

SRM VALLIAMMAI ENGINEERING COLLEGE
(An Autonomous Institution)

S.R.M. Nagar, Kattankulathur - 603203

DEPARTMENT OF MATHEMATICS

QUESTION BANK



I SEMESTER
(Common to all Branches)

MA3122 - MATRICES AND CALCULUS
Regulation – 2023

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DEPARTMENT OF MATHEMATICS

SUBJECT: MA3122 - MATRICES AND CALCULUS

SEM / YEAR: I / I year B.E./ B.Tech.(Common to all Branches)

UNIT I - MATRICES

Eigen values and Eigen vectors of a real matrix - Characteristic equation - Properties of Eigen values and Eigen vectors - Statement and Applications of Cayley-Hamilton Theorem - Reduction of a quadratic form into canonical form by orthogonal transformation.

Q.No.	Question	BT Level	Competence	Course Outcome
PART – A				
1.	Find the sum of the eigen values of $2A$, if $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	BTL-1	Remembering	CO 1
2.	Find the sum and product of the eigen values of $A = \begin{bmatrix} 2 & -2 & 2 \\ -2 & -1 & -1 \\ 2 & -1 & -1 \end{bmatrix}$	BTL-1	Remembering	CO 1
3.	Find the constants a and b such that the matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3, -2 be the eigen values of A	BTL-1	Remembering	CO 1
4.	Find the eigen values of A^{-1} if $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$	BTL-4	Analyzing	CO 1
5.	Prove that the eigen values of A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \dots, \frac{1}{\lambda_n}$	BTL-2	Understanding	CO 1
6.	If the sum of two eigen values and the trace of a 3×3 matrix are equal, find the value of $ A $	BTL-2	Understanding	CO 1
7.	If the eigen values of the matrix A of order 3×3 are 2, 3 and 1, then find the determinant of A	BTL-1	Remembering	CO 1
8.	Find the characteristic equation of $A = \begin{bmatrix} 1 & -2 \\ 5 & 4 \end{bmatrix}$	BTL-1	Remembering	CO 1
9.	Find the characteristic equation of $A = \begin{bmatrix} 1 & -2 \\ 5 & 4 \end{bmatrix}$	BTL-1	Remembering	CO 1
10.	For the given matrix A of order 3, $ A =32$, and two eigen values are 8 & 2. Find the sum of the Eigenvalues.	BTL-2	Understanding	CO 1
11.	Two eigen values of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to unity each.	BTL-1	Remembering	CO 1

12.	If the characteristic equation of a matrix is $\lambda^2 - 3\lambda - 10 = 0$, then find the the eigen values of the matrix $10A^{-1} - 2I$.	BTL-4	Analyzing	CO 1
13.	Find the sum and product of the eigen values of $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$	BTL-1	Remembering	CO 1
14.	The product of the 2 eigen values of $A = \begin{bmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 14. Find the 3 rd eigen value.	BTL-2	Understanding	CO 1
15.	Write any 2 applications of Cayley Hamilton theorem	BTL-2	Understanding	CO 1
16.	If $A = \begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}$ find A^3 using Cayley Hamilton theorem	BTL-3	Applying	CO 1
17.	State Cayley-Hamilton theorem.	BTL-1	Remembering	CO 1
18.	Use Cayley Hamilton theorem to find A^{-1} if $A = \begin{bmatrix} 2 & 1 \\ 1 & -5 \end{bmatrix}$	BTL-1	Remembering	CO 1
19.	Find the quadratic form corresponding to the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	BTL-3	Applying	CO 1
20.	Define Index, Signature and Rank.	BTL-1	Remembering	CO 1
21.	Find the matrix corresponding to the quadratic form $x^2 + y^2 + z^2$	BTL-1	Remembering	CO 1
22.	Find the Rank, Index, Signature and Nature of given matrix $A = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 5 & 2 \\ 0 & 0 & -1 \end{bmatrix}$	BTL-1	Remembering	CO 1
23.	Determine the nature of the following quadratic form $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - 9x_3^2$	BTL-2	Understanding	CO 1
24.	Find the matrix corresponding to the quadratic form $2xy - 2yz + 2xz$.	BTL-4	Analyzing	CO 1
25.	Find the matrix associated with the quadratic form $x_1^2 + 4x_2^2 + 2x_1x_2$.	BTL-4	Analyzing	CO1
PART – B				
1.(a)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$	BTL-2	Understanding	CO 1
1.(b)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$	BTL-2	Understanding	CO 1
2.(a)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$	BTL-2	Understanding	CO 1

2.(b)	Obtain the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$	BTL-4	Analyzing	CO 1
3.(a)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$	BTL-2	Understanding	CO 1
3.(b)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$	BTL-2	Understanding	CO 1
4.(a)	Find the Characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^4 .	BTL-4	Analyzing	CO 1
4.(b)	Using Cayley-Hamilton theorem evaluate the matrix $A^4 + A^3 - 18A^2 - 39A + 2I$ given the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$	BTL -3	Applying	CO 1
5.	Use Cayley-Hamilton theorem to find the value of the matrix given by $A^6 - 5A^5 + 8A^4 - 2A^3 - 9A^2 - 35A + 6I$ if the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$	BTL-4	Analyzing	CO 1
6.(a)	Using Cayley-Hamilton theorem evaluate the matrix $A^4 - 4A^3 - 5A^2 + A + 2I$ given the matrix $A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$	BTL-3	Applying	CO 1
6.(b)	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -2 \\ -1 & 1 & 2 \end{bmatrix}$	BTL -4	Analyzing	CO 1
7.	Verify Cayley-Hamilton theorem and hence find A^{-1} of $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$	BTL-1	Remembering	CO 1
8.	Use Cayley-Hamilton theorem to find the value of the matrix given by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ if the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$	BTL -3	Applying	CO 1
9.	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and also find A^{-1} .	BTL-4	Analyzing	CO 1
10.	Determine the nature of the quadratic form $2xy + 2yz - 2xz$ by reducing it into canonical form by orthogonal transformation	BTL-4	Analyzing	CO 1

11.	Diagonalize the matrix $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ using an orthogonal transformation.	BTL-4	Analyzing	CO 1
12.	Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal reduction.	BTL-3	Applying	CO 1
13.	Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form by an orthogonal reduction.	BTL-2	Understanding	CO 1
14.	Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}$ using an orthogonal transformation.	BTL -4	Analyzing	CO 1
15.	Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ into canonical form by an orthogonal reduction.	BTL-1	Remembering	CO 1
16.	Reduce the quadratic form $2x_1^2 + 5x_2^2 + 3x_3^2 + 4x_1x_2$ into canonical form by an orthogonal reduction.	BTL-1	Remembering	CO 1
17.	Determine the nature of the quadratic form $2xy - 2yz + 2xz$ by reducing it into canonical form by orthogonal transformation	BTL -4	Analyzing	CO 1
18.	Diagonalize the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ using an orthogonal transformation.	BTL -3	Applying	CO 1

UNIT-II DIFFERENTIAL CALCULUS OF ONE VARIABLE

Limit of a function - Continuity - Derivatives - Differentiation rules (sum, product, quotient, chain rules) - Implicit differentiation - Rolle's Theorem and Mean Value theorem -Taylor's series- Maxima and Minima of functions of one variable.

Q.No.	Question	BT Level	Competence	Course Outcome
PART – A				
1.	Evaluate $\lim_{t \rightarrow 1} \frac{t^4 - 1}{t^3 - 1}$	BTL -1	Remembering	CO 2
2.	Check whether $\lim_{x \rightarrow 3} \frac{3x+9}{ x+3 }$ exist	BTL -2	Understanding	CO 2
3.	Find the domain of a function $y = \sqrt{x+4}$	BTL -1	Remembering	CO 2
4.	Find $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$	BTL -1	Remembering	CO 2
5.	If the function $f(x) = \begin{cases} \frac{x^2-16}{x-4} & \text{if } x \neq 4 \\ c & \text{if } x = 4 \end{cases}$ is continuous, what is the value of c?	BTL -2	Understanding	CO 2
6.	If $f(x) = xe^x$ find $f'(x)$	BTL -2	Understanding	CO 2
7.	Show that $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ using Squeeze theorem.	BTL -3	Applying	CO 2

8.	Predict the values of a and b so that the function f given by $f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax + b & \text{if } 3 < x < 5 \\ 7 & \text{if } x \geq 5 \end{cases}$ continuous at x=3 and x=5.	BTL -3	Applying	CO 2
9.	Compute $\frac{d}{dx}((x)^{\sin x})$	BTL -4	Analyzing	CO 2
10.	Find the equation of the tangent line to the curve $y = x^4 + 2x^2 - x$ at the point (1,2)	BTL -3	Applying	CO 2
11.	Point out $\frac{dy}{dx}$, if $y = \ln x$	BTL -3	Applying	CO 2
12.	Calculate $\frac{d}{dx}((x)^{\sqrt{x}})$	BTL -2	Understanding	CO 2
13.	Estimate $\frac{d}{dx}(\cosh^{-1}x)$	BTL -3	Applying	CO 2
14.	Estimate y' if $x^3 + y^3 = 6xy$	BTL -2	Understanding	CO 2
15.	Estimate $\frac{d}{dx}((\sin x)^{\cos x})$	BTL -4	Analyzing	CO 2
16.	Evaluate $\frac{d}{dx}(\sinh x)$	BTL -4	Analyzing	CO 2
17.	Using Rolle's theorem find the value of c for the function $f(x) = (x-a)(b-x), a \leq x \leq b, a \neq b$	BTL -4	Analyzing	CO 2
18.	Verify Lagrange's mean value theorem for the function $f(x) = x^3, [-2, 2]$	BTL -2	Understanding	CO 2
19.	Using Rolle's theorem find the value of c for the function $f(x) = \sqrt{1-x^2}, 1 \leq x \leq 1$	BTL -4	Analyzing	CO 2
20.	Verify Lagrange's mean value theorem for the function $f(x) = \frac{1}{x}, [1, 2]$	BTL -1	Remembering	CO 2
21.	Find the critical numbers of the function $f(x) = 2x^3 - 3x^2 - 36x$	BTL -3	Applying	CO 2
22.	Find the critical points of $y = 5x^2 - 6x$	BTL -3	Applying	CO 2
23.	Find the Taylor's series expansion of the function $f(x) = \sin x$ about the point $x = \frac{\pi}{2}$	BTL -3	Applying	CO 2
24.	Find the critical points of $y = 5x^3 - 6x$	BTL -2	Understanding	CO 2
25.	Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so where?	BTL -4	Analyzing	CO 2
PART – B				
1.(a)	For what value of the constant "c" in the function "f" continuous on $(-\infty, \infty)$, $f(x) = \begin{cases} cx^2 + 2x & ; x < 2 \\ x^3 - cx & ; x \geq 2 \end{cases}$	BTL -4	Analyzing	CO 2
1.(b)	Obtain y'' if $x^4 + y^4 = 16$	BTL -4	Analyzing	CO 2
2.(a)	Point out the domain where the function f is continuous Also find the number at which the function f is discontinuous	BTL -4	Analyzing	CO 2

	when $f(x) = \begin{cases} 1 + x^2 & \text{if } x \leq 0 \\ 2 - x & \text{if } 0 < x \leq 2 \\ (x - 2)^2 & \text{if } x > 2 \end{cases}$			
2.(b)	Find $\frac{dy}{dx}$ if $y = x^2 e^{2x}$.	BTL -3	Applying	CO 2
3.(a)	Find the values of a and b that make f continuous on $(-\infty, \infty)$ $f(x) = \begin{cases} \frac{x^3-8}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$	BTL -3	Applying	CO 2
3.(b)	Find y' if $x = a(\cos\theta + \log \tan \frac{\theta}{2})$, $y = a \sin\theta$.	BTL -3	Applying	CO 2
4.(a)	Find y' for $\cos(xy) = 1 + \sin y$.	BTL -3	Applying	CO 2
4.(b)	Find the derivative of $f(x) = \cos^{-1}\left(\frac{b+acosx}{a+bcosx}\right)$	BTL -3	Applying	CO 2
5.(a)	Find $\frac{dy}{dx}$ for the following functions $e^x + e^y = e^{x+y}$.	BTL -3	Applying	CO 2
5.(b)	Verify Rolle's theorem for the following $f(x) = 2x^3 - 5x^2 - 4x + 3, x \in [\frac{1}{2}, 3]$.	BTL -3	Applying	CO 2
6.(a)	For what value of the constant b, is the function f continuous on $(-\infty, \infty)$ if $f(x) = \begin{cases} bx^2 + 2x & \text{if } x < 2 \\ x^3 - bx & \text{if } x \geq 2 \end{cases}$	BTL -3	Applying	CO 2
6.(b)	Find $\frac{dy}{dx}$, when $y = \tan^{-1}\left(\frac{acosx+bsinx}{bcosx-asinx}\right)$	BTL -4	Analyzing	CO 2
7.	Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing. Also find the local maximum, local minimum, concavity and the inflection points of f(x).	BTL -4	Analyzing	CO 2
8.(a)	Verify mean value theorem for the following $f(x) = x^3 - 5x^2 - 3x, x \in [1, 3]$.	BTL -3	Applying	CO 2
8.(b)	Find the Taylor's series expansion of $f(x) = \frac{1}{1+x}$ about $x=0$.	BTL -3	Applying	CO 2
9.	For the function $f(x) = x^4 - 4x^3$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity and the inflection points.	BTL -4	Analyzing	CO 2
10.	If $f(x) = 2x^3 + 3x^2 - 36x$, find the intervals on which it is increasing or decreasing, local maximum, local minimum values, concavity and the inflection points of f(x).	BTL -4	Analyzing	CO 2

11.	Find the tangent line to the equation $x^3 + y^3 = 6xy$ at the point (3,3) and at what point the tangent line is horizontal in the first quadrant.	BTL -3	Applying	CO 2
12.(a)	Find the absolute maximum and minimum of $f(x) = x - 2\tan^{-1}x \in [0, 4]$.	BTL -4	Analyzing	CO 2
12.(b)	Verify Lagrange's mean value theorem for the following $f(x) = 2x^3 + x^2 - x - 1, x \in [0, 2]$.	BTL -3	Applying	CO 2
13.	Use second derivative test to examine the relative maxima for $f(x) = x(12 - 2x)^2$.	BTL -4	Analyzing	CO 2
14.(a)	Examine the local extreme of $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$. Also discuss the concavity and find the inflection points	BTL -4	Analyzing	CO 2
14.(b)	Verify Rolle's theorem for the following $f(x) = x(x - 1)(x - 2), x \in [0, 2]$.	BTL -3	Applying	CO 2
15.	For the function $y = 2 + 2x^2 - x^4$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity.	BTL -4	Analyzing	CO 2
16.(a)	Examine the local extreme of $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$. Also discuss the concavity and find the inflection points.	BTL -4	Analyzing	CO 2
16.(b)	Find the Taylor's series expansion of $f(x) = \tan^{-1}x$ about $x = 0$.	BTL -3	Applying	CO 2
17.	Find the local maximum and minimum values of $f(x) = \sqrt{x} - \sqrt[4]{x}$ using both first and second derivatives tests.	BTL -4	Analyzing	CO 2
18.(a)	Calculate the absolute maximum and minimum of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ in $[-2, 3]$.	BTL -3	Applying	CO 2
18.(b)	Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$.	BTL -3	Applying	CO 2

UNIT-III: DIFFERENTIAL CALCULUS OF SEVERAL VARIABLES

Partial derivatives - Total derivatives - Jacobians and properties - Taylor's series for functions of two variables - Maxima and Minima of functions of two variables -Lagrange's method of undetermined multipliers.

Q.No.	Question	BT Level	Competence	Course Outcome
PART – A				
1.	Find the value of $\frac{du}{dt}$, given $u = \log(x + y + z)$, $x = e^t$, $y = \sin t, z = \cos t$.	BTL -2	Understanding	CO 3
2.	Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3ax^2y$.	BTL -1	Remembering	CO 3

3.	Find $\frac{\partial r}{\partial x}$, if $x = r \cos \theta$ & $y = r \sin \theta$.	BTL -1	Remembering	CO 3
4.	If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$.	BTL -1	Remembering	CO 3
5.	Find $\frac{du}{dt}$ if $u = \frac{x}{y}$, where $x = e^t, y = \log t$.	BTL -2	Understanding	CO 3
6.	Find $\frac{du}{dt}$ if $u = \sin \left(\frac{x}{y} \right)$, where $x = e^t, y = t^2$	BTL -2	Understanding	CO 3
7.	Find the value of $\frac{du}{dt}$, given $u = x^2 + y^2, x = at^2, y = 2at$.	BTL -3	Applying	CO 3
8.	If $u = x^3y^2 + x^2y^3$ where $x = at^2$ and $y = 2at$, then find $\frac{du}{dt}$.	BTL -3	Applying	CO 3
9.	If $u = \frac{y^2}{2x}$ and $v = \frac{x^2+y^2}{2x}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.	BTL -4	Analyzing	CO 3
10.	If $x = uv, y = \frac{u}{v}$. Find $\frac{\partial(x,y)}{\partial(u,v)}$.	BTL -3	Applying	CO 3
11.	Write any two properties of Jacobians.	BTL -3	Applying	CO 3
12.	Find the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$, if $x = r \cos \theta$ & $y = r \sin \theta$, $u = 2xy, v = x^2 - y^2$ without actual substitution.	BTL -2	Understanding	CO 3
13.	Expand $xy + 2x + y + 2$ in powers of $(x-2)$ and $(y+1)$ using Taylor's series up to first degree term.	BTL -1	Remembering	CO 3
14.	Find the Taylor series expansion of x^y near the point $(1, 1)$ up to first term.	BTL -4	Analyzing	CO 3
15.	If $x = u^2 - v^2, y = 2uv$ find the Jacobian of x, y with respect to u and v .	BTL -4	Analyzing	CO 3
16.	If $u = \frac{x+y}{1-xy}$ and $v = x + y$, find $\frac{\partial(u,v)}{\partial(x,y)}$.	BTL -2	Understanding	CO 3
17.	State the necessary and sufficient condition for $f(x, y)$ to have a relative maximum at a point (a, b) .	BTL -4	Analyzing	CO 3
18.	Find the minimum value of $f = x^2 + y^2$ subject to the constraint $x = 1$.	BTL -3	Applying	CO 3
19.	Find the Stationary points of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.	BTL -4	Analyzing	CO 3
20.	If $u = x+y, y = uv$ then find $\frac{\partial(x,y)}{\partial(u,v)}$.	BTL -2	Understanding	CO 3
21.	Find the Stationary points of $x^2 - xy + y^2 - 2x + y$.	BTL -2	Understanding	CO 3
22.	Find the stationary points of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.	BTL -4	Analyzing	CO 3
23.	If $u = x^2+1, v = y^2-2$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.	BTL -2	Understanding	CO 3
24.	Find the nature of the stationary point $(1,1)$ of the function $f(x,y)$ if $f_{yy} = 6x^3y, f_{xx} = 6y^3x, f_{xy} = 9x^2y$.	BTL -4	Analyzing	CO 3
25.	Find the minimum point of $f(x, y) = x^2 + y^2 + 6x + 12$.	BTL -3	Applying	CO 3

PART – B				
1.	If $u = \log(x^2 + y^2) + \tan^{-1}x$, then prove that $u_{xx} + u_{yy} = 0$.	BTL -3	Applying	CO 3
2.(a)	If $u = \frac{yz}{x}, v = \frac{zx}{y}$ and $w = \frac{xy}{z}$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	BTL -3	Applying	CO 3
2.(b)	A flat circular plate is heated so that the temperature at any point (x, y) is $u(x, y) = x^2 + 2y^2 - x$. Find the coldest point on the plate.	BTL -3	Applying	CO 3
3.	A rectangular box open at the top, is to have a volume of 32cc. Find the dimensions of box which requires least amount of material for its construction.	BTL -3	Applying	CO 3
4.(a)	Find $\frac{du}{dt}$ if $u = x^3 + y^3$, $x = a \cos t$ and $y = b \sin t$ and verify the result.	BTL -4	Analyzing	CO 3
4.(b)	Find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ if $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$.	BTL -3	Applying	CO 3
5.	If $x + y + z = u, y + z = uv, z = uvw$, prove $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.	BTL -3	Applying	CO 3
6.(a)	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.	BTL -4	Analyzing	CO 3
6.(b)	If $u = f(x - y, y - z, z - x)$, then show $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	BTL -3	Applying	CO 3
7.	Expand $x^3y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ using Taylor's series up to third degree terms.	BTL -3	Applying	CO 3
8.(a)	If $z = e^{ax+by} \cdot f(ax - by)$ then prove that $b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$.	BTL -3	Applying	CO 3
8.(b)	Find the extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.	BTL -3	Applying	CO 3
9.	Find the Taylors series expansion of $e^x \cos y$ at the point $(0, \frac{\pi}{2})$ up to third degree terms.	BTL -4	Analyzing	CO 3
10.	Find the maximum volume of the largest rectangular parallelopiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	BTL -3	Applying	CO 3
11.	Find the shortest and longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$.	BTL -4	Analyzing	CO 3
12.(a)	If $u = xyz, v = x^2 + y^2 + z^2$ and $w = x + y + z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.	BTL -4	Analyzing	CO 3
12.(b)	Expand Taylor's series of $x^3 + y^3 + xy^2$ in powers of $(x - 1)$ and $(y - 2)$ up to the second-degree term.	BTL -3	Applying	CO 3

13.	Find the dimension of the rectangular box without a top of maximum capacity, whose surface area is 108 sq. cm.	BTL -4	Analyzing	CO 3
14.(a)	Examine $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values.	BTL -4	Analyzing	CO 3
14.(b)	Expand $e^x \log(1+y)$ in powers of x & y up to terms of second -degree using Taylor's series.	BTL -3	Applying	CO 3
15.	Discuss the extreme values of $f(x, y) = x^3 y^2 (1 - x - y)$.	BTL -4	Analyzing	CO 3
16.(a)	Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.	BTL -3	Applying	CO 3
16.(b)	Expand e^{xy} in powers of $(x-1)$ and $(y-1)$ upto second degree terms by Taylor's series.	BTL -3	Applying	CO 3
17.	Find the Taylors series expansion of $e^x \sin y$ at the point $(-1, \frac{\pi}{4})$ up to third degree terms .	BTL -4	Analyzing	CO 3
18.(a)	If $u = x + y + z, u^2 v = y + z, u^3 w = z$, prove $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1}{u^5}$.	BTL -3	Applying	CO 3
18.(b)	Find the minimum value of $x^2 y z^3$ subject to $2x + y + 3z = a$.	BTL -3	Applying	CO 3

UNIT-IV:MULTIPLE INTEGRALS

Double integrals in Cartesian and polar coordinates - Change of order of integration - Area enclosed by plane curves - Change of variables in Polar coordinates - Triple integrals - Volume of solids.

Q.No.	Question	BT Level	Competence	Course Outcome
PART – A				
1.	Change the order of integration $\int_0^1 \int_{y^2}^y f(x, y) dx dy$.	BTL -3	Applying	CO 4
2.	Change the order of integration $\int_0^\infty \int_x^\infty f(x, y) dx dy$.	BTL -3	Applying	CO 4
3.	Evaluate $\int_0^2 \int_0^x \frac{xdy}{x^2+y^2}$.	BTL -2	Understanding	CO 4
4.	Evaluate $\int \int dx dy$ over the region bounded by $x = 0, x = 2, y = 0$ and $y = 2$.	BTL -2	Understanding	CO 4
5.	Evaluate $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{xdy}{1+x^2+y^2}$.	BTL -1	Remembering	CO 4
6.	Evaluate $\int_2^3 \int_1^2 \frac{xdy}{xy}$.	BTL -1	Remembering	CO 4
7.	Evaluate $\int_0^2 \int_0^1 4xy dx dy$.	BTL -1	Remembering	CO 4
8	Find the limits of integration in the double integral $\iint_R f(x, y) dx dy$ where R is in the first quadrant and bounded	BTL -3	Applying	CO 4

	$x=1, y=0, y^2 = 4x.$			
9.	Find the limits of integration in the double integral $\iint_R f(x,y)dxdy$ where R is in the first quadrant and bounded $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ $x = 0, y = 0,$	BTL -2	Understanding	CO 4
10.	Find the limits of integration in the double integral $\iint_R f(x,y)dxdy$ where R is in the first quadrant and bounded $x = 0, y = 0, x + y = 1.$	BTL -2	Understanding	CO 4
11.	Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(\theta + \phi) d\theta d\phi.$	BTL -4	Analyzing	CO 4
12.	Find the area bounded by the lines $x = 0, y = 1$ and $y = x.$	BTL -2	Understanding	CO 4
13.	Evaluate $\int \int \int (x + y + z)dxdydz$ over the region bounded by $x = 0, x = 1, y = 0$ and $y = 1, z = 0, z = 1.$	BTL -4	Analyzing	CO 4
14.	Evaluate $\int_0^{\pi} \int_0^{\sin\theta} r dr d\theta.$	BTL -3	Applying	CO 4
15.	Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dy dx.$	BTL -4	Analyzing	CO 4
16.	Evaluate $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y} dx dy.$	BTL -2	Understanding	CO 4
17.	Evaluate $\int_1^3 \int_3^4 \int_1^4 xyz dz dy dx.$	BTL -4	Analyzing	CO 4
18.	Evaluate $\int_a^b \int_c^d \int_f^g e^{x+y+z} dz dy dx.$	BTL -1	Remembering	CO 4
19.	Evaluate $\int_0^1 dx \int_0^2 dy \int_0^3 (x + y + z) dz$	BTL -4	Analyzing	CO 4
20.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy(x + y) dy dx.$	BTL -3	Applying	CO 4
21.	Evaluate $\int_0^5 \int_0^2 (x^2 + y^2) dx dy.$	BTL -2	Understanding	CO 4
22.	Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2) dy dx$	BTL -4	Analyzing	CO 4
23.	Evaluate $\int_0^{\pi} \int_0^5 r^4 \sin\theta dr d\theta$	BTL -4	Analyzing	CO 4
24.	Evaluate $\int_0^{\pi} \int_0^a r dr d\theta.$	BTL -3	Applying	CO 4
25.	Write down the double integral to find the area of the circles $r = 2\sin\theta, r = 4\sin\theta.$	BTL -2	Understanding	CO 4
PART – B				
1.(a)	Find the area bounded by parabolas $y = 4 - x$ and $y^2 = x$ by double integration.	BTL -2	Understanding	CO 4
1.(b)	Using double integral find the area of the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$	BTL -4	Applying	CO 4

2.	Change the order of integration $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy dx dy$ and hence evaluate it.	BTL -1	Remembering	CO 4
3.	Change the order of integration $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y dx dy$ and hence evaluate it.	BTL -3	Analyzing	CO 4
4.	Evaluate $\iint xy dx dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.	BTL -4	Applying	CO 4
5.	Change the order of integration $\int_0^1 \int_y^{2-y} xy dx dy$ and hence evaluate it.	BTL -2	Understanding	CO 4
6.(a)	Change the order of integration $\int_0^2 \int_0^{\sqrt{4-y^2}} xy dx dy$ and hence evaluate it.	BTL -2	Understanding	CO 4
6.(b)	Find the area included between the curves $y^2 = 4x$ and $x^2 = 4y$.	BTL -4	Applying	CO 4
7.	Find the volume of Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	BTL -2	Understanding	CO 4
8.	Find the area common to the cardioids $r = a(1 + \cos \theta)$ and $r = a(1 - \cos \theta)$.	BTL -4	Analyzing	CO 4
9.	Evaluate $\iiint \frac{dx dy dz}{x^2+y^2+z^2}$, taken throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$.	BTL -3	Applying	CO 4
10.(a)	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$.	BTL -3	Applying	CO 4
10.(b)	Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) dx dy dz$.	BTL -3	Applying	CO 4
11.	Find the value of $\iiint xyz dx dy dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \leq a^2$.	BTL -4	Analyzing	CO 4
12.	Find the volume of Sphere $x^2 + y^2 + z^2 = a^2$.	BTL -2	Understanding	CO 4
13.	Change the order of integration $\int_0^a \int_y^a \frac{x dx dy}{\sqrt{x^2+y^2}}$ and hence evaluate it.	BTL -2	Understanding	CO 4
14.	Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$, $z = 0$.	BTL -2	Understanding	CO 4
15.	By changing in to polar Co – ordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$. Hence find the value of $\int_0^\infty e^{-x^2} dx$.	BTL -2	Understanding	CO 4
16.(a)	Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.	BTL -3	Applying	CO 4
16.(b)	Evaluate $\iint (x^2 + y^2) dx dy$ over the area bounded by the parabola $y^2 = 4x$ and its latus rectum.	BTL -2	Understanding	CO 4
17.	By change the order of integration and	BTL -4	Analyzing	CO 4

	evaluate $\int_0^2 \int_{x^2}^{2-x} xy dy dx$.			
18.	Change the integral into polar coordinates $\int_0^a \int_0^x \frac{x^3}{\sqrt{x^2+y^2}} dx dy$ and hence evaluate it.	BTL -3	Applying	CO 4

UNIT-V: VECTOR CALCULUS

Gradient and directional derivative – Divergence and curl – Vector identities – Irrotational and Solenoidal vector fields – Vector Integration Green's, Gauss divergence and Stoke's theorems – Verification and application in evaluating line, surface and volume integrals.

Q.No.	Question	BT Level	Competence	Course Outcome
PART – A				
1.	What is the value of m if the vector $\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + mz)\vec{k}$ is solenoidal.	BTL -3	Applying	CO 5
2.	Show that the vector $\vec{F} = (x + 2y)\vec{i} + (y - 2z)\vec{j} + (x - 2z)\vec{k}$ is solenoidal.	BTL -1	Remembering	CO 5
3.	Find the maximum directional derivative of $\phi = xyz^2$ at $(1, 2, 3)$.	BTL -2	Understanding	CO 5
4.	If $\phi = x^3 + yz$ then find $grad \phi$.	BTL -1	Remembering	CO 5
5.	If $\phi = x^3 + y^3 + z^3$ then find $grad \phi$ at $(2, 0, 4)$.	BTL -2	Understanding	CO 5
6.	Find the Directional derivative of $\phi = 4xz^2 + x^2yz$ at $(0, 2, 4)$ in the direction $2\vec{i} + 3\vec{j} + 4\vec{k}$.	BTL -1	Remembering	CO 5
7.	Give the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$ at $(2, 3, 4)$.	BTL -2	Understanding	CO 5
8.	Find the unit normal to the surface $xy^3z^2 = 4$ at $(1, 1, 3)$	BTL -3	Applying	CO 5
9.	What is the value of a, b, c if the vector $\vec{F} = (x + y + az)\vec{i} + (bx + 2y - z)\vec{j} + (-x + cy + 2z)\vec{k}$ may be irrotational.	BTL -3	Applying	CO 5
10.	Is the position vector $r \rightarrow = xi \rightarrow + yj \rightarrow + zk \rightarrow$ irrotational? Justify.	BTL -3	Applying	CO 5
11.	If $\phi = 3xy - yz$, Find $grad \phi$ at $(1, 0, 1)$.	BTL -3	Applying	CO 5
12.	Show that $\nabla(r^n) = nr^{n-2}\vec{r}$.	BTL -2	Understanding	CO 5
13.	Show that the vector $\vec{F} = 3y^3z^2\vec{i} + 4x^2z^2\vec{j} - 3x^2y^2\vec{k}$ is solenoidal.	BTL -4	Analyzing	CO 5
14.	Show that the vector $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy -$	BTL -3	Applying	CO 5

	$2xz + 2z)\vec{k}$ is solenoidal.			
15.	Show that $\text{curl}(\text{grad}\phi) = 0$.	BTL -3	Applying	CO 5
16.	If $\vec{r}$ is the position vector, Find $\text{div}\vec{r}$.	BTL -2	Understanding	CO 5
17.	Give the unit normal vector to the surface $xyz = 4$ at $(2, 1, 1)$.	BTL -4	Analyzing	CO 5
18.	Find the unit normal to the surface $x^2 + y^2 - z = 10$ at $(2,4,6)$	BTL -3	Applying	CO 5
19.	Evaluate using Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$.	BTL -2	Understanding	CO 5
20.	If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, Evaluate $\int \vec{F} \cdot d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from the point $(0,0)$ to the point $(1,3)$.	BTL -4	Analyzing	CO 5
21.	If $\vec{F} = (x^2)\vec{i} + (xy^2)\vec{j}$, evaluate $\int \vec{F} \cdot d\vec{r}$ from $(0,0)$ to $(2,2)$ along the path $y = x$.	BTL -3	Applying	CO 5
22.	Using Green's theorem evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the square enclosed by the lines $x = 0, y = 0, x = 3, y = 4$.	BTL -2	Understanding	CO 5
23.	State Greens theorem.	BTL -4	Analyzing	CO 5
24.	State Stokes theorem.	BTL -1	Remembering	CO 5
25.	State Gauss Divergence theorem.	BTL -4	Analyzing	CO 5
PART – B				
1.	Find the Directional Derivative of $\phi = 3x^2yz + 4xz^2 + xyz$ at $(1, 2, 3)$ in the direction of $2\vec{i} + \vec{j} - \vec{k}$.	BTL -2	Understanding	CO 5
2.(a)	Calculate the angle between the normals to the surface $xy = z^2$ at the points $(4, 1, 2)$ and $(3, 3, -3)$.	BTL -2	Understanding	CO 5
2.(b)	Find the work done when a force $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ moves a particle in the XY plane from $(0,0,0)$ to $(1,1,1)$ along the parabola $x = y^2$.	BTL -3	Analyzing	CO 5
3.	Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the point $(6, 4, 3)$.	BTL -4	Applying	CO 5
4.(a)	Find the value of a, b, c so that the vector $\vec{F} = (x + y + az)\vec{i} + (bx - 2y - z)\vec{j} + (-x + cy + 2z)\vec{k}$ may be irrotational. Also find its scalar potential.	BTL -2	Understanding	CO 5
4.(b)	Find the work done by the force $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ where C is the arc of the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$.	BTL -4	Applying	CO 5

5.	Find the scalar potential, if the vector field $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$ is irrotational.	BTL -2	Understanding	CO 5
6.(a)	Find the constants a and b , so that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ may cut orthogonally at the point $(1,-1,2)$.	BTL -3	Analyzing	CO 5
6.(b)	Find the unit normal to the surface $x^2y + 2xz^2 = 8$ at the point $(1, 0, 2)$.	BTL -2	Understanding	CO 5
7.	Find the Directional Derivative of $\phi = xy^2\vec{i} + zy^2\vec{j} + xz^2\vec{k}$ at $(2, 0, 3)$ in the direction of the outward normal to the sphere $x^2 + y^2 + z^2$ at the point $(3,2,1)$.	BTL -3	Analyzing	CO 5
8.(a)	Find the work done by the force when it moves a particle from $(1,-2, 1)$ to $(3, 1,4)$ along any path?	BTL -4	Applying	CO 5
8.(b)	Find the values of a and b so that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ may cut orthogonally at $(1,-1, 2)$.	BTL -1	Remembering	CO 5
9.	Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - 3 = z$ at the points $(2,-1, 2)$.	BTL -2	Understanding	CO 5
10.(a)	Verify Green's theorem in the plane for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by $x = 0, y = 0, x + y = 1$.	BTL -3	Analyzing	CO 5
10.(b)	Find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.	BTL -4	Applying	CO 5
11.	Prove that area bounded by a simple closed curve is given by $\frac{1}{2} \int_C (xdy - ydx)$. Hence find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.	BTL -2	Understanding	CO 5
12.	Verify Green's theorem in the plane for $\int_C [(xy + y^2)dx + (x^2)dy]$ where 'c' is the closed region bounded by $x = y$ and $y = x^2$.	BTL -2	Understanding	CO 5
13.	Verify Gauss divergence theorem for $\vec{F} = (x^3)\vec{i} + (y^3)\vec{j} + z^3\vec{k}$ where S is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$.	BTL -3	Applying	CO 5
14	Verify Gauss divergence theorem for $\vec{F} = (x^2)\vec{i} + (y^2)\vec{j} + (z^2)\vec{k}$ where s is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$.	BTL -4	Analyzing	CO 5
15.	Verify Gauss divergence theorem for $\vec{F} = (x^3 - yz)\vec{i} - (2x^2y)\vec{j} + 2z\vec{k}$ where s is the surface of the cuboid formed by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	BTL -3	Applying	CO 5
16	Verify Stokes theorem for $\vec{F} = (xy + y^2)\vec{i} + x^2\vec{j}$ in the XOY plane bounded by $x = y$ and $y = x^2$.	BTL -4	Analyzing	CO 5

17.	Verify the Stokes theorem is verified for $\vec{F} = (x^2)\vec{i} + xy\vec{j}$ integrated round the square those sides formed $x = 0, x = a, y = 0, y = a$ in the plane $z = 0$	BTL -2	Understanding	CO 5
18.	Show that Stokes theorem is verified for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ where S is the surface of the cube formed by $x = 0, x = 2, y = 0, y = 2, \text{ and } z = 2$ above the XY- plane.	BTL -4	Analyzing	CO 5

