)	Diagonalise the matrix A= $\begin{pmatrix}10 & -2 & -5\\-2 & 2 & 3\\-5 & 3 & 5\end{pmatrix}$ by orthogonal reduction.	BTL-1	Remembering
7(a)	Using Cayley-Hamilton theorem for adj(A) for A= $\begin{pmatrix}1 & 2 & -1\\0 & 1 & -1\\1 & -1 & 2\end{pmatrix}$	BTL-1	Remembering
7(b)	Determine a diagonal matrix orthogonally similar to the real symmetric matrix $\begin{pmatrix}8 & -6 & 2\\-6 & 7 & -4\\2 & -4 & 3\end{pmatrix}$	BTL-2	Understanding
8(a)	Use Cayley-Hamilton theorem to find the value of the matrix given by $A^8 - 5A^7 + 7A^6$ -$3A^5$ + $5A^4 - 5A^3 + 8A^2 - 2A + I$ if the matrix A= $\begin{pmatrix}2 & 1 & 1\\0 & 1 & 0\\1 & 1 & 2\end{pmatrix}$	BTL-4	Analyzing
8(b)	Determine a diagonal matrix orthogonally similar to the real symmetric matrix $\begin{pmatrix}6 & -2 & 3\\-2 & 3 & -1\\2 & -1 & 3\end{pmatrix}$	BTL-1	Remembering
9(a)	Find the Eigen values of A and hence find A^nC_n is positive integer given that A=$\begin{pmatrix}1 & 2\\4 & 3\end{pmatrix}$	BTL-5	Evaluating
9(b)	The Eigen vectors of a 3*3 real symmetric matrix A corresponding to the eigenvalues 2,3,6 are $(1, \ 0, \ -1)^T, (1, \ 1, \ 1)^T, (1, \ -2, \ 1)^T$ respectively. Find the matrix A.	BTL-1	Remembering

10	Reduce the quadratic form $8x^2+7y^2+3z^2-12xy+4xz-8yz$ into canonical form by orthogonal reduction and find its nature.	BTL-5	Evaluating
11	Reduce the quadratic form $6{x_1}^2+3{x_2}^2+3{x_3}^2-4x_1x_2-2x_2x_3+4x_3x_1$ into canonical form by an orthogonal reduction and find the rank, index, signature, and nature.	BTL-2	Understanding
12	Reduce the quadratic form $3{x_1}^2+2{x_2}^2+3{x_3}^2-2x_1x_2-2x_2x_3-2x_3x_1$ into canonical form by an orthogonal reduction .	BTL-3	Applying
13	Reduce the quadratic form $2{x_1}^2+{x_2}^2+{x_3}^2+2x_1x_2-2x_1x_3-4x_2x_3$ into canonical form by an orthogonal reduction and find the rank, index, signature, and nature.	BTL-1	Remembering
14	Reduce the quadratic form $2{x_1}^2+5{x_2}^2+3{x_3}^2+4x_1x_2$ into canonical form by an orthogonal reduction .Also discuss its nature.	BTL-1	Remembering

UNIT II- VECTORCALCULUS

Gradient and directional derivative – Divergence and curl – Vector identities – Irrotational and Solenoidal vector fields – Line integral over a plane curve – Surface integral – Area of a curved surface – Volume integral – Green's, Gauss divergence and Stoke's theorems – Verification and application in evaluating line, surface and volume integrals.

PART-A

1	Find div(curl φ), if $\varphi = x^3\vec{\imath}+y^3\vec{\jmath}+z^3\vec{k}$	BTL-1	Remembering
2	Find the Directional derivative of $f = xyz$ at (1,1,1)in the direction $\vec{\imath}+\vec{\jmath}+\vec{k}$.	BTL-1	Remembering
3	Find the Directional derivative of $\varphi = 4xz^2+x^2yz$ at (1,-2,1) in the direction $2\vec{\imath}+3\vec{\jmath}+4\vec{k}$.	BTL-1	Remembering
4	Is the position vector $\vec{r} = x\vec{i}+y\vec{j}+z\vec{k}$ irrotational? Justify.	BTL-1	Remembering
5	What is the value of *m* if the vector $\vec{F} = (x+3y)\vec{\imath}+(y-2z)\vec{\jmath}+(x+mz)\vec{k}$ is solenoidal	BTL-1	Remembering
6	What is the value of a, b, c if the vector $\vec{F} = (x+y+az)\vec{\imath}+(by+2y-z)\vec{\jmath}+(-x+cy+2z)\vec{k}$ may be irrotational	BTL-1	Remembering
7	Give the unit normal vector to the surface $xyz = 2$ at (2, 1, 1).	BTL-2	Understanding
8	Give the unit normal vector to the surface $x^2+y^2+z^2 = 1$ at (1,1,1).	BTL-2	Understanding
9	If $\varphi = 3xy-yz$, Find $grad\ \varphi$ at (1,1,1).	BTL-2	Understanding
10	If $\vec{r}$ is the position vector, Find $div\ \vec{r}$.	BTL-2	Understanding
11	Show that $\nabla(r^n) = nr^{n-2}\vec{r}$.	BTL-3	Applying
12	Show that the vector $\vec{F} = (y^2-z^2+3yz-2x)\vec{\imath}+(3xz+2xy)\vec{\jmath}+(3xy-2xz+2z)\vec{k}$ is solenoidal .	BTL-3	Applying
13	Show that $curl(grad\ \varphi) = 0$.	BTL-3	Applying
14	If $\vec{F} = 3xy\vec{\imath}-y^2\vec{\jmath}$, Evaluate$\int \vec{F}.d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from the point (0,0) to the point (1,2).	BTL-4	Analyzing
15	If $\vec{F} = (x^2)\vec{\imath}+(xy^2)\vec{\jmath}$, evaluate $\int \vec{F}.d\vec{r}$ from (0,0) to (1,1) along the path $y = x$.	BTL-3	Applying
16	Using Green's theorem evaluate $\int_C [(2x^2-y^2)dx+(x^2+y^2)\,dy$ where C is the boundary of the square enclosed by the lines $x = 0, y = 0, x = 2, y = 3$.	BTL-6	Creating
17	State Gauss Divergence theorem.	BTL-5	Evaluating
18	Evaluate using Gauss Divergence theorem for $\vec{F} = 4xz\vec{\imath}-y^2\vec{\jmath}++yz\vec{k}$ taken over the cube $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$.	BTL-5	Evaluating
19	State Stokes theorem.	BTL-6	Creating
20	State Greens theorem	BTL-6	Creating

PART-B

1(a)	Find the Directional Derivative of $\varphi = 3x^2yz\ \ +4xz^2+xyz$ at (1, 2, 3) in the direction of	BTL-1	Remembering

	$2\vec{i}+\vec{j}-\vec{k}$.		
1(b)	If $\vec{F}=(3x^2+6y)\vec{i}-14yz\vec{j}+20z^2\vec{k}$, evaluate the line integral $\int \vec{F}.d\vec{r}$ from (0,0,0) to (1,1,1) along the path $x=t, y=t^2, z=t^3$	BT L-2	Understanding
2(a)	Find the values of a and b so that the surfaces $ax^3-by^2z=(a+3)x^2$ and $4x^2y-z^3=11$ may cut orthogonally at (2,-1,-3).	BT L-1	Remembering
2(b)	Evaluate $\iint_s \vec{F}.d\vec{s}$, where $\vec{F}=12x^2y\vec{i}-3yz\vec{j}+2z\vec{k}$ and s is the portion of the plane $x+y+z=1$ included in the first octant.	BT L-2	Understanding
3(a)	Calculate the angle between the normals to the surface $xy=z^2$ at the points (4,1,2) and(3,3,-3).	BT L-1	Remembering
3(b)	Find the work done when a force $\vec{F}=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ moves a particle in the XY pane from(0,0,0) to (1,1,1) along the parabola $x=y^2$. Is the workdone different when the path is the straight line $x=y$?	BT L-3	Applying
4(a)	Find the angle between the surface $x^2+y^2+z^2=9$ and $x^2+y^2-3=z$ at the points (2,-1,2).	BT L-2	Understanding
4(b)	Find the work done by the force $\vec{F}=\left(2xy+z^3\right)\vec{\imath}+\left(x^2\right)\vec{\jmath}+5xz^2\vec{k}$ when it moves a particle from (1,-2,1) to (3,1,4) along any path?	BT L-4	Analyzing
5(a)	Find the values of a and b so that the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ may cut orthogonally at (1,-1,2).	BT L-3	Applying
5(b)	Find the value of n such that $r^n r$ is both solenoidal and irrotational	BT L-5	Evaluating
6(a)	Show that div $(\vec{u}\text{ X }\vec{v})=\vec{v}$.curl $\vec{u}-\overline{u}$.curl $\vec{v}$	BT L-1	Remembering
6(b)	Find the value of a,b,c so that the vector $\vec{F}=(x+y+az)\vec{i}+(bx-2y-z)\vec{j}+(-x+cy+2z)\vec{k}$ may be irrotational. Also find its scalar potential	BT L-3	Applying
7(a)	Prove that div$(\varphi F)=\varphi\ div\ f+\nabla\varphi.f$ where $\emptyset$ is a scalar point function and f is a vector point function.	BT L-1	Remembering
7(b)	Find its scalar potential, if the vector field $\vec{F}=(x^2+xy^2)\vec{i}+(y^2+x^2y)\vec{j}$ is irrotational	BT L-6	Creating
8(a)	Prove that the area bounded by a simple closed curve c is given by $\frac{1}{2}\int_c[(xy+y^2)dx+(x^2)dy]$ Hence find the area of ellipse $x=acos\theta, y=bsin\theta$	BT L-3	Applying
8(b)	Apply Green's theorem to evaluate $\int_c[(y-\sin x)dx+(\cos x)dy]$ where c is the plane triangle enclosed by the lines $y=0, x=\frac{\pi}{2}, y=\frac{\pi}{2}x$	BT L-1	Remembering
9(a)	Verify Green's theorem in the plane for $\int_c[(xy+y^2)dx+(x^2)dy]$ where c is a closed of the region bounded by $x=y$ and $y=x^2$.	BT L-3	Applying
9(b)	Verify Gauss divergence theorem for $\vec{F}=(x^3)\vec{\imath}+(y^3)\vec{\jmath}+z^3\vec{k}$ where s is the surface of the cuboid formed by the planes x=0, x=a, y=0, y=a, z=0,z =a.	BT L-5	Evaluating
10(a)	Verify Green's theorem in the plane for $\int_c[(3x^2-8y^2)dx+(4y-6xy)dy]$ where C is the boundary of the region bounded by x = 0, y = 0, x + y = 1.	BT L-3	Applying
10(b)	Verify Stoke's theorem for $\vec{F}=(xy+y^2)\vec{i}+x^2\vec{j}$ in the xoy-plane bounded $x=y$ and $y=x^2$.	BT L-6	Creating
11	Show that Stokes theorem is verified for $\vec{F}=(y-z+2)\vec{\imath}+(yz+4)\vec{\jmath}-xz\vec{k}$ where S is the surface of the cube formed by $x=0, x=2, y=0, y=2, and\ z=2$ above the xy-plane.	BT L-3	Applying
12	Verify stokes theorem for $\vec{F}=(2x-y)\vec{\imath}-(yz^2)\vec{\jmath}-y^2\vec{k}$ where s is the upper hemisphere $x^2+y^2+z^2=a^2$ and c is the boundary .	BT L-1	Remembering
13	Verify Gauss divergence theorm for $\vec{F}=(x^2)\vec{\imath}+(y^2)\vec{\jmath}+z^2\vec{k}$ where s is the surface of the cuboid formed by the planes $x=0, x=a, y=0, y=b, z=0, z=c$.	BT L-3	Applying
14	Verify Gauss divergence theorem for $\vec{F}=(x^3-yz)\vec{\imath}-(2x^2y)\vec{\jmath}+2\vec{k}$ where s is the surface of the cuboid formed by the planes $x=0, x=a, y=0, y=a, z=0, z=a$.	BT L-3	Applying
	UNIT III ANALYTIC FUNCTIONS		

Analytic functions – Necessary and sufficient conditions for analyticity in Cartesian and polar coordinates - Properties – Harmonic conjugates – Construction of analytic function - Conformal mapping – Mapping by functions $w = z + C, Cz, 1/z, z^2$ - Bilinear transformation.

PART-A

1	Examine if f(z)= z^3 analytic ?	BTL-1	Remembering
2	Identify the constants a,b,c if $f(z) = x + ay + i(bx + cy)$ is analytic.	BTL-1	Remembering
3	Define conformal mapping.	BTL-1	Remembering
4	Can $u = 3x^2y - y^3$ be the real part of an analytic function? Justify your answer	BTL-1	Remembering
5	State necessary and sufficient condition for f(z) to be analytic.	BTL-1	Remembering
6	Identify the invariant point of the bilinear transformation $w = \frac{2z+6}{z+7}$	BTL-1	Remembering
7	Estimate the invariant points of the transformation $w = \frac{z-1}{z+1}$	BTL-2	Understanding
8	Estimate the invariant point of the bilinear transformation $w = \frac{z-1-i}{z+2}$	BTL-2	Understanding
9	Give the image of the circle $\|z\| = 3$ under the transformation w = 5z.	BTL-2	Understanding
10	If f(z) = $r^2(cos2\theta + i\ sinP\theta)$ is analytic, then find the value of 'p'.	BTL-2	Understanding
11	Show that $\|z\|^2$ is not analytic at any point.	BTL-3	Applying
12	Show that an analytic function in a region R with constant imaginary part is constant.	BTL-3	Applying
13	Show that $u = 2x - x^3 + 3xy^2$ is harmonic.	BTL-3	Applying
14	If f(z) is an analytic function whose real part is constant, Point out f(z) is a constant function.	BTL-4	Analyzing
15	Explain that a bilinear transformation has at most 2 fixed points.	BTL-4	Analyzing
16	Examine whether the function xy^2 can be real part of analytic function.	BTL-4	Analyzing
17	Test the analyticity of the function $f(z) = e^{-z}$	BTL-5	Evaluating
18	Evaluate the image of hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$	BTL-5	Evaluating
19	Formulate the critical points of the transformation $w = z + \frac{1}{z}$	BTL-6	Creating
20	Formulate the bilinear transformation which maps $z = 0, -i, -1$ into $w = i, 1, 0$ respectively.	BTL-6	Creating

PART-B

1(a)	Given that $u = \frac{sin2x}{cosh2y - cos2x}$, Estimate the analytic function.	BTL-2	Understanding
1(b)	Find the image of $\|z\| = 2$ under the transformation $(i) w = z + 3 + 2i (ii)\ w = 3z$	BTL-4	Analysing
2(a)	Estimate the analytic function $w = u + iv$ if $u = e^x(xcos2y - ysin2y)$.	BTL-2	Understanding
2(b)	Formulate the image of $\|z + 1\| = 1$ under the map $w = 1/z$.	BTL-6	Creating
3(a)	Estimate the analytic function $f(z) = u + iv$ given the imaginary part is $v = x^2 - y^2 + \frac{x}{x^2+y^2}$.	BTL-2	Understanding
3(b)	Prove that an analytic function with constant modulus is constant.	BTL-1	Remembering
4(a)	Show that the function $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. Find also the conjugate harmonic function v.	BTL-3	Applying
4(b)	Show that the transformation $w = \frac{1}{z}$ maps, in general, circles and straight lines into circles and straight lines. Point out the circles and straight lines are transformed into straight lines and circles respectively.	BTL-1	Remembering

5(a)	Estimate the analytic function $w = u + iv$ if $u - v = e^x(cosy - siny)$	BTL-2	Understanding
5(b)	Under the transformation w = $\frac{1}{z}$, find the image of a region (i) $x > c$ where $c > 0$, (ii) $y > c$, where $c < 0$	BTL-4	Analysing
6(a)	Solve the bilinear transformation that maps the point $z_1 = i., z_2 = -1$, $z_3 = 1$into the points $w_1 = 0, w_2 = 1, w_3 = \infty$ respectively.	BTL-5	Evaluating
6(b)	Identify the image of the infinite strip $(i) 0 \le y \le \frac{1}{2}$ $(ii) \frac{1}{4} < y < \frac{1}{2}$ under the transformation $= 1/z$.	BTL-2	Understanding
7(a)	Determine the analytic function $w = u + iv$ given that $3u + 2v = y^2 - x^2 + 16xy.$	BTL-4	Analysing
7(b)	Formulate the image of $\|z - 2\| = 2$ under the map $w = z^2$	BTL-6	Creating
8(a)	Point out the bilinear transformation that maps the point $z_1 = 1$, $z_2 = i, z_3 =$ -1 into the points $w_1 = i, w_2 = 0, w_3 = -i$ respectively.	BTL-3	Applying
8(b)	Show that the image of $\|z - 1\| = 1$ under the transformation $w = z^2$ is the cardioid $R = 2(1 + cos\varphi)$	BTL-1	Remembering
9(a)	Give the bilinear transformation which maps $z = 1,0,-1$ into $w = 0,-1,\infty$ respectively. What are the invariant points of the transformation?	BTL-2	Understanding
9(b)	If $u = x^2 - y^2, v = -\frac{y}{x^2+y^2}$, prove that u and v are harmonic functions but u+iv is not an analytic function.	BTL-5	Evaluating
10(a)	Identify the bilinear transformation that maps $1 + i, -i, 2 - i$ at the z-plane into the points $0, 1, i$ of the w-plane.	BTL-1	Remembering
10(b)	Determine the analytic function $f(z) = u + iv$ such that $2u + v = e^x(cosy - siny)$		
11(a)	Identify the bilinear mapping which maps $-1,0,1$ of the z-plane onto $-1, i, 1$ of the w-plane. Show that under this mapping the upper half of z- plane maps onto the interior of unit circle $\|w\| = 1$.	BTL-1	Remembering
11(b)	If $w = f(z)$ is analytic then Show that $\left(\left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)\right)$ log f(z) = 0.	BTL-6	Creating
12(a)	Identify the bilinear mapping which maps $1, i, -1$ of the z-plane onto $0,1,\infty$ of the w-plane. Show that the transformation maps the interior of the unit circle of the z-plane onto the upper half of the w-plane.	BTL-1	Remembering
12(b)	If f (z) is a regular function of z, Show that $\nabla^2\|f(z)\|^2 = 4\|f'(z)\|^2$.	BTL-3	Applying
13(a)	If $w = u(x, y) + iv(x, y)$ is an analytic function , show that the curves of the family $u(x, y) = a$ and the curves of the family $v(x, y) = b$, cut orthogonally where a and b are varying constants.	BTL-2	Understanding
13(b)	Solve the bilinear transformation that maps the point $z_1 = 0, z_2 = 1, z_3 = \infty$ into the points $w_1 = i, w_2 = 1, w_3 = -i$ respectively.	BTL-5	Analyzing
14(a)	If f (z) = u + iv is an analytic function of z, then formulate that $\nabla^2[\log\|f'(z)\|] = 0$.	BTL-6	Creating
14(b)	Solve the bilinear transformation that maps the point $z_1 = 0, z_2 = 1, z_3 = \infty$ into the points $w_1 = -5, w_2 = -1, w_3 = 3$ respectively. What are invariant points of transformation.	BTL-5	Analyzing

UNIT IV COMPLEX INTEGRATION

Line integral - Cauchy's integral theorem – Cauchy's integral formula – Taylor's and Laurent's series – Singularities – Residues – Residue theorem – Application of residue theorem for evaluation of real integrals – Use of circular contour and semicircular contour.

PART –A

1	State Cauchy's integral theorem	BTL-1	Remembering
2	Identify the type of singularity of function $Sin\left(\frac{1}{1-z}\right)$.	BTL-1	Remembering
3	State Cauchy's residue theorem and Cauchy's integral formula	BTL-1	Remembering

4	Identify the value of $\int_C e^z dz$, where C is $\|z\| = 1$?	BTL-1	Remembering
5	Estimate the residue of the function $f(z)=\frac{4}{z^3(z-2)}$ at a simple pole	BTL-2	Understanding
6	Give the Laurent's series of $f(z)=\frac{1}{z(1-z)}$ $valid\ in\ the\ region\ \|z+1\|<1$.	BTL-2	Understanding
7	Give the Laurent's series expansion of $f(z)=\frac{e^z}{(z-1)^2}$ about $z = 1$.	BTL-2	Understanding
8	Give the Taylor's series for $f(z)=Sin\,z\ about\ z=\frac{\pi}{4}$.	BTL-2	Understanding
9	Calculate the residue at $z = 0$ of $f(z)=\frac{1-e^z}{z^3}$	BTL-3	Applying
10	Calculate the residue of the function $\frac{z-3}{(z+1)(z+2)}$ at poles.	BTL-3	Applying
11	Determine the residues at poles of the function $f(z)=\frac{z+4}{(z-1)(z-2)}$.	BTL-3	Applying
12	Expand $\frac{1}{z(z-1)}$ as Laurent's series about z = 0 in the annulus $0 < \|z\| < 1$.	BTL-4	Analyzing
13	Obtain the expansion of $\log(1+z)\ when\ \|z\|<1.$	BTL-4	Analyzing
14	Evaluate $\int_C \frac{z}{(z-2)} dz$ where C is a)$\|z\| = 1$ b) $\|z\| = 3$	BTL-4	Analyzing
15	Evaluate $\oint_C \frac{z+2}{z} dz\ \ where\ C\ is\ the\ circle\ \|z\|=2$ in the z –plane.	BTL-5	Evaluating
16	Evaluate $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3} dz\ where\ C\ is\ \|z\|=4$ using Cauchy's integral formula.	BTL-5	Evaluating
17	Integrate $\int_C \frac{dz}{z+4}$ where C is the circle $\|Z\|=2$.	BTL-6	Creating
18	Integrate $\int_C \frac{e^z}{z-1} dz\ if\ C\ is\ \|z\| = 2.$	BTL-6	Creating
19	Expand $f(z) = \frac{1}{z^2}$ as Taylor's series about the point $z = 2$	BTL-4	Analyzing
20	Find the residues of $f(z) = \frac{z+2}{(z-2)(z+1)^2}$ about each singularity.	BTL-5	Evaluating
PART -B			
1(a)	Find the Taylor's series to represent the function $\frac{z^2-1}{(z+2)(z+3)} in \|z\|<2.$	BTL -4	Analyzing
1(b)	Using contour integration estimate $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$ a> 0, b > 0.	BTL -2	Understanding
2(a)	Identify the Taylor's series to represent the function $\frac{1}{(z+2)(z+3)} in \|z\|<2.$	BTL -1	Remembering
2(b)	Using Contour Integration evaluate $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+4)(x^2+9)}$	BTL -2	Understanding
3(a)	Identify the Laurent's series expansion of $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region $\|z\| < 2$ and $2 < \|z\| < 3$	BTL -1	Remembering
3(b)	Apply the calculus of residues to prove that $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}$	BTL 3	Applying
4(a)	Identify the Laurent's series expansion for the function $f(z) = \frac{4z}{(z^2-1)(z-4)}$ in the regions $2 < \|z-1\| < 3\ and\ \|z-1\| > 4$	BTL -1	Remembering
4(b)	Apply the calculus of residues to prove that $\int_0^{\infty} \frac{dx}{(x^4+a^4)} = \frac{\pi\sqrt{2}}{4a^3}$	BTL 3	Applying

5(a)	Using Laurent's series, find $\frac{1}{z(z-1)}$ valid in (i) $\lvert z+1\rvert < 1$ (ii) $1 < \lvert z+1\rvert < 2$ (iii) $\lvert z+1\rvert > 1$	BTL -2	Understanding
5(b)	Evaluate using contour integration $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$	BTL -5	Evaluating
6(a)	Identify the Laurent's series of $f(z) = \frac{(z+3)}{(z-1)(z-4)}$, valid in $\lvert z\rvert > 2$ and $0 < \lvert z-1\rvert < 1$	BTL -1	Remembering
6(b)	Apply the calculus of residues to evaluate $\int_0^{\infty} \frac{x\sin mx}{x^2+a^2} dx, a > 0, m > 0$	BTL 3	Applying
7(a)	Identify the Laurent's series expansion for the function $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < \lvert z+1\rvert < 3$.	BTL -1	Remembering
7(b)	Apply the calculus of residues to evaluate $\int_0^{\infty} \frac{x\sin x}{(x^2+1)(x^2+4)} dx$	BTL 3	Applying
8(a)	Evaluate using contour integration $\int_0^{\infty} \frac{\cos ax}{(x^2+b^2)^2} dx, a > 0, b > 0$	BTL -5	Evaluating
8(b)	Expand as Laurent's series of the function $\frac{z}{(z^2-3z+2)}$ in the regions (i) $\lvert z\rvert < 1$ (ii) $1 < \lvert z\rvert < 2$ (iii) $\lvert z\rvert > 3$	BTL -4	Analyzing
9(a)	Evaluate using contour integration $\int_0^{\infty} \frac{\cos x}{(x^2+a^2)(x^2+b^2)} dx, a > b > 0$	BTL -5	Evaluating
9(b)	Applying Cauchy's integral formula solve $\int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$, C is the circle $\lvert z\rvert = 3$.	BTL -3	Applying
10(a)	Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b\sin\theta)} (a > 0, b > 0)$, using contour integration	BTL -5	Evaluating
10(b)	Using Cauchy's integral formula calculate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle $\lvert z+1+i\rvert = 2$	BTL -3	Applying
11(a)	Formulate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$, using the method of contour integration.	BTL -6	Creating
11(b)	Evaluate using Cauchy's integral formula $\int_C \frac{(z+1)}{(z-3)(z-1)} dz$ where C is the circle $\lvert z\rvert = 2$	BTL -5	Evaluating
12(a)	Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)} (a > 0, b > 0)$, using contour integration	BTL -5	Evaluating
12(b)	Evaluate $\int_C \frac{z^2 dz}{(z-1)^2(z+2)}$ where C is $\lvert z\rvert = 3$.	BTL -5	Evaluating
13(a)	Evaluate $\int_0^{2\pi} \frac{\cos 3\theta d\theta}{(5+4\cos\theta)}$ using contour integration	BTL -5	Evaluating
13(b)	If $f(a) = \int_C \frac{3z^2+7z+1}{(z-a)} dz$ where C is the circle $\lvert z\rvert = 2$, Identify f(3), f(1), $f'(1-i)$, $f''(1-i)$.	BTL -1	Remembering
14(a)	Evaluate $\int_0^{2\pi} \frac{d\theta}{(13+12\cos\theta)} (a > 0, b > 0)$, using contour integration	BTL -5	Evaluating
14(b)	Evaluate $\int_C \frac{z dz}{(z-1)(z-2)^2}$ where C is the circle $\lvert z-2\rvert = \frac{1}{2}$	BTL -4	Analyzing

UNIT V LAPLACE TRANSFORM

Existence conditions – Transforms of elementary functions – Transform of unit step function and unit impulse function – Basic properties – Shifting theorems -Transforms of derivatives and integrals – Initial and final value theorems – Inverse transforms – Convolution theorem – Transform of periodic functions – Application to solution of linear second order ordinary differential equations with constant coefficients.

PART-A

1	State the sufficient conditions for the existence of Laplace transform.	BTL-1	Remembering
2	State first and second shifting theorem.	BTL-1	Remembering
3	State and prove change of scale property	BTL-1	Remembering
4	State Initial value and final value theorems.	BTL-1	Remembering
5	State Convolution theorem	BTL-1	Remembering

6	Tell whether $L\left[\dfrac{\cos t}{t}\right]$ exist? Justify.	BTL-1	Remembering
7	Find the inverse Laplace transform of F(s) = $\dfrac{1}{s(s-2)}$	BTL-2	Understanding
8	Estimate L[t cost]	BTL-2	Understanding
9	Estimate $L\left[\dfrac{\sin at}{t}\right]$	BTL-2	Understanding
10	Find $L^{-1}\left[\cot^{-1} s\right]$	BTL-2	Understanding
11	Apply and verify the initial value theorem for the function f(t) = 3 e^{-2t}	BTL-3	Applying
12	Apply and verify the final value theorem of the function f(t) = t^2 e^{-3t}	BTL-3	Applying
13	Give example of two functions for which Laplace Transform do not exist?	BTL-3	Applying
14	Verify initial value theorem for the function $1+e^{-2t}$.	BTL-4	Analyzing
15	Find $L\left[\dfrac{e^{at}-e^{-bt}}{t}\right]$	BTL-4	Analyzing
16	Find $L^{-1}\left[\log\dfrac{s+1}{s-1}\right]$	BTL-4	Analyzing
17	Evaluate $L^{-1}\left[\dfrac{1}{(s+2)^4}\right]$	BTL-5	Evaluating
18	Evaluate $L\left[\dfrac{e^t}{t}\right]$	BTL-5	Evaluating
19	Formulate L[t sinh2t]	BTL- 6	Creating
20	Formulate $L^{-1}\left[\dfrac{1}{s(s-4)}\right]$	BTL- 6	Creating
	PART-B		
1(a)	Estimate L[f(t)], if f(t)= $\begin{cases} \sin\omega t, \ for\ 0<t<\dfrac{\pi}{\omega} \\ 0, \ for \dfrac{\pi}{\omega}<t<\dfrac{2\pi}{\omega} \end{cases}$ and $f\left(t+\dfrac{2\pi}{\omega}\right)=f(t)$, for all t.	BTL-2	Understanding
1(b)	Identify the Inverse Laplace transform of $\left[\tan^{-1}\left(\dfrac{2}{s}\right)+\cot^{-1}\left(\dfrac{s}{3}\right)\right]$	BTL-1	Remembering
2(a)	Give L [f(t)], if f(t)= $\begin{cases} t, \ for\ 0\le t\le c \\ 2c-t, \ for\ c<t<2c \end{cases}$ and $f(t+2c) = f(t)$), for all t.	BTL-2	Understanding
2(b)	Find the inverse Laplace Transform of $\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$	BTL-5	Evaluating
3(a)	Identify the Laplace transform of the square- wave function of period a defined as $f(t)=\begin{cases} 1, when\ 0<t<a/2 \\ -1, when\ a/2<t<a \end{cases}$	BTL-1	Remembering
3(b)	Find L[t cost sinh2t]	BTL-4	Analyzing
4(a)	Find the Laplace transform of the square- wave function of period a defined as f(t)= $\begin{cases} 1, when\ 0<t<1 \\ 0, \ when\ 1<t<2 \end{cases}$ and $f(t+2) = f(t)$), for all t.	BTL-5	Evaluating

4(b)	Find f(t), if $L(f(t)) = \frac{s}{(s+2)^2}$	BTL-5	Evaluating
5(a)	Identify the Laplace Transform of the function $[t\ Sin3t\ Cos\ 2t]$	BTL-1	Remembering
5(b)	Give L [f(t)], if f(t)= $\begin{cases} \frac{4Et}{P} - E, \ for\ 0 \le t \le \frac{P}{2} \\ 3E - \frac{4E}{P}t,\ for \frac{P}{2} < t < P \end{cases}$ and $f(t+P) = f(t)$), for all t.	BTL-5	Evaluating
6(a)	Find the Laplace transform of f(t) if $f(t) = e^t, 0 < t < 2\pi$ and $f(t+2\pi) = f(t)$	BTL-4	Analyzing
6(b)	Identify the inverse of Laplace Transform of the function $\left[\frac{1-\cos t}{t}\right]$	BTL-1	Remembering
7(a)	Apply initial and final value theorem for the verification of the function f(t) = 1+ e^{-t} (sint + cost).	BTL-3	Applying
7(b)	Using Convolution theorem, Evaluate $L^{-1}\left[\frac{1}{s(s^2+1)}\right]$	BTL-5	Evaluating
8(a)	Using convolution theorem, find $L^{-1}\left[\frac{4}{(s^2+2s+5)^2}\right]$	BTL-4	Analyzing
8(b)	Give the general solution of (D^2+ 9) y = cos2t, given that y (0) = 1, y$\left(\frac{\pi}{2}\right)$ = -1.	BTL-2	Understanding
9(a)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$	BTL- 6	Creating
9(b)	Give the general solution of (D^2 + 4D + 4) y = e^{-t}, given that y (0) =0,y' (0) =0.	BTL-2	Understanding
10(a)	Solve y''-3y'+2y= 4t + e^{3t} when y' (0) = -1and y (0) = 1 using Laplace transforms.	BTL-5	Evaluating
10(b)	Apply convolution theorem, find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$	BTL-3	Applying
11(a)	Formulate and solve using Laplace transforms, (D^2+D) y = t^2+2t, given that y = 4, y'= -2 when t = 0	BTL-5	Evaluating
11(b)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$	BTL- 6	Creating
12(a)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1}\left[\frac{1}{(s\ +1)(s^2+1\ )}\right]$	BTL- 6	Creating
12(b)	using Laplace transforms, solve y''-3y'-4y= $2e^{-t}$ when y' (0) = 1and y(0)= 1.	BTL-5	Evaluating
13(a)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1}\left[\frac{2}{(s\ +1)(s^2+4\ )}\right]$	BTL- 6	Creating
13(b)	Using Laplace transforms, solve y''-2y'+y= e^t when y' (0) = -1and y(0)=2	BTL-4	Analyzing
14(a)	Using Laplace transforms, solve y''-3y'+2y= 1 when y' (0) = 1and y(0)=0	BTL-1	Remembering
14(b)	Evaluate $L\left[\frac{\cos at - \cos bt}{t}\right]$	BTL-5	Evaluating