

SRM VALLIAMMAI ENGINEERING COLLEGE

(An Autonomous Institution)

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ISO 9001:2015 Certified Institution

DEPARTMENT OF INFORMATION TECHNOLOGY

QUESTION BANK



I SEMESTER

MA3126 - ADVANCED MATHEMATICS FOR DATA SCIENCE

Regulation – 2023

Academic Year 2026- 2027

Prepared by

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QUESTION BANK**SUBJECT CODE: MA3126****SUBJECT TITLE: ADVANCED MATHEMATICS FOR DATA SCIENCE****YEAR /SEMESTER: I Year / I Semester M.Tech.**

UNIT I MATRIX THEORY				
Eigen values, Eigen vectors, and Diagonalization, singular value decomposition. Positive definite Matrices-Minima, Maxima and saddle points, Test of Positive definiteness, semi-definite and indefinite Matrices.				
Q.No.	Question	BT - Level	Competence	Course Outcomes
PART-A				
1	Find the characteristic equation of the matrix A is $\begin{pmatrix} 2 & 5 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{pmatrix}$	BTL-1	Remembering	CO 1
2	Find the eigen values of $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$	BTL-2	Understanding	CO 1
3	Two eigen values of the matrix $A = \begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$ are 0 and 1. Find the third eigen value.	BTL-2	Understanding	CO 1
4	Find the characteristic equation of $A = \begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$	BTL-1	Remembering	CO 1
5	Find the eigen values of A^2 if $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$	BTL-1	Remembering	CO 1
6	If the eigen values of the matrix A of order 3×3 are 2,3 and 1, then find the determinant of A	BTL-2	Understanding	CO 1
7	If the sum of 2 eigen values and the trace of a 3×3 matrix are equal, find the value of $ A $	BTL-1	Remembering	CO 1
8	Find the sum of the eigen values of $2A$, if $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	BTL-2	Understanding	CO 1
9	The product of the 2 eigen values of $A = \begin{pmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 14. Find the 3 rd eigen value.	BTL-2	Understanding	CO 1
10	Find the sum and product of the eigen values of $A = \begin{pmatrix} 2 & -2 & 2 \\ -2 & -1 & -1 \\ 2 & -1 & -1 \end{pmatrix}$	BTL-2	Understanding	CO 1
11	Find the sum and product of the eigen values of $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$	BTL-1	Remembering	CO 1
12	Find the constants a and b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3,-2 be the eigen values of A	BTL-1	Remembering	CO 1
13	Find the matrix corresponding to the quadratic form $2xy - 2yz + 2xz$.	BTL-1	Remembering	CO 1
14	Find the quadratic form corresponding to the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	BTL-2	Understanding	CO 1
15	Write the 3×3 matrix corresponding to the function $x_1^2 + x_2^2 + x_3^2 - 2x_1x_3 - 2x_1x_2 + 2x_2x_3$	BTL-2	Understanding	CO 1

16	If the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ are 2,-2 then find the eigenvalues of A^T .	BTL-2	Understanding	CO 1
17	Write down the condition for minimum of the function $ax^2 + 2bxy + cy^2$	BTL-2	Understanding	CO 1
18	Test for minima or maxima for the function $2x^2 + 4xy + y^2$	BTL-2	Understanding	CO 1
19	Define positive definite matrix	BTL-2	Understanding	CO 1
20	State Singular value decomposition theorem	BTL-2	Understanding	CO 1
21	Decide for the positive definiteness of $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ using sub matrices	BTL-1	Remembering	CO 1
22	If $A = \begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$ Find the eigen values of A^2 .	BTL-2	Understanding	CO 1
23	If 1,1,5 are the eigen values of $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ Find the Eigen values if $5A$.	BTL-2	Understanding	CO 1
24	What are the tests available to test the positive definiteness of a matrix	BTL-2	Understanding	CO 1
25	Decide for the positive definiteness of $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 2 \end{pmatrix}$ using eigen values	BTL-2	Understanding	CO 1

PART-B

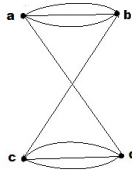
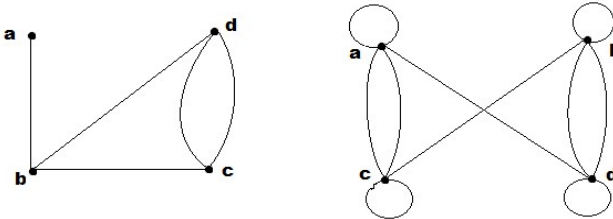
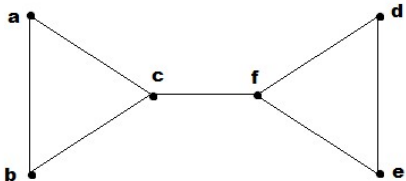
1	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$	BTL-3	Applying	CO 1
2	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$	BTL-3	Applying	CO 1
3	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$	BTL-3	Applying	CO 1
4	Obtain the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$.	BTL-4	Analyzing	CO 1
5	Obtain the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	BTL-2	Understanding	CO 1
6	Reduce the matrix $A = \begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$ to diagonal form	BTL-4	Analyzing	CO 1
7	Diagonalize the matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 5 \end{pmatrix}$ by means of an orthogonal transformation.	BTL-2	Understanding	CO 1
8	Diagonalize the matrix $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$ by means of an orthogonal transformation.	BTL-4	Analyzing	CO 1

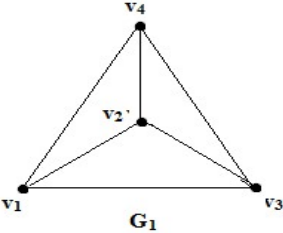
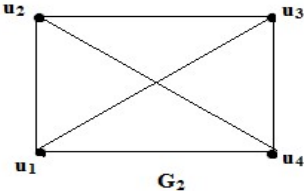
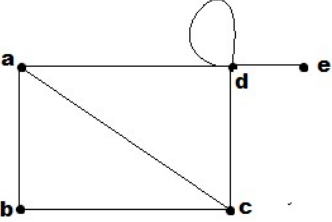
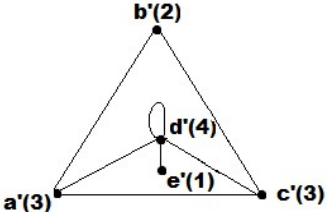
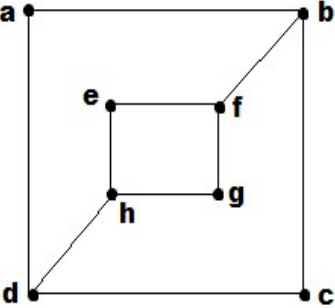
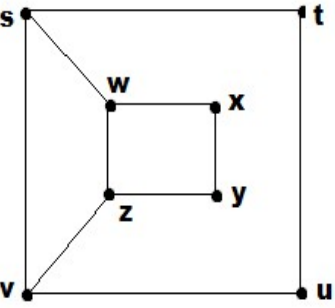
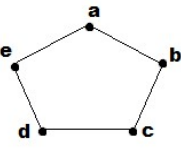
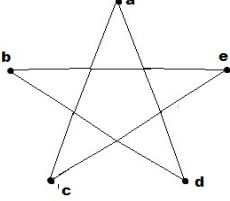
9	Diagonalize the matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ by means of an orthogonal transformation.	BTL-4	Analyzing	CO 1
10	Reduce the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ to diagonal form using orthogonal transformation.	BTL-4	Analyzing	CO 1
11	Find the singular value decomposition of $\begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}$	BTL-2	Understanding	CO 1
12	Find the singular value decomposition of $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$	BTL-4	Analyzing	CO 1
13	Prepare the singular value decomposition of $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$	BTL-2	Understanding	CO 1
14	Discuss the positive semi definiteness of the matrix $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$ using all the tests	BTL-2	Understanding	CO 1
15	Test whether $A^T A$ is positive definite if $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$	BTL-2	Understanding	CO 1
16	Discuss the positive definiteness of the matrix $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ using all the tests	BTL-2	Understanding	CO 1
17	Check the matrix for positive definite $A = \begin{pmatrix} 5 & 6 \\ 6 & 7 \end{pmatrix}$	BTL-3	Applying	CO 1
18	The Eigen vectors of a 3X3 real symmetric matrix A corresponding to the eigen values 1,2,4 are $(1, 0, 0)^T, (0, 1, 1)^T, (0, 1, -1)^T$ respectively Create the matrix A.	BTL-3	Applying	CO 1

UNIT II GRAPH THEORY

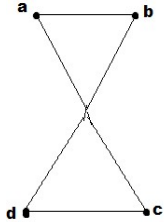
Graphs and graph models – Graph terminology and special types of graphs - Graph Isomorphism- Planar graphs - graph coloring-Hamilton circuits and Euler cycles.

Q.No.	Question	BT - Level	Competence	Course Outcomes
PART – A				
1.	Define graphs	BTL-1	Remembering	CO 2
2.	Define a regular graph. Can a complete graph be a regular graph?	BTL-2	Understanding	CO 2
3.	Define complete graph and draw K_5 .	BTL-2	Understanding	CO 2
4.	Define strongly connected graph.	BTL-1	Remembering	CO 2
5.	State the handshaking theorem.	BTL-1	Remembering	CO 2
6.	Define isomorphism of two graphs.	BTL-2	Understanding	CO 2
7.	When is a simple graph G bipartite? Give an example.	BTL-1	Remembering	CO 2
8.	How many edges are there in a graph with 10 vertices each of degree 3?	BTL-2	Understanding	CO 2
9.	How many edges does a graph have if it has vertices of degree 5, 2, 2, 2, 2, 1? Draw such a graph.	BTL-2	Understanding	CO 2
10.	Define connected graph and a disconnected graph with example.	BTL-2	Understanding	CO 2

11.	Let G be the graph with 10 vertices. If four vertices has degree four and six vertices has degree five , then find the number of edges of G.	BTL-1	Remembering	CO 2
12.	Use an incidence matrix represents the graph. 	BTL-1	Remembering	CO 2
13.	Draw the graph with the following adjacency matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	BTL-1	Remembering	CO 2
14.	Give an Example of a bipartite graph which is Hamiltonian but not Eulerian.	BTL-2	Understanding	CO 2
15.	What should be the degree of each vertex of a graph G if it has Hamilton circuit?	BTL-2	Understanding	CO 2
16.	Define complete bipartite graph.	BTL-2	Understanding	CO 2
17.	State the necessary and sufficient conditions for the existence of an Eulerian path in a connected graph.	BTL-2	Understanding	CO 2
18.	Define Hamiltonian path.	BTL-2	Understanding	CO 2
19.	Show that there does not exist a graph with 5 vertices with degrees 1, 3, 4, 2, 3 respectively.	BTL-2	Understanding	CO 2
20.	Give an example of a graph which is Eulerian but not Hamiltonian	BTL-2	Understanding	CO 2
21.	Define graph coloring	BTL-1	Remembering	CO 2
22.	Represent the given graph using an adjacency matrix. 	BTL-2	Understanding	CO 2
23.	An undirected graph G has 16 edges and all the vertices are of degree 2. Find the number of vertices.	BTL-2	Understanding	CO 2
24.	Define planar graph.	BTL-2	Understanding	CO 2
25.	Does the graph have a Hamilton path? If so find such a path. 	BTL-2	Understanding	CO 2
PART – B				

1.	Define isomorphism. Establish an isomorphism for the following graphs.  	BTL -3	Applying	CO 2
2.	 	BTL -3	Applying	CO 2
3.	The adjacency matrices of two pairs of graph as given below. Examine the isomorphism of G and H by finding a permutation matrix. $A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	BTL -3	Applying	CO 2
4.	 	BTL -4	Analyzing	CO 2
5. (a)	 	BTL -4	Analyzing	CO 2
5. (b)	Write the adjacency matrix of the digraph $G = \left\{ (v_1, v_3), (v_1, v_2), (v_2, v_4), (v_3, v_1), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_4, v_2), (v_4, v_3) \right\}$. Also draw the graph.	BTL -3	Applying	CO 2
6.	Examine whether the following pair of graphs are isomorphic or not. Justify your answer.	BTL -4	Analyzing	CO 2

7.	Give an example of a graph which is (i) Eulerian but not Hamiltonian (ii) Hamiltonian but not Eulerian (iii) Both Eulerian and Hamiltonian (iv) Not Eulerian and not Hamiltonian	BTL-3	Applying	CO 2
8. (a)	Prove that a connected graph G is Euler graph if and only if every vertex of G is of even degree.	BTL -4	Analyzing	CO 2
8. (b)	If G is a graph with n vertices and $(G) \geq \frac{n-1}{2}$, then G is connected.	BTL -4	Analyzing	CO 2
9.	Define Isomorphism between the two graphs. Are the simple graphs with the following adjacency matrices isomorphic? $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$	BTL -3	Applying	CO 2
10.	Find an Euler path or an Euler Circuit if it exists, in each of the three graphs given below.	BTL -3	Applying	CO 2
11.	Find the number of paths of length 4 from the vertex D to E in the undirected graph given below.	BTL -3	Applying	CO 2
12.(a)	The sum of all vertex degree is equal to twice the number of edges (or) the sum of the degrees of the vertices of G is even.	BTL -3	Applying	CO 2
12.(b)	Prove that the complement of a disconnected graph is connected.	BTL -3	Applying	CO 2
13.	How many paths of length four are there from a to d in the simple graph G given below.	BTL -4	Analyzing	CO 2

				
14.	Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $(n - k)(n - k - 1)/2$	BTL -4	Analyzing	CO 2
15.	Prove that the following statements are equivalent for a connected graph G . 1. G is Eulerian 2. Every vertex has even degree 3. The set of edges of G can be partitioned into cycles	BTL -3	Applying	CO 2
16. (a)	The sum of all vertex degree is equal to twice the number of edges (or) the sum of the degrees of the vertices of G is even.	BTL -3	Applying	CO 2
16.(b)	Prove that the complement of a disconnected graph is connected.	BTL -3	Applying	CO 2
17.(a)	If a graph G has exactly two vertices of odd degree there is a path joining these two vertices.	BTL -2	Understanding	CO 2
17.(b)	Define (i) Complete graph. (ii) Complete bipartite graph with example.	BTL -2	Understanding	CO 2
18.(a)	Represent each of the following graphs with an adjacency matrix (i). K_4 , (ii). $K_{1,4}$ (iii). C_4 (iv). W_4	BTL -4	Analyzing	CO 2
18.(b)	Show that a graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other is in V_2 .	BTL -2	Understanding	CO 2

UNIT-III COMBINATORICS

The pigeonhole principle – Permutations and combinations - Permutations and Combinations with and without repetition- Specialized techniques to solve combinatorial enumeration problems - Inclusion and exclusion principle and its applications

Q.No.	Question	BT - Level	Competence	Course Outcomes
PART – A				
1.	In how many ways can all the letters in MATHEMATICAL be arranged?	BTL-1	Remembering	CO 3
2.	How many permutations are there in the word MISSISSIPPI?	BTL-2	Understanding	CO 3
3.	How many different bit strings are there of length seven?	BTL-2	Understanding	CO 3
4.	How many permutations of $\{a,b,c,d,e,f,g\}$ starting with a ?	BTL-1	Remembering	CO 3
5.	In how many ways can first, second and third prize in Pie-baking contest be given to 15 participants?	BTL-1	Remembering	CO 3
6.	How many permutations are there in the word MALAYALAM?	BTL-2	Understanding	CO 3
7.	How many 7-digit numbers can be formed using the digits 1,2,0,2,4,2 and 4?	BTL-1	Remembering	CO 3
8.	State the Pigeonhole principle.	BTL-2	Understanding	CO 3
9.	State Generalized Pigeonhole Principle.	BTL-2	Understanding	CO 3
10.	How many cards must be selected from a deck of 52 cards to guarantee that atleast 3 cards of the same suit are chosen?	BTL-2	Understanding	CO 3
11.	How many 16-bit strings are there containing exactly 5 zeros?	BTL-1	Remembering	CO 3
12.	From 10 programmers, in how many ways can five be selected when a particular programmer is included every time?	BTL-1	Remembering	CO 3
13.	Define permutations and combinations.	BTL-1	Remembering	CO 3

14.	How many permutations of the letters ABCDEFGH contain the string ABC.	BTL-2	Understanding	CO 3
15.	In how many ways can all the letters in the word “INDIA” be arranged?	BTL-2	Understanding	CO 3
16.	Find the number of distinct permutations that can be formed from the word “RADAR”.	BTL-2	Understanding	CO 3
17.	In how many ways can 5 persons be selected from amongst 10 persons?	BTL-2	Understanding	CO 3
18.	How many bit strings of length ‘n’ contain exactly ‘k’ 1’s.	BTL-2	Understanding	CO 3
19.	State the principle of inclusion and exclusion.	BTL-2	Understanding	CO 3
20.	What is the number of arrangements of all the six letters in the word PEPPER.	BTL-2	Understanding	CO 3
21.	Find the number of 5- permutations of a set with nine elements.	BTL-1	Remembering	CO 3
22.	In how many ways can a set of five letters to be selected from the English Alphabet.	BTL-2	Understanding	CO 3
23.	Show that among 13 children, there are atleast two children who were born in the same month.	BTL-2	Understanding	CO 3
24.	Find the number of distinct permutations that can be formed from the word “UNUSUAL”.	BTL-2	Understanding	CO 3
25.	Show that if seven colours are used to paint 50 cars, atleast eight cars will have the same colour.	BTL-2	Understanding	CO 3

PART – B

1.	In a survey of 120 passengers, an Airline found that 52 enjoyed wine with their meals, 75 enjoyed mixed drinks and 62 enjoyed iced tea. 35 enjoyed any given pair these beverages and 20 passengers enjoyed all of them. Find the no. of passengers who enjoyed (i) Only tea (ii) Only one of the three (iii) Exactly two of the three beverages None of the drinks	BTL -2	Understanding	CO 3
2.(a)	Prove that in a group of six people at least three must be mutual friends or at least three must be mutual strangers.	BTL -2	Understanding	CO 3
2.(b)	Determine the number of positive integer n, $1 \leq n \leq 2000$ that are not divisible by 2, 3 or 5 but are divisible by 7.	BTL -3	Applying	CO 3
3.	How many permutations of the letters A B C D E F G contain (a) the string BCD (b) the string CFGA (c) the strings BA and GF (d) the strings ABC and DE (e) the strings ABC and CDE (f) the strings CBA and BED?	BTL -4	Analyzing	CO 3
4.	In how many ways can 20 students out of a class of 30 be selected for an extracurricular activity, if (a) Rama refuses to be selected? (b) Raja insists on being selected? (c) Gopal and Govind insist on being selected? (d) either Gopal or Govind or both get selected? (e) just one of Gopal and Govind gets selected?	BTL -4	Analyzing	CO 3
5.	How many bits of string of length 10 contains (i) Exactly four 1’s (ii) At most four 1’s (iii) At least four 1’s (iv) An equal number of 0’s and 1’s	BTL -3	Applying	CO 3
6.	There are 2500 students in a college, of these 1700 have taken a course in C, 1000 have taken a course Pascal and 550 have taken a course in networking. Further 750 have taken courses in both C and Pascal, 400 have taken courses in both C and networking, 275 have taken courses in both Pascal and Networking. If 200 of these students have taken courses in C, Pascal and Networking.	BTL -4	Analyzing	CO 3

	(i) How many of these 2500 students have taken a course in any of these three courses. (ii) How many of these 2500 students have not taken a course in any of these three courses.			
7.	From a club consisting of six men and seven women, in how many ways we select a committee of (1) 3 men and 4 women? (2) 4 persons which has at least one woman? (3) 4 persons that has at most one man? (4) 4 persons that has both sexes?	BTL -3	Applying	CO 3
8.	There are three piles of identical red, blue and green balls, where each pile contains at least 10 balls. In how many ways can 10 balls be selected (1) If there is no restriction? (2) If at least 1 red ball must be selected? (3) If at least 1 red, at least 2 blue and at least 3 green balls must be selected? (4) If at most 1 red ball is selected?	BTL -3	Applying	CO 3
9.	Determine n such that $1 \leq n \leq 100$ which are not divisible by 5 or by 7.	BTL -3	Applying	CO 3
10.	Find the number of integers between 1 to 100 that are not divisible by any of the integers (i) 2,3,5 or 7 (ii) 2,3,5 but not by 7.	BTL -3	Applying	CO 3
11.	Find the number of integers between 1 to 250 that are not divisible by any of the integers 2,3,5 and 7	BTL -3	Applying	CO 3
12.	A Committee of 5 is to be selected from 6 boys and 5 girls. Determine the number of ways of selecting the committee if it is to consist of at least 1 boy and 1 girl.	BTL -2	Understanding	CO 3
13.	A survey of 550 television watchers produced the following information: 285 watch football game, 195 watch hockey game, 115 watch baseball game, 45 watch football and baseball games, 70 watch football and hockey games, 50 watch hockey and baseball games, 100 do not watch any of the three games. Then (a) How many people in the survey watch all three games? (b) How many people watch exactly one of the three games?	BTL -3	Applying	CO 3
14.	A survey of 500 from a school produced the following information. 200 play volleyball, 120 play hockey, 60 play both volley ball and hockey. How many are not playing either volley ball or Hockey.	BTL -3	Applying	CO 3
15.	A total of 1232 students have taken a course in Spanish, 879 have taken a course in French and 114 have taken a course in Russian. Further 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian and 14 have taken courses in both French and Russian. If 2092 students have taken at least one Spanish, French and Russian, how students have taken a course in all three languages?	BTL -4	Analyzing	CO 3
16.(a)	A team of 11 players is to be chosen from 15 members. In how many ways can this be done if (i) One particular player is always included. (ii) Two such players have always to be included.	BTL -3	Applying	CO 3
16.(b)	A box contains 6 white balls and 5 red balls. Find the number of ways 4 balls can be drawn from the box if (i) They can be any colour. (ii) Two must be white and two red. (iii) They must all be the same colour.	BTL -3	Applying	CO 3
17.	Out of 100 students in a college, 38 play tennis, 57 play cricket and 31 play hockey, 9 play cricket and Hockey, 10 play Hockey and Tennis, 12 play tennis and cricket. How many play (i) All the three	BTL -3	Applying	CO 3

	games (ii) Just one game (iii) Tennis and cricket but not Hockey. Assume that each student plays at least one game.			
18.	In a survey of 100 students, it was found that 30 studied Mathematics, 54 studied statistics, 25 studied operation research, 1 studied all the three subjects. 20 studied Mathematics and statistics, 3 studied mathematics and operation Research and 15 studied statistics and operation research. (i) How many students studied none of these subjects? (ii) How many students studied only Mathematics?	BTL -4	Analyzing	CO 3

UNIT IV SINGLE VARIABLE OPTIMIZATION TECHNIQUES

Introduction-Formulation of optimization problems, classification of optimization problems-overview of analytical solution for unconstrained optimization problems- constrained optimization-convex set-convex functions-convex optimization problem- search methods – Overview of single variable search method.

Q.No.	Question	BT - Level	Competence	Course Outcomes
PART - A				
1.	What is an objective function?	BTL-1	Remembering	CO 4
2.	Define Local optimal point.	BTL-2	Understanding	CO 4
3.	Define constraints in an optimization problem	BTL-2	Understanding	CO 4
4.	Write down the algorithms in solving single variable optimization problems	BTL-1	Remembering	CO 4
5.	Describe Direct search method in solving single variable optimization problems	BTL-1	Remembering	CO 4
6.	Describe gradient based search method in solving single variable optimization problems	BTL-2	Understanding	CO 4
7.	Define optimization	BTL-1	Remembering	CO 4
8.	Define global optimal point	BTL-2	Understanding	CO 4
9.	Define inflection point	BTL-2	Understanding	CO 4
10.	Write down the condition for a point $x=a$ to be a minimum point	BTL-2	Understanding	CO 4
11.	Estimate the inflection point of $f(x) = x^3$	BTL-1	Remembering	CO 4
12.	Define concave and convex function theory	BTL-1	Remembering	CO 4
13.	Check whether the function $f(x) = 10 - x^2$ is concave or convex	BTL-1	Remembering	CO 4
14.	Check whether the function $f(x) = 2x^3 - 3x^2$ is concave or convex	BTL-2	Understanding	CO 4
15.	Check whether the function $f(x) = x^3 - 2x^2$ is concave or convex	BTL-2	Understanding	CO 4
16.	Evaluate the optimum point for $f(x) = x^3 - 10x - 2x^2 + 10$	BTL-2	Understanding	CO 4
17.	Identify the optimum point for $f(x) = e^x - x^3$	BTL-2	Understanding	CO 4
18.	List the uses of the classical optimization	BTL-2	Understanding	CO 4
19.	What are the types of optimization problems	BTL-2	Understanding	CO 4
20.	Find the optimal value for $f(x) = x^2 - 4x + 2$	BTL-2	Understanding	CO 4
21.	What are the five steps involved in optimization problems	BTL-1	Remembering	CO 4
22.	Check whether the function $f(x) = 5 - 3x^2$ is concave or convex	BTL-2	Understanding	CO 4
23.	Check whether the function $f(x) = x^4 + x^3$ is concave or convex	BTL-2	Understanding	CO 4
24.	A company manufactures two products A and B. It takes 30 minutes to process one unit of product A and 15 minutes for one unit of product B. The maximum machine time available is 35 hours per week. One unit of product A requires 2 kg of raw material, while product B requires 3 kg raw material per unit. The available raw material is limited to 180 kg per week. The products A and B have unlimited market potential and sell for Rs.200 and Rs. 500 per unit respectively. If the manufacturing costs for	BTL-2	Understanding	CO 4

	products A and B are $2x^2, 3y^2$ respectively Formulate it as an optimization problem				
25.	60 m^3 of a granular product are to be transported across a river in a ferry. The transportation cost across the river in Rs 100 per trip, irrespective of the amount transported, but there is restriction on the number of trips, which should not be more than 40. The cost of the container depends upon its dimensions as given below: Cost of bottom= Rs.10per square meter Cost of front and back sides = Rs. 10 per square meter And cost of the ends = Rs. 20 per square metre Thus if a small container is used, cost of container will be less, but the number of trips will be more, and if a large container is used, but the number of trips will be less. Find the dimensions of the container so that the sum of the container cost and transportation cost is minimized	BTL-2	Understanding	CO 4	
	PART -B				BTL-
1.	Perform a test for an optimization for the function $f(x) = x^4$	BTL -3	Applying	CO 4	
2.	Perform a test for an optimization for the function $f(x) = x^3$	BTL -4	Analyzing	CO 4	
3.	Check whether the function (i) $f(x) = x^4 + 6x^2 + 12x$ (ii) $f(x) = x^4 + x^2$ is concave or convex or neither.	BTL -3	Applying	CO 4	
4.	Solve : Minimize $f(x) = x^2 + \frac{54}{x}$ at (0,5) using exhaustive search method	BTL -4	Analyzing	CO 4	
5.	Use golden section search method in order to minimize the function $f(x) = x^2 - 6x + 15$ in the interval (0,4)	BTL -3	Applying	CO 4	
6.	Use golden section search method in order to minimize the function $f(x) = 4x^3 + x^2 - 7x + 14$ in the interval (0,1)	BTL -4	Analyzing	CO 4	
7.	Maximize $f(x) = 20x - 3x^2 - x^4$ using one- dimensional search procedure	BTL -3	Applying	CO 4	
8.	Maximize $f(x) = 6x - x^2$ using one- dimensional search procedure. Take the initial upper bound and lower bounds as 4.8 and 0 and error tolerance $\epsilon = 0.04$	BTL -4	Analyzing	CO 4	
9.	Minimize $f(x) = x^4 - x^2 - 4x$ using one- dimensional search procedure. Take the initial upper bound and lower bounds as 2 and 0 and error tolerance $\epsilon = 0.01$	BTL -3	Applying	CO 4	
10.	Use the one dimensional search procedure to interactively solve, approximately the following equation: Minimize $f(x) = x^3 + 2x - 2x^2 - .025x^4$ Use an error intolerance $\epsilon = 0.04$ and the initial lower and upper bounds as 0 and 2.4	BTL -4	Analyzing	CO 4	
11.	Discuss about the golden section method in solving single variate optimization problem	BTL -3	Applying	CO 4	
12.	Minimize $f(x) = x^2 + \frac{54}{x}$ using bisection method at (2,5) and $\epsilon = 10^{-3}$	BTL -4	Analyzing	CO 4	
13.	Use the golden section search method to find the value of x that minimizes $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ in the range [0,2].Locate this value of x to within a range of 0.3	BTL -3	Applying	CO 4	
14.	Discuss about the Exhaustive search method in solving single variate optimization problem	BTL -4	Analyzing	CO 4	
15.	Minimize $f(x) = x^2 + \frac{54}{x}$ using interval halving method at (0,5) and $\epsilon = 10^{-3}$	BTL -3	Applying	CO 4	
16.	Minimize $f(x) = x^2 + \frac{54}{x}$ using Golden section search method at (0,5)	BTL -4	Analyzing	CO 4	
17.	Maximize $f(x) = 12x - 3x^4 - 2x^6$ using one dimensional search method	BTL -3	Applying	CO 4	

18.	Minimize $f(x) = x^2 + \frac{16}{x}$ using interval halving method at (0,5) to an accuracy of 0.1	BTL -4	Analyzing	CO 4	
UNIT V MULTI – VARIABLE OPTIMIZATION TECHNIQUES Search methods for Multivariable unconstrained problems - Kuhn- Tucker conditions- Optimality criteria-direct search methods- evolutionary search method, Hook-Jeeves pattern search method, gradient based methods – Cauchy’s steepest descent method, Newton’s method.					
Q.No.	Question	BTL-1	Remembering	Course Outcomes	
	PART - A				BTL-
1.	Define stationary point	BTL-2	Understanding	CO 5	
2.	State the condition for a minimum point for the function $f(x_1, x_2, \dots, x_n)$	BTL-2	Understanding	CO 5	
3.	State the condition for a maximum point for the function $f(x_1, x_2, \dots, x_n)$	BTL-2	Understanding	CO 5	
4.	State the condition for inflection point for the function $f(x_1, x_2, \dots, x_n)$	BTL-1	Remembering	CO 5	
5.	Define single variable search methods	BTL-2	Understanding	CO 5	
6.	Define Multivariable unconstrained problems search methods	BTL-2	Understanding	CO 5	
7.	Define unidirectional search in multivariable optimization problem	BTL-1	Remembering	CO 5	
8.	What is multivariable optimization	BTL-1	Remembering	CO 5	
9.	Determine whether the function $f(x) = x_1x_2 - x_1^2 - x_2^2$ is convex, concave or neither	BTL-2	Understanding	CO 5	
10.	Discuss whether the function $f(x) = 3x_1 + 4x_2 + 2x_1^2 + x_2^2 - 2x_1x_2$ is convex, concave or neither	BTL-1	Remembering	CO 5	
11.	Find out whether the function $f(x) = x_1^2 + 3x_1x_2 + x_2^2$ is convex, concave or neither	BTL-2	Understanding	CO 5	
12.	Find out whether the function $f(x) = x_1^2 - 2x_1x_2 + x_2^2$ is convex, concave or neither	BTL-2	Understanding	CO 5	
13.	State Kuhn-Tucker condition	BTL-2	Understanding	CO 5	
14.	How do you solve unconstrained optimization problems?	BTL-1	Remembering	CO 5	
15.	Define Newton’s method	BTL-1	Remembering	CO 5	
16.	What are the gradient based methods available to solve multi variable unconstrained problems	BTL-1	Remembering	CO 5	
17.	State the methods to solve multivariate optimization problem	BTL-2	Understanding	CO 5	
18.	Define direct search method in solving unconstrained optimization problems	BTL-2	Understanding	CO 5	
19.	Define indirect search method in solving unconstrained optimization problems	BTL-2	Understanding	CO 5	
20.	Define random jumping method	BTL-2	Understanding	CO 5	
21.	Define random walk method	BTL-2	Understanding	CO 5	
22.	What are the Direct search methods methods available to solve multi variable unconstrained problems	BTL-2	Understanding	CO 5	
23.	What are the random search methods methods available to solve multi variable unconstrained problems	BTL-2	Understanding	CO 5	
24.	Define unidirectional search method	BTL-1	Remembering	CO 5	
25.	Define optimality criteria	BTL-2	Understanding	CO 5	
	PART- B				
1.	Solve the following two-variable unconstrained non-linear problem using search procedure Maximize $f(x_1, x_2) = 2x_1x_2 + 2x_2 - x_1^2 - 2x_2^2$, $(\frac{1}{2}, \frac{1}{2})$ may be taken as the starting trial solution.	BTL -3	Applying	CO 5	

	Draw the path of the trial solution by solving the system of linear equations by setting $\nabla f(x) = 0$			
2.	Minimize $f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2$ in $(-2, 2)$ using Random jumping method.	BTL -3	Applying	CO 5
3.	Solve $f(x) = 4x_1^2 + x_2^2 - 2x_1x_2$ by Newton's method	BTL -3	Applying	CO 5
4.	Solve the following two-variable unconstrained non-linear problem using search procedure Maximize $f(x_1, x_2) = 2x_1x_2 + x_2 - x_1^2 - 2x_2^2$	BTL -4	Analyzing	CO 5
5.	Starting from the initial trial solution $(x_1, x_2) = (0, 0)$, interactively apply two iterations of the gradient search procedure to the following two-variable unconstrained problem. Also determine the exact solution by solving $\nabla f(x) = 0$, Maximize $f(x) = 8x_1 - x_1^2 - 12x_2 - 2x_2^2 + 2x_1x_2$	BTL -4	Analyzing	CO 5
6.	Solve the following two-variable unconstrained non-linear problem using search procedure Maximize $f(x_1, x_2) = x_1 - x_2 + 2x_1x_2 + 2x_1^2 + x_2^2$	BTL -3	Applying	CO 5
7.	Minimize $(x_1, x_2) = x_1^3 + x_2^2 + 2x_1^2 + 4x_2^2$ using univariate search method with the initial point as $(2, 2)$	BTL -3	Applying	CO 5
8.	Apply Newton's method to Minimize : $f(x_1, x_2) = x_1^2 + x_2^2 + 14x_1 + 14x_2 + 100$ with initial point as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	BTL -4	Analyzing	CO 5
9.	Apply Random search method to Minimize: $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$	BTL -3	Applying	CO 5
10.	Apply Newton's method to solve: $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$	BTL -3	Applying	CO 5
11.	Apply Cauchy's steepest decent method to solve: Minimize : $f(x_1, x_2) = x_1^2 - x_1x_2 + x_2^2$, the error not exceed by 0.05 for function approximation $X_1 = \left(1, \frac{1}{2}\right)$	BTL -4	Analyzing	CO 5
12.	Minimize: $f(x) = (1 - x_1)^2 + (2 - x_2)^2$ using simplex search method with initial point as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and scaling factor $\alpha = 2$	BTL -3	Applying	CO 5
13.	Discuss Cauchy's steepest decent method	BTL -4	Analyzing	CO 5
14.	Apply Hooke-Jeeves pattern search method to solve: Minimize : $(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$	BTL -3	Applying	CO 5
15.	Minimize $f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2$ in $(-2, 2)$ using Random walk method.	BTL -3	Applying	CO 5
16.	Discuss the procedure of Gradient search in Multivariable unconstrained problems	BTL -2	Understanding	CO 5
17.	Minimize $(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using univariate search method with the starting point as $(0, 0)$	BTL -3	Applying	CO 5
18.	Discuss the step by step process involved in simplex search method.	BTL -2	Understanding	CO 5