

VALLIAMMAI ENGINEERING COLLEGE

S.R.M. Nagar, Kattankulathur - 603203

DEPARTMENT OF MATHEMATICS

QUESTION BANK



**II SEMESTER
(COMMON TO ALL BRANCHES)**

MA 8251- MATHEMATICS –II

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SUBJECT : MA8251- ENGINEERING MATHEMATICS-II

SEM / YEAR: II SEMESTER / I YEAR (COMMON TO ALL BRANCHES)

UNIT I MATRICES			
Eigen values and Eigenvectors of a real matrix – Characteristic equation – Properties of Eigen values and Eigenvectors – Cayley-Hamilton theorem – Diagonalization of matrices – Reduction of a quadratic form to canonical form by orthogonal transformation – Nature of quadratic forms.			
Q.No.	Question	BT Level	Competence
PART-A			
1	Obtain the eigen values of A^3 where $A = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	BTL-1	Remembering
2	Find the eigen values of $2A^2$ if $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$	BTL-1	Remembering
3	Find the sum and product of the eigen values of $A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{pmatrix}$	BTL-1	Remembering
4	Find the sum and squares of the eigen values of $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$	BTL-1	Remembering
5	The product of the 2 eigen values of $A = \begin{pmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 14. Find the 3 rd eigen value.	BTL-1	Remembering
6	If the sum of 2 eigen values and the trace of a 3×3 matrix are equal, find the value of $ A $	BTL-1	Remembering
7	State Cayley-Hamilton theorem.	BTL-2	Understanding
8	Use Cayley Hamilton theorem to find A^{-1} if $A = \begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}$	BTL-2	Understanding
9	If $A = \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$ find A^3 using Cayley Hamilton theorem	BTL-2	Understanding
10	Write any 2 applications of Cayley Hamilton theorem	BTL-2	Understanding
11	Prove that the eigen values of A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \dots, \frac{1}{\lambda_n}$	BTL-3	Applying
12	Prove that sum of eigen values of a matrix is equal to its trace.	BTL-3	Applying
13	Find the sum of the eigen values of $2A$, if $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	BTL-3	Applying
14	Find the constants a and b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3, -2 be the eigen values of A	BTL-4	Analyzing
15	For what values of c, the eigen values of the matrix $\begin{pmatrix} 1 & 2 \\ c & 4 \end{pmatrix}$ are real and unequal, real and equal, complex conjugates.	BTL-4	Analyzing

5(b)	Obtain the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$	BTL-5	Evaluating
6(a)	Verify that the matrix $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ satisfies the characteristic equation and hence find A^4 .	BTL-4	Analyzing
6(b)	Diagonalise the matrix $A = \begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$ by orthogonal reduction.	BTL-1	Remembering
7(a)	Using Cayley-Hamilton theorem for $\text{adj}(A)$ for $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 2 \end{pmatrix}$	BTL-1	Remembering
7(b)	Determine a diagonal matrix orthogonally similar to the real symmetric matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	BTL-2	Understanding
8(a)	Use Cayley-Hamilton theorem to find the value of the matrix given by $A^8 - 5A^7 + 7A^6 - 3A^5 + 5A^4 - 5A^3 + 8A^2 - 2A + I$ if the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$	BTL-4	Analyzing
8(b)	Determine a diagonal matrix orthogonally similar to the real symmetric matrix $\begin{pmatrix} 6 & -2 & 3 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$	BTL-1	Remembering
9(a)	Find the Eigen values of A and hence find A^n (n is positive integer) given that $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$	BTL-5	Evaluating
9(b)	The Eigen vectors of a 3X3 real symmetric matrix A corresponding to the eigenvalues 2,3,6 are $(1, 0, -1)^T, (1, 1, 1)^T, (1, -2, 1)^T$ respectively. Find the matrix A.	BTL-1	Remembering
10	Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal reduction and find its nature.	BTL-5	Evaluating
11	Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form by an orthogonal reduction and find the rank, index, signature, and nature.	BTL-2	Understanding
12	Reduce the quadratic form $3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_3x_1$ into canonical form by an orthogonal reduction .	BTL-3	Applying
13	Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ into canonical form by an orthogonal reduction and find the rank, index, signature, and nature.	BTL-1	Remembering
14	Reduce the quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_3$ into canonical form by an orthogonal reduction .Also discuss its nature.	BTL-1	Remembering

UNIT II- VECTORCALCULUS

Gradient and directional derivative – Divergence and curl – Vector identities – Irrotational and Solenoidal vector fields – Line integral over a plane curve – Surface integral – Area of a curved surface – Volume integral – Green’s,

Gauss divergence and Stoke's theorems – Verification and application in evaluating line, surface and volume integrals.			
PART-A			
1	Find $\nabla\phi$, if $\phi = x^2 + y^2 + z^2$ at (1, -1, 1).	BTL-1	Remembering
2	Find the Directional derivative of $f = xyz$ at (1,1,1) in the direction $\vec{i} + \vec{j} + \vec{k}$.	BTL-1	Remembering
3	Find the Directional derivative of $\phi = 4xz^2 + x^2yz$ at (1,-2,1) in the direction $2\vec{i} + 3\vec{j} + 4\vec{k}$.	BTL-1	Remembering
4	State Gauss Divergence theorem	BTL-1	Remembering
5	State Stokes theorem.	BTL-1	Remembering
6	State Greens theorem	BTL-1	Remembering
7	Give the unit normal vector to the surface $xyz = 2$ at (2, 1, 1).	BTL-2	Understanding
8	Give the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$ at (1,1,1).	BTL-2	Understanding
9	If $\phi = 3xy - yz$, Give $grad \phi$ at (1,1,1).	BTL-2	Understanding
10	If \vec{r} is the position vector, Give $div \vec{r}$.	BTL-2	Understanding
11	Show that $\nabla(r^n) = nr^{n-2}\vec{r}$.	BTL-3	Applying
12	Show that the vector $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is solenoidal	BTL-3	Applying
13	Show that $curl(grad \phi) = 0$.	BTL-3	Applying
14	If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from the point (0,0) to the point (1,2).	BTL-4	Analyzing
15	If $\vec{F} = (x^2)\vec{i} + (xy^2)\vec{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ from (0,0) to (1,1) along the path $y = x$.	BTL-3	Applying
16	Using Green's theorem evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the square enclosed by the lines $x = 0, y = 0, x = 2, y = 3$.	BTL-6	Creating
17	Is the position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ irrotational? Justify.	BTL-5	Evaluating
18	Evaluate using Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$.	BTL-5	Evaluating
19	What is the value of m if the vector $\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + mz)\vec{k}$ is solenoidal	BTL-6	Creating
20	What is the value of a, b, c if the vector $\vec{F} = (x + y + az)\vec{i} + (by + 2y - z)\vec{j} + (-x + cy + 2z)\vec{k}$ may be irrotational.	BTL-6	Creating
PART-B			
1(a)	Find the Directional Derivative of $\phi = 3x^2yz + 4xz^2 + xyz$ at (1, 2, 3) in the direction of $2\vec{i} + \vec{j} - \vec{k}$.	BTL-1	Remembering
1(b)	If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20z^2\vec{k}$, evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ from (0,0,0) to (1,1,1) along the path $x = t, y = t^2, z = t^3$	BTL-2	Understanding
2(a)	Find the values of a and b so that the surfaces $ax^3 - by^2z = (a+3)x^2$ and	BTL-1	Remembering

	$4x^2y - z^3 = 11$ may cut orthogonally at $(2, -1, -3)$.		
2(b)	Evaluate $\iint_s \vec{F} \cdot d\vec{s}$, where $\vec{F} = 12x^2y \vec{i} - 3yz \vec{j} + 2z \vec{k}$ and s is the portion of the plane $x + y + z = 1$ included in the first octant	BTL-2	Understanding
3(a)	Calculate the angle between the normals to the surface $xy = z^2$ at the points $(4, 1, 2)$ and $(3, 3, -3)$.	BTL-1	Remembering
3(b)	Find the work done when a force $\vec{F} = (x^2 - y^2 + x) \vec{i} - (2xy + y) \vec{j}$ moves a particle in the XY plane from $(0, 0, 0)$ to $(1, 1, 1)$ along the parabola $x = y^2$. Is the work done different when the path is the straight line $x = y$?	BTL-3	Applying
4(a)	Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - 3 = z$ at the points $(2, -1, 2)$.	BTL-2	Understanding
4(b)	Find the work done by the force $\vec{F} = (2xy + z^3) \vec{i} + (x^2) \vec{j} + 5xz^2 \vec{k}$ when it moves a particle from $(1, -2, 1)$ to $(3, 1, 4)$ along any path?	BTL-4	Analyzing
5(a)	Find the values of a and b so that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ may cut orthogonally at $(1, -1, 2)$.	BTL-3	Applying
5(b)	Verify Green's theorem in the plane for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by $x = 0, y = 0, x + y = 1$.	BTL-5	Evaluating
6(a)	Show that $\text{div}(\vec{u} \times \vec{v}) = \vec{v} \cdot \text{curl} \vec{u} - \vec{u} \cdot \text{curl} \vec{v}$	BTL-1	Remembering
6(b)	Verify Green's theorem in the plane for $\int_C [(xy + y^2)dx + (x^2)dy]$ where c is a closed of the region bounded by $x = y$ and $y = x^2$.	BTL-3	Applying
7(a)	Prove that $\text{div}(\phi F) = \phi \text{div} f + \nabla \phi \cdot f$ where ϕ is a scalar point function and f is a vector point function.	BTL-1	Remembering
7(b)	Prove that the area bounded by a simple closed curve c is given by $\frac{1}{2} \int_C [(xy + y^2)dx + (x^2)dy]$ Hence find the area of ellipse.	BTL-6	Creating
8(a)	Find the value of n such that $r^n r$ is both solenoidal and irrotational	BTL-3	Applying
8(b)	Apply Green's theorem to evaluate $\int_C [(y - \sin x)dx + (\cos x)dy]$ where c is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}, y = \frac{\pi}{2}x$	BTL-1	Remembering
9(a)	Find the value of a, b, c so that the vector $\vec{F} = (x + y + az) \vec{i} + (bx - 2y - z) \vec{j} + (-x + cy + 2z) \vec{k}$ may be irrotational. Also find its scalar potential	BTL-3	Applying
9(b)	Verify Stokes theorem for $\vec{F} = (x^2 - y^2) \vec{i} + 2xy \vec{j}$ in the rectangular region of $z = 0$ plane bounded by the lines $x = 0, y = 0, x = a$ and $y = b$.	BTL-5	Evaluating
10(a)	Find its scalar potential, if the vector field $\vec{F} = (x^2 + xy^2) \vec{i} + (y^2 + x^2y) \vec{j}$ is irrotational.	BTL-3	Applying
10(b)	Verify Stoke's theorem for $\vec{F} = (xy + y^2) \vec{i} + x^2 \vec{j}$ in the xoy-plane bounded $x = y$ and $y = x^2$.	BTL-6	Creating

11	Show that Stokes theorem is verified for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ where S is the surface of the cube formed by $x = 0, x = 2, y = 0, y = 2, \text{ and } z = 2$ above the xy -plane.	BTL-3	Applying
12	Verify stokes theorem for $\vec{F} = (2x - y)\vec{i} - (yz^2)\vec{j} - y^2\vec{k}$ where S is the upper hemisphere $x^2 + y^2 + z^2 = a^2$ and c is the boundary	BTL-1	Remembering
13	Verify Gauss divergence therm for $\vec{F} = (x^2)\vec{i} + (y^2)\vec{j} + z^2\vec{k}$ where s is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$.	BTL-3	Applying
14	Verify Gauss divergence theorem for $\vec{F} = (x^3 - yz)\vec{i} - (2x^2y)\vec{j} + 2\vec{k}$ where s is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$.	BTL-3	Applying

UNIT III ANALYTIC FUNCTIONS

Analytic functions – Necessary and sufficient conditions for analyticity in Cartesian and polar coordinates - Properties – Harmonic conjugates – Construction of analytic function - Conformal mapping – Mapping by functions $w = z + C, Cz, 1/z, z^2$ - Bilinear transformation.

PART-A

1	Examine if $f(z) = z^3$ analytic ?	BTL-1	Remembering
2	Identify the constants a, b, c if $f(z) = x + ay + i(bx + cy)$ is analytic.	BTL-1	Remembering
3	Define conformal mapping.	BTL-1	Remembering
4	Can $u = 3x^2y - y^3$ be the real part of an analytic function? Justify your answer	BTL-1	Remembering
5	State necessary and sufficient condition for $f(z)$ to be analytic.	BTL-1	Remembering
6	Identify the invariant point of the bilinear transformation $w = \frac{2z+6}{z+7}$	BTL-1	Remembering
7	Estimate the invariant points of the transformation $w = \frac{z-1}{z+1}$	BTL-2	Understanding
8	Estimate the invariant point of the bilinear transformation $w = \frac{z-1-i}{z+2}$	BTL-2	Understanding
9	Give the image of the circle $ z = 3$ under the transformation $w = 5z$.	BTL-2	Understanding
10	Under the transformation $w = \frac{1}{z}$ give the image of the circle $ z - 1 = 1$ in the complex plane.	BTL-2	Understanding
11	Show that $ z ^2$ is not analytic at any point.	BTL-3	Applying
12	Show that an analytic function in a region R with constant imaginary part is constant.	BTL-3	Applying
13	Show that $u = 2x - x^3 + 3xy^2$ is harmonic.	BTL-3	Applying
14	If $f(z)$ is an analytic function whose real part is constant, Point out $f(z)$ is a constant function.	BTL-4	Analyzing
15	Explain that a bilinear transformation has at most 2 fixed points.	BTL-4	Analyzing

16	Examine whether the function xy^2 can be real part of analytic function.	BTL-4	Analyzing
17	Test the analyticity of the function $f(z) = e^{-z}$	BTL-5	Evaluating
18	Evaluate the image of hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$	BTL-5	Evaluating
19	Formulate the critical points of the transformation $w = z + \frac{1}{z}$	BTL-6	Creating
20	Formulate the bilinear transformation which maps $z = 0, -i, -1$ into $w = i, 1, 0$ respectively.	BTL-6	Creating
PART-B			
1(a)	Given that $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$, Estimate the analytic function.	BTL-2	Understanding
1(b)	Find the image of $ z = 2$ under the transformation (i) $w = z + 3 + 2i$ (ii) $w = 3z$	BTL-4	Analysing
2(a)	Estimate the analytic function $w = u + iv$ if $u = e^x(x \cos 2y - y \sin 2y)$.	BTL-2	Understanding
2(b)	Formulate the image of $ z + 1 = 1$ under the map $w = 1/z$.	BTL-6	Creating
3(a)	Estimate the analytic function $f(z) = u + iv$ given the imaginary part is $v = x^2 - y^2 + \frac{x}{x^2 + y^2}$.	BTL-2	Understanding
3(b)	Prove that an analytic function with constant modulus is constant.	BTL-1	Remembering
4(a)	Show that the function $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. Find also the conjugate harmonic function v .	BTL-3	Applying
4(b)	Show that the transformation $w = \frac{1}{z}$ maps, in general, circles and straight lines into circles and straight lines. Point out the circles and straight lines are transformed into straight lines and circles respectively.	BTL-1	Remembering
5(a)	Estimate the analytic function $w = u + iv$ if $u - v = e^x(\cos y - \sin y)$	BTL-2	Understanding
5(b)	Under the transformation $w = \frac{1}{z}$, find the image of a region (i) $x > c$ where $c > 0$, (ii) $y > c$, where $c < 0$	BTL-4	Analysing
6(a)	Solve the bilinear transformation that maps the point $z_1 = i, z_2 = -1, z_3 = 1$ into the points $w_1 = 0, w_2 = 1, w_3 = \infty$ respectively	BTL-5	Evaluating
6(b)	Identify the image of the infinite strip (i) $0 \leq y \leq \frac{1}{2}$ (ii) $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = 1/z$.	BTL-2	Understanding
7(a)	Determine the analytic function $w = u + iv$ given that $3u + 2v = y^2 - x^2 + 16xy$.	BTL-4	Analysing
7(b)	Formulate the image of $ z - 2 = 2$ under the map $w = z^2$	BTL-6	Creating

8(a)	Point out the bilinear transformation that maps the point $z_1 = 1, z_2 = i, z_3 = -1$ into the points $w_1 = i, w_2 = 0, w_3 = -i$ respectively.	BTL-3	Applying
8(b)	Show that the image of $ z - 1 = 1$ under the transformation $w = z^2$ is the cardioid $R = 2(1 + \cos\phi)$	BTL-1	Remembering
9(a)	Give the bilinear transformation which maps $z = 1, 0, -1$ into $w = 0, -1, \infty$ respectively. What are the invariant points of the transformation?	BTL-2	Understanding
9(b)	If $u = x^2 - y^2, v = -\frac{y}{x^2 + y^2}$, prove that u and v are harmonic functions but $u + iv$ is not an analytic function.	BTL-5	Evaluating
10(a)	Identify the bilinear transformation that maps $1 + i, -i, 2 - i$ at the z -plane into the points $0, 1, i$ of the w -plane.	BTL-1	Remembering
10(b)	Determine the analytic function $f(z) = u + iv$ such that $2u + v = e^x(\cos y - \sin y)$		
11(a)	Identify the bilinear mapping which maps $-1, 0, 1$ of the z -plane onto $-1, i, 1$ of the w -plane. Show that under this mapping the upper half of z -plane maps onto the interior of unit circle $ w = 1$.	BTL-1	Remembering
11(b)	If $w = f(z)$ is analytic then, show that $\left[\left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)\right] \log f(z) = 0$	BTL-6	Creating
12(a)	Identify the bilinear mapping which maps $1, i, -1$ of the z -plane onto $0, 1, \infty$ of the w -plane. Show that the transformation maps the interior of the unit circle of the z -plane onto the upper half of the w -plane.	BTL-1	Remembering
12(b)	If $f(z)$ is a regular function of z , Show that $\nabla^2 f(z) ^2 = 4 f'(z) ^2$.	BTL-3	Applying
13(a)	If $w = u(x, y) + iv(x, y)$ is an analytic function, show that the curves of the family $u(x, y) = a$ and the curves of the family $v(x, y) = b$, cut orthogonally where a and b are varying constants.	BTL-2	Understanding
13(b)	Formulate the bilinear transformation that maps the point $z_1 = 0, z_2 = 1, z_3 = \infty$ into the points $w_1 = i, w_2 = 1, w_3 = -i$ respectively	BTL-5	Analyzing
14(a)	If $f(z) = u + iv$ is an analytic function of z , then formulate that $\nabla^2[\log f'(z)] = 0$.	BTL-6	Creating
14(b)	Derive the bilinear transformation that maps the point $z_1 = 0, z_2 = 1, z_3 = \infty$ into the points $w_1 = -5, w_2 = -1, w_3 = 3$ respectively. What are invariant points of transformation?	BTL-5	Analyzing

UNIT IV COMPLEX INTEGRATION

Line integral - Cauchy's integral theorem – Cauchy's integral formula – Taylor's and Laurent's series – Singularities – Residues – Residue theorem – Application of residue theorem for evaluation of real integrals – Use of circular contour and semicircular contour.

PART -A

1	State Cauchy's integral theorem	BTL-1	Remembering
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2	Identify the type of singularity of function $\sin\left(\frac{1}{1-z}\right)$.	BTL-1	Remembering
3	State Cauchy's residue theorem and Cauchy's integral formula	BTL-1	Remembering
4	Identify the value of $\int_C e^z dz$, where C is $ z = 1$?	BTL-1	Remembering
5	Estimate the residue of the function $f(z) = \frac{4}{z^3(z-2)}$ at a simple pole	BTL-2	Understanding
6	Give the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the region $ z+1 < 1$.	BTL-2	Understanding
7	Give the Laurent's series expansion of $f(z) = \frac{e^z}{(z-1)^2}$ about $z = 1$.	BTL-2	Understanding
8	Give the Taylor's series for $f(z) = \sin z$ about $z = \frac{\pi}{4}$.	BTL-2	Understanding
9	Calculate the residue at $z = 0$ of $f(z) = \frac{1-e^z}{z^3}$	BTL-3	Applying
10	Calculate the residue of the function $\frac{z-3}{(z+1)(z+2)}$ at poles.	BTL-3	Applying
11	Determine the residues at poles of the function $f(z) = \frac{z+4}{(z-1)(z-2)}$.	BTL-3	Applying
12	Expand $\frac{1}{z(z-1)}$ as Laurent's series about $z = 0$ in the annulus $0 < z < 1$.	BTL-4	Analyzing
13	Obtain the expansion of $\log(1+z)$ when $ z < 1$.	BTL-4	Analyzing
14	Evaluate $\int_C \frac{z}{(z-2)} dz$ where C is a) $ z = 1$ b) $ z = 3$	BTL-4	Analyzing
15	Evaluate $\oint_C \frac{z+2}{z} dz$ where C is the circle $ z = 2$ in the z -plane.	BTL-5	Evaluating
16	Evaluate $\frac{1}{2\pi i} \int_C \frac{z^2+5}{z-3} dz$ where C is $ z = 4$ using Cauchy's integral formula.	BTL-5	Evaluating
17	Integrate $\int_C \frac{dz}{z+4}$ where C is the circle $ z = 2$.	BTL-6	Creating
18	Integrate $\int_C \frac{e^z}{z-1} dz$ if C is $ z = 2$.	BTL-6	Creating
19	Expand $f(z) = \frac{1}{z^2}$ as Taylor's series about the point $z = 2$	BTL-4	Analyzing
20	Find the residues of $f(z) = \frac{z+2}{(z-2)(z+1)^2}$ about each singularity.	BTL-5	Evaluating
PART - B			
1(a)	Find the Taylor's series to represent the function $\frac{z^2-1}{(z+2)(z+3)}$ in $ z < 2$.	BTL-4	Analyzing
1(b)	Using contour integration estimate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ $a > 0, b > 0$.	BTL-2	Understanding

2(a)	Identify the Taylor's series to represent the function $\frac{1}{(z+2)(z+3)}$ in $ z < 2$.	BTL-1	Remembering
2(b)	Using Contour Integration evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+4)(x^2+9)}$	BTL-2	Understanding
3(a)	Identify the Laurent's series expansion of $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region $ z < 2$ and $2 < z < 3$	BTL-1	Remembering
3(b)	Apply the calculus of residues to prove that $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}$	BTL 3	Applying
4(a)	Identify the Laurent's series expansion for the function $f(z) = \frac{4z}{(z^2-1)(z-4)}$ in the regions $2 < z-1 < 3$ and $ z-1 > 4$	BTL-1	Remembering
4(b)	Apply the calculus of residues to prove that $\int_0^{\infty} \frac{dx}{(x^4+a^4)} = \frac{\pi\sqrt{2}}{4a^3}$	BTL 3	Applying
5(a)	Using Laurent's series, find $\frac{1}{z(z-1)}$ valid in (i) $ z+1 < 1$ (ii) $1 < z+1 < 2$ (iii) $ z+1 > 1$	BTL-2	Understanding
5(b)	Evaluate using contour integration $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$	BTL-5	Evaluating
6(a)	Identify the Laurent's series of $f(z) = \frac{(z+3)}{(z-1)(z-4)}$, valid in $ z > 2$ and $0 < z-1 < 1$	BTL-1	Remembering
6(b)	Apply the calculus of residues to evaluate $\int_0^{\infty} \frac{x \sin mx}{x^2+a^2} dx, a > 0, m > 0$	BTL 3	Applying
7(a)	Identify the Laurent's series expansion for the function $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$.	BTL-1	Remembering
7(b)	Apply the calculus of residues to evaluate $\int_0^{\infty} \frac{x \sin x}{(x^2+1)(x^2+4)} dx$	BTL 3	Applying
8(a)	Evaluate using contour integration $\int_0^{\infty} \frac{\cos ax}{(x^2+b^2)^2} dx, a > 0, b > 0$	BTL-5	Evaluating
8(b)	Expand as Laurent's series of the function $\frac{z}{(z^2-3z+2)}$ in the regions (i) $ z < 1$ (ii) $1 < z < 2$ (iii) $ z > 3$	BTL-4	Analyzing
9(a)	Evaluate using contour integration $\int_0^{\infty} \frac{\cos x}{(x^2+a^2)(x^2+b^2)} dx, a > b > 0$	BTL-5	Evaluating
9(b)	Applying Cauchy's integral formula solve $\int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$, C is the circle $ z = 3$.	BTL-3	Applying
10(a)	Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b \sin \theta)}$ ($a > 0, b > 0$), using contour integration	BTL-5	Evaluating
10(b)	Using Cauchy's integral formula calculate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle $ z+1+i = 2$	BTL-3	Applying
11(a)	Formulate $\int_0^{2\pi} \frac{d\theta}{13+5 \sin \theta}$, using the method of contour integration.	BTL-6	Creating
11(b)	Evaluate using Cauchy's integral formula $\int_C \frac{(z+1)}{(z-3)(z-1)} dz$ where C	BTL-5	Evaluating

	is the circle $ z = 2$		
12(a)	Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)}$ ($a > 0, b > 0$), using contour integration	BTL-5	Evaluating
12(b)	Evaluate $\int_C \frac{z^2 dz}{(z-1)^2(z+2)}$ where C is $ z = 3$.	BTL-5	Evaluating
13(a)	Evaluate $\int_0^{2\pi} \frac{\cos 3\theta d\theta}{(5+4\cos\theta)}$ using contour integration	BTL-5	Evaluating
13(b)	If $f(z) = \int_C \frac{3z^2 + 7z + 1}{(z-a)} dz$ where C is the circle $ z = 2$, Identify $f(3)$, $f(1)$, $f'(1-i)$, $f''(1-i)$.	BTL-1	Remembering
14(a)	Evaluate $\int_0^{2\pi} \frac{d\theta}{(13+12\cos\theta)}$ ($a > 0, b > 0$), using contour integration	BTL-5	Evaluating
14(b)	Evaluate $\int_C \frac{z dz}{(z-1)(z-2)^2}$ where C is the circle $ z-2 = \frac{1}{2}$	BTL-4	Analyzing

UNIT V LAPLACE TRANSFORM

Existence conditions – Transforms of elementary functions – Transform of unit step function and unit impulse function – Basic properties – Shifting theorems -Transforms of derivatives and integrals – Initial and final value theorems – Inverse transforms – Convolution theorem – Transform of periodic functions – Application to solution of linear second order ordinary differential equations with constant coefficients.

PART-A

1	State the sufficient conditions for the existence of Laplace transform.	BTL-1	Remembering
2	State first and second shifting theorem.	BTL-1	Remembering
3	State and prove change of scale property	BTL-1	Remembering
4	State Initial value and final value theorems.	BTL-1	Remembering
5	State Convolution theorem	BTL-1	Remembering
6	Tell whether $L\left[\frac{\cos t}{t}\right]$ exist? Justify.	BTL-1	Remembering
7	Find the inverse Laplace transform of $F(s) = \frac{1}{s(s-2)}$	BTL-2	Understanding
8	Estimate $L[t \cos t]$	BTL-2	Understanding
9	Estimate $L\left[\frac{\sin at}{t}\right]$	BTL-2	Understanding
10	Find $L^{-1}[\cot^{-1} s]$	BTL-2	Understanding
11	Apply and verify the initial value theorem for the function $f(t) = 3e^{-2t}$	BTL-3	Applying
12	Apply and verify the final value theorem of the function $f(t) = t^2 e^{-3t}$	BTL-3	Applying
13	Give example of two functions for which Laplace Transform do not exist?	BTL-3	Applying
14	Verify initial value theorem for the function $1+e^{-2t}$.	BTL-4	Analyzing

15	Find $L^{-1} \left[\frac{e^{at} - e^{-bt}}{t} \right]$	BTL-4	Analyzing
16	Find $L^{-1} \left[\log \frac{s+1}{s-1} \right]$	BTL-4	Analyzing
17	Evaluate $L^{-1} \left[\frac{1}{(s+2)^4} \right]$	BTL-5	Evaluating
18	Evaluate $L^{-1} \left[\frac{3s}{2s+9} \right]$	BTL-5	Evaluating
19	Formulate $L^{-1} \left[\frac{3s+2}{s^2-4} \right]$	BTL-6	Creating
20	Formulate $L^{-1} \left[\frac{1}{s(s-4)} \right]$	BTL-6	Creating

PART-B

1(a)	Estimate $L[f(t)]$, if $f(t) = \begin{cases} \sin \omega t, & \text{for } 0 < t < \frac{\pi}{\omega} \\ 0, & \text{for } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ and $f\left(t + \frac{2\pi}{\omega}\right) = f(t)$, for all t .	BTL-2	Understanding
1(b)	Identify the Inverse Laplace transform of $\left[\tan^{-1} \left(\frac{2}{s} \right) + \cot^{-1} \left(\frac{s}{3} \right) \right]$	BTL-1	Remembering
2(a)	Give $L[f(t)]$, if $f(t) = \begin{cases} t, & \text{for } 0 \leq t \leq c \\ 2c - t, & \text{for } c < t < 2c \end{cases}$ and $f(t+2c) = f(t)$, for all t .	BTL-2	Understanding
2(b)	Find the inverse Laplace Transform of $\log \left(\frac{s^2+a^2}{s^2+b^2} \right)$	BTL-5	Evaluating
3(a)	Identify the Laplace transform of the square-wave function of period a defined as $f(t) = \begin{cases} 1, & \text{when } 0 < t < a/2 \\ -1, & \text{when } a/2 < t < a \end{cases}$	BTL-1	Remembering
3(b)	Find $L^{-1} \left[\frac{5s+3}{(s^2+2s+5)(s-1)} \right]$	BTL-4	Analyzing
4(a)	Find the Laplace transform of the square-wave function of period a defined as $f(t) = \begin{cases} K, & \text{when } 0 < t < a \\ -K, & \text{when } a < t < 2a \end{cases}$ and $f(t+2a) = f(t)$, for all t .	BTL-5	Evaluating
4(b)	Find $f(t)$, if $L(f(t)) = \frac{s}{(s+2)^2}$	BTL-5	Evaluating
5(a)	Identify the Laplace Transform of the function $[t \sin 3t \cos 2t]$	BTL-1	Remembering

5(b)	Give $L[f(t)]$, if $f(t) = \begin{cases} t, & \text{for } 0 \leq t \leq 1 \\ 2-t, & \text{for } 1 < t < 2 \end{cases}$ and $f(t+2) = f(t)$, for all t .	BTL-5	Evaluating
6(a)	Find the Laplace transform of $f(t)$ if $f(t) = e^t, 0 < t < 2\pi$ and $f(t+2\pi) = f(t)$	BTL-4	Analyzing
6(b)	Identify the Laplace Transform of the function $\left[\frac{1 - \cos t}{t} \right]$	BTL-1	Remembering
7(a)	Apply initial and final value theorem for the verification of the function $f(t) = 1 + e^{-t} (\sin t + \cos t)$.	BTL-3	Applying
7(b)	Using Convolution theorem, Evaluate $L^{-1} \left[\frac{1}{s(s^2 + 1)} \right]$	BTL-5	Evaluating
8(a)	Using convolution theorem, find $L^{-1} \left[\frac{4}{(s^2 + 2s + 5)^2} \right]$	BTL-4	Analyzing
8(b)	Give the general solution of $(D^2 + 9)y = \cos 2t$, given that $y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$.	BTL-2	Understanding
9(a)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1} \left[\frac{s^2}{(s^2 + 1)(s^2 + 4)} \right]$	BTL-6	Creating
9(b)	Give the general solution of $(D^2 + 4D + 4)y = e^{-t}$, given that $y(0) = 0, y'(0) = 0$.	BTL-2	Understanding
10(a)	Solve $y'' - 3y' + 2y = 4t + e^{3t}$ when $y'(0) = -1$ and $y(0) = 1$ using Laplace transforms.	BTL-5	Evaluating
10(b)	Apply convolution theorem, find $L^{-1} \left[\frac{s}{(s^2 + a^2)^2} \right]$	BTL-3	Applying
11(a)	Formulate and solve using Laplace transforms, $(D^2 + D)y = t^2 + 2t$, given that $y = 4, y' = -2$ when $t = 0$	BTL-5	Evaluating
11(b)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1} \left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right]$	BTL-6	Creating
12(a)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1} \left[\frac{1}{(s + 1)(s^2 + 1)} \right]$	BTL-6	Creating
12(b)	Using Laplace transforms, solve $y'' - 3y' - 4y = 2e^{-t}$ when $y'(0) = 1$ and $y(0) = 1$.	BTL-5	Evaluating
13(a)	Using Convolution theorem calculate the inverse Laplace	BTL-6	Creating

	transform of $L^{-1}\left[\frac{2}{(s+1)(s^2+4)}\right]$		
13(b)	Using Laplace transforms, solve $y''-2y'+y = e^t$ when $y'(0) = -1$ and $y(0) = 2$	BTL-4	Analyzing
14(a)	Identify the Inverse Laplace transform of $\left[\frac{3s^2 + 16s + 26}{s(s^2 + 4s + 13)}\right]$	BTL-1	Remembering
14(b)	Evaluate $L\left[\frac{\cos at - \cos bt}{t}\right]$	BTL-5	Evaluating

