SRM VALLIAMMAI ENGINEERING COLLEGE

(An Autonomous Institution)

SRM Nagar, Kattankulathur – 603 203

DEPARTMENT OF ELECTRONICS AND INSTRUMENTATION ENGINEERING

M.E. CONTROL AND INSTRUMENTATION ENGINEERING

QUESTION BANK

III SEMESTER

1913302-OPTIMAL CONTROL

Regulation – 2019

Academic Year 2020 – 21

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QUESTION BANK

SUBJECT : OPTIMAL CONTROL

SEM / YEAR : III / II

UNIT I - INTRODUCTION TO OPTIMAL CONTROL			
Statement of optimal Control problem - problem formulation and forms of optimal control performance measures - various methods of optimization - Linear programming – nonlinear programming.			
PART A			
Q.No	**Question**	**BT Level**	**Competence**
1.	Write the steps involved for solving the problem using optimal control.	**BTL 2**	Understand
2.	Write the limitation of the transfer function approach .	**BTL 5**	Evaluate
3.	Classify the Components of a Modern Control System.	**BTL 3**	Apply
4.	Define optimal control.	**BTL 1**	Remember
5.	Differentiate linear programming and non linear programming.	**BTL 2**	Understand
6.	Define minimum principle.	**BTL 1**	Remember
7.	Formulate Performance Index for Fuel-Optimal Control System.	**BTL 4**	Analyze
8.	Write necessary condition for Lagrange Multipliers and the Hamiltonian.	**BTL 6**	Create
9.	List various methods of optimization.	**BTL 1**	Remember
10.	Formulate Performance Index for Time-Optimal Control System.	**BTL 2**	Remember
11.	Write sufficient conditions for Lagrange Multipliers and the Hamiltonian.	**BTL 6**	Create
12.	What is the major requirement of making the output of the system small in output regulator problem?	**BTL 4**	Analyze
13.	Draw the block diagram of Classical Control Configuration.	**BTL 3**	Apply
14.	Differentiate static and dynamic optimization.	**BTL 4**	Analyze
15.	Draw the block diagram of modern Control Configuration	**BTL 3**	Apply
16.	What is the main objective of optimal control.	**BTL 1**	Remember
17.	Write state equation.	**BTL 5**	Evaluate
18.	What are the requirements for the formulation of optimal control.	**BTL 1**	Remember
19.	Define optimization.	**BTL 1**	Remember
20.	What do you mean by performance index?	**BTL 2**	Understand
PART B			
1.	Write short notes on the following		
	i) Performance Index for Time-Optimal Control System **(3)**	**BTL 3**	Apply
	ii) Performance Index for Fuel-Optimal Control System **(5)**	**BTL 3**	Apply

	iii) Performance Index for Minimum-Energy Control System	**(5)**		
2.	Explain classical and modern control with single input and single output with block diagram.	**(13)**	**BTL 2**	Understand
3.	With block diagram explain the components of modern control system.	**(13)**	**BTL 1**	Remember
4.	Write short notes on the following			
	i) Performance Index for Terminal Control System	**(6)**	**BTL 1**	Remember
	ii) Performance Index for General Optimal Control System	**(7)**	**BTL 1**	Remember
5.	Discuss about optimization with neat block diagram.	**(13)**	**BTL 4**	Analyze
6.	Derive equation for Quadratic Performance Index with Linear Constraint.	**(13)**	**BTL 1**	Remember
7.	Derive equation for Quadratic Surface with Linear Constraint	**(13)**	**BTL 3**	Apply
8.	Describe about formal statement of optimal control System with block diagram.	**(13)**	**BTL 2**	Understand
9.	Discuss about the historical tour of optimal control in brief.	**(13)**	**BTL 1**	Remember
10.	Write the historical tour for calculas of variation in detail.	**(13)**	**BTL 5**	Evaluate
11.	Explain the effect of Changes in Constraints in brief.	**(13)**	**BTL 4**	Analyze
12.	Explain the necessary and sufficient conditions for Lagrange Multipliers and the Hamiltonian.	**(13)**	**BTL 4**	Analyze
13.	Write the historical tour for optimal control theory in detail.	**(13)**	**BTL 2**	Understand
14.	Explain optimization without constraints in detail.	**(13)**	**BTL 6**	Create
PART C				
1.	Generalize the steps in constrained minimization by the method of steepest descent.	**(15)**	**BTL 5**	Evaluate
2.	Formulate various performance index for different optimal control systems.	**(15)**	**BTL 5**	Evaluate
3.	Discuss about the requirement for the formulation of optimal control problem.	**(15)**	**BTL 6**	Create
4.	Derive equation for optimazation by scalar manipulation.	**(15)**	**BTL 6**	Create

UNIT II CALCULUS OF VARIATIONS

Basic concepts – variational problem - Extreme functions with conditions - variational approach to optimal control systems.

PART A

Q.No	**Question**	**BT Level**	**Competence**
1.	Distinguish between function and functional.	**BTL 4**	Analyze
2.	Write the Sufficient Condition to determine the nature of optimization.	**BTL 6**	Create
3.	What do you mdean by Calculus of variations?	**BTL 1**	Remember
4.	Examine differential of a function.	**BTL 3**	Apply
5.	Analyze Optimum of a Function.	**BTL 4**	Analyze
6.	Analyze Optimum of a Functional.	**BTL 4**	Analyze
7.	Define Increment of a Function.	**BTL 1**	Remember

8.	Evaluate Fixed-End Time System.		**BTL 5**	Evaluate
9.	Express Extrema of Functions with Conditions.		**BTL 2**	Understand
10.	Define Increment of a Functional.		**BTL 1**	Remember
11.	Evaluate Fixed-End State System.		**BTL 5**	Evaluate
12.	Define Fixed-Final Time and Free-Final State System.		**BTL 1**	Remember
13.	Examine variation of a function.		**BTL 3**	Apply
14.	List the Variational techniques to Optimal Control Systems.		**BTL 1**	Remember
15.	Express Extrema of Functionals with Conditions.		**BTL 2**	Understand
16.	Express Free-Final Time and Dependent Free-Final State System.		**BTL 2**	Understand
17.	Express Free-Final Time and Independent Free-Final State.		**BTL 2**	Understand
18.	What do you mean by second variation?		**BTL 3**	Apply
19.	List the Features of Lagrange Multiplier.		**BTL 1**	Remember
20.	Write Euler-Lagrange Equation.		**BTL 6**	Create
	PART B			
1.	Explain function and functional in calculas of variation with example.	**(13)**	**BTL 2**	Understand
2.	**i)** Find the increment of the function $f(t) = (t_1 + t_2)^2$	**(7)**	**BTL 1**	Remember
	ii) Find the increment of the functional $J = \int_{t_0}^{t_f} \left[2x^2(t) + 1\right] dt.$	**(6)**	**BTL 2**	Understand
3.	**i)** Find the increment and the derivative of the function j (t). $f(t) = t^2 + 2t.$	**(5)**	**BTL 5**	Evaluate
	ii) Given the functional $J(x(t)) = \int_{t_0}^{t_f} [2x^2(t) + 3x(t) + 4]dt,$ Evaluate the variation of the functional.	**(8)**	**BTL 4**	Analyze
4.	With example explain increment of a function in detail.	**(13)**	**BTL 5**	Evaluate
5.	Write equations for differential of a function with example.	**(13)**	**BTL 6**	Create
6.	Explain optimum of function with minimum and maximum of a function f(t).	**(13)**	**BTL 1**	Remember
7.	Explain Variational Approach to Optimal Control Systems.	**(13)**	**BTL 4**	Analyze
8.	With example explain increment of a functional in detail.	**(13)**	**BTL 3**	Apply
9.	Feneralize the steps involved in optimal solution to the fixed-end time and fixed-end state system	**(13)**	**BTL 4**	Analyze
10.	Design step by step procedure for Extrema of Functionals with Conditions.	**(13)**	**BTL 1**	Remember
11.	Write equations for variation of a function with example.	**(13)**	**BTL 3**	Apply
12.	Comment on Euler-Lagrange Equation in brief.	**(13)**	**BTL 2**	Understand
13.	Explain different cases for Euler-Lagrange Equation.	**(13)**	**BTL 1**	Remember
14.	Find the minimum length between any two points	**(13)**	**BTL 4**	Analyze

	using optimal control.		
	PART C		
1.	Find the optimum of **(15)** $$J = \int_0^2 \left[\dot{x}^2(t) - 2tx(t)\right] dt$$ that satisfy the boundary (initial and final) conditions x(0) = 1 and x(2) = 5.	**BTL 5**	Evaluate
2.	A manufacturer wants to maximize the volume of the material stored in a circular tank subject to the condition that the material used for the tank is limited (constant). Thus, for a constant thickness of the material, the manufacturer wants to minimize the volume of the material used and hence part of the cost for the tank. Estimate using Direct Method. **(15)**	**BTL 5**	Evaluate
3.	Discuss about Terminal Cost Problem in Variational Approach to Optimal Control Systems. **(15)**	**BTL 4**	Analyze
4.	Explain Different Types of Systems in Variational Approach to Optimal Control Systems. **(15)**	**BTL 4**	Analyze

UNIT III LINEAR QUADRATIC OPTIMAL CONTROL SYSTEM

Problem formulation - finite time LQR - infinite time LQR - Linear Quadratic tracking system – LQR with a specified degree of stability.

PART A

Q.No	**Question**	**BT Level**	**Competence**
1.	Define optimal control problem.	**BTL 1**	Remember
2.	How to linearize the cost function around the optimal solution?	**BTL 3**	Apply
3.	What is optimization?	**BTL 1**	Remember
4.	What is the necessary condition for the optimal value to be a local minimum?	**BTL 2**	Understand
5.	Define state space model.	**BTL 3**	Apply
6.	Write the energy equation of the control signal.	**BTL 2**	Understand
7.	Write the Riccati equation for the optimal state feedback.	**BTL 5**	Evaluate
8.	Define optimal state feedback.	**BTL 1**	Remember
9.	State Lyapunov equation.	**BTL 1**	Remember
10.	What is quadratic convergence?	**BTL 4**	Analyze
11.	What is linear quadratic problem?	**BTL 1**	Remember
12.	Define feasible control.	**BTL 2**	Understand
13.	Write the condition for the system to be completely controllable.	**BTL 6**	Create
14.	What is unique optimal control ?	**BTL 1**	Remember
15.	List out the constraints to perform the cost function using Lagrange's multipliers.	**BTL 2**	Understand
16.	Define nonlinear system with an example.	**BTL 4**	Analyze
17.	Write the necessary condition for a solution to be optimal using Pontryagin .	**BTL 6**	Create

18.	Draw the block diagram of state space model.		**BTL 4**	Analyze
19.	Define LQR criterion.		**BTL 5**	Evaluate
20.	What is the necessary condition for the optimal input?		**BTL 3**	Apply
	PART B			
1.	Discuss about the plant and the quadratic performance index with particular reference to physical significance.	**(13)**	**BTL 1**	Remember
2.	Explain various matrices in the cost functional and their implications	**(13)**	**BTL 5**	Evaluate
3.	With neat neat block diagram explain State and Costate System.	**(13)**	**BTL 2**	Understand
4.	Explain Symmetric Property of the Riccati Coefficient Matrix.	**(13)**	**BTL 3**	Apply
5.	Explain Finite-Time Linear Quadratic Regulator in brief.	**(13)**	**BTL 6**	Create
6.	With block diagram explain Closed-Loop Optimal Control Implementation.	**(13)**	**BTL 3**	Apply
7.	Derive equations for LQR System for General Performance Index	**(13)**	**BTL 1**	Remember
8.	Formulate Analytical Solution to the Matrix Differential Riccati Equation	**(13)**	**BTL 1**	Remember
9.	Derive equation for Infinite-Time LQR System I.	**(13)**	**BTL 2**	Understand
10.	Interpret Riccati Coefficient and find the minimum cost.	**(13)**	**BTL 4**	Analyze
11.	Discuss various Stability Issues of Time-Invariant Regulator.	**(13)**	**BTL 4**	Analyze
12.	Prove the theorm that the optimal value of the performance index (PI) is given by $J^*(\mathbf{x}^*(t),t) = \frac{1}{2}\mathbf{x}^{*\prime}(t)\mathbf{P}(t)\mathbf{x}^*(t).$	**(13)**	**BTL 2**	Understand
13.	Derive equation for Infinite-Interval Regulator System.	**(13)**	**BTL 3**	Apply
14.	Explain about Linear Quadratic Tracking System for Finite-Time Case.	**(13)**	**BTL 2**	Understand
	PART C			
1.	Derive equation for Infinite-Time LQR System II.	**(15)**	**BTL 4**	Analyze
2.	Discuss about the implementation of the Closed-Loop Optimal Control for Infinite Final Time.	**(15)**	**BTL 5**	Evaluate
3.	Given a second order plant $\dot{x}_1(t) = x_2(t), \quad x_1(0) = 2$ $\dot{x}_2(t) = -2x_1(t) + x_2(t) + u(t), \quad x_2(0) = -3$ and the performance index $J = \frac{1}{2}\int_0^\infty \left[2x_1^2(t) + 6x_1(t)x_2(t) + 5x_2^2(t) + 0.25u^2(t)\right] dt,$ Obtain the feedback optimal control law	**(15)**	**BTL 5**	Evaluate
4.	Discuss various steps involved in Finite-Time Linear Quadratic Regulator	**(15)**	**BTL 6**	Evaluate

UNIT IV DISCRETE TIME OPTIMAL CONTROL SYSTEM			
Variational calculus for DT system – DT optimal control system - DT linear state regulator system -- DT linear quadratic tracking system			
PART A			
Q.No	**Question**	**BT Level**	**Competence**
1.	Define the term variation calculus for discrete time system.	**BTL 4**	Analyze
2.	Write the steps involved in the optimization of a function.	**BTL 1**	Remember
3.	Demonstrate the term discrete-time version of the Euler-Lagrange equation.	**BTL 6**	Create
4.	Write the transversality condition.	**BTL 3**	Apply
5.	Define the fixed end point system.	**BTL 1**	Remember
6.	Demonstrate the free final point system.	**BTL 2**	Understand
7.	State the state and costate system.	**BTL 5**	Evaluate
8.	Explain the open loop optimal control.	**BTL 1**	Remember
9.	What is meant by performance index?	**BTL 1**	Remember
10.	Explain Kalman gain.	**BTL 3**	Apply
11.	Write the Kalman gain matrix.	**BTL 5**	Evaluate
12.	Define optimal cost function	**BTL 1**	Remember
13.	Expand DRE.	**BTL 2**	Understand
14.	State Frequency domain interpretation.	**BTL 3**	Apply
15.	Generalize the term closed loop optimal controller.	**BTL 4**	Analyze
16.	How do you derive the ricatti coefficient?	**BTL 6**	Create
17.	Draw the open-loop characteristics polynomial of the system.	**BTL 2**	Understand
18.	Define optimal state.	**BTL 4**	Analyze
19.	State the Hamilton Jacobi Bellman equation.	**BTL 2**	Understand
20.	Give some examples discrete time system.	**BTL 1**	Remember
PART B			
1.	Consider the minimization of a functional **(13)** $J(x(k_0), k_0) = J = \sum_{k=k_0}^{k_f-1} [x(k)x(k+1) + x^2(k)]$ subject to the boundary conditions x(0) = 2, and x(10) = 5.	**BTL 6**	Create
2.	Explain step by step procedure for Discrete-Time Optimal Control Systems. **(13)**	**BTL 6**	Create
3.	For a second order system **(13)** $\dot{x}_1(t) = x_2(t)$ $\dot{x}_2(t) = -2x_1(t) + 3u(t)$ with performance index $J = 0.5x_1^2(\pi/2) + \int_0^{\pi/2} 0.5u^2(t)dt$ and boundary conditions $\mathbf{x}(0) = [0 \;\; 1]'$ and $\mathbf{x}(t_f)$ is free, find the optimal control.	**BTL 2**	Understand

4.	Explain Extremization of a Functional in Variational Calculus for Discrete-Time Systems	**(13)**	**BTL 4**	Analyze
5.	(i) Explain Functional with Terminal Cost. (ii) Derive th equation for Functional with Terminal Cost. **(7)**	**(6)**	**BTL 5**	Evaluate
6.	Explain Fixed-Final State control with example. Explain Open-Loop Optimal Control with example.**(6)**	**(7)**	**BTL 1**	Remember
7.	Derive equation for Discrete-Time Linear State Regulator System	**(13)**	**BTL 3**	Apply
8.	Derive equation for Closed-Loop Optimal Control by Matrix Difference Riccati Equation	**(13)**	**BTL 1**	Remember
9.	With block diagram explain Closed-Loop Optimal Controller for Linear Discrete-Time Regulator	**(13)**	**BTL 3**	Apply
10.	(i) Derive equation for Steady-State Regulator System. (ii) Draw the block diagram for Steady-State regukator system.**(8)**	**(5)**	**BTL 5**	Evaluate
11.	Find the analytical Solution to the Riccati Equation	**(13)**	**BTL 4**	Analyze
12.	Derive equation for Discrete-Time Linear Quadratic Tracking System	**(13)**	**BTL 5**	Evaluate
13.	With block diagram describe about Implementation of Discrete-Time Optimal Tracker	**(13)**	**BTL 5**	Evaluate
14.	With block diagram explain about Closed-Loop Optimal Control for Discrete-Time Steady-State Regulator System	**(13)**	**BTL 4**	Analyze
PART C				
1.	Explain Free-Final State and Open-Loop Optimal Control	**(15)**	**BTL 6**	Evaluate
2.	Estimate frequency domain interpretation.	**(15)**	**BTL 5**	Evaluate
3.	Evaluate Optimal Cost Function or Linear Discrete-Time Regulator	**(15)**	**BTL 5**	Evaluate
4.	Consider the minimization of the performance index (PI) $J(k_0) = \frac{1}{2}\sum_{k=k_0}^{k_f-1} u^2(k),$ subject to the boundary conditions $x(k_0 = 0) = 1, \quad x(k_f = 10) = 0$ for a simple scalar system $x(k+1) = x(k) + u(k).$	**(15)**	**BTL 5**	Evaluate

UNIT V PONTRYAGIN MINIMUM PRINCIPLE
Pontryagin minimum principle - Dynamic programming – Hamilton - Jacobi - Bellman equation - LQR system using HJB equation – Time optimal control – fuel optimal control system optimal Control system with constraints.
PART A

Q.No	Question	BT Level	Competence
1.	Define Pontryagin Minimum Principle.	**BTL 1**	Remember
2.	Define multistage process.	**BTL 3**	Apply
3.	Give some example for constrained system.	**BTL 3**	Apply
4.	Write the three relation for control, state and costate.	**BTL 5**	Evaluate
5.	Formulate the formulation of the Hamiltonian.	**BTL 1**	Remember
6.	Draw the diagram of an optimal control function constrained by boundary.	**BTL 4**	Analyze
7.	Define the principle of optimality.	**BTL 1**	Remember
8.	What is meant by dynamic programming?	**BTL 2**	Understand
9.	Give an example for backward solution for multistage decision process.	**BTL 5**	Evaluate
10.	Define forward solution.	**BTL 2**	Understand
11.	Demonstrate the optimal control system with constraint.	**BTL 4**	Analyze
12.	Write the HJB equation.	**BTL 3**	Apply
13.	What is the role of HJB equation?	**BTL 6**	Create
14.	Show the Bellmen equation.	**BTL 1**	Remember
15.	Write the steps involved for the solution of the TOC system.	**BTL 6**	Create
16.	Define the term Time-optimal control. is meant the STOC.	**BTL 2**	Understand
17.	What is meant the STOC.	**BTL 1**	Remember
18.	What is the condition for Normal Time-optimal control system?	**BTL 2**	Understand
19.	State Bang-Bang control law.	**BTL 4**	Analyze
20.	Classify the types of Time-optimal control.	**BTL 1**	Remember

PART B				
1.	Minimizing a scalar function $H = u^2 - 6u + 7$ subject to the constraint relation $\|u\| \leq 2, \longrightarrow -2 \leq u \leq +2.$	**(13)**	**BTL 4**	Analyze
2.	Illustrate Constrained (Admissible) Controls with a graph	**(13)**		
3.	Write step by step procedure for the Principle of Optimality.	**(13)**	**BTL 5**	Evaluate
4.	Find the Hamilton-Jacobi-Bellman equation for the system $\dot{x}_1(t) = x_2(t)$ $\dot{x}_2(t) = -2x_2(t) - 3x_1^2(t) + u(t)$ with the performance index as $J = \frac{1}{2}\int_0^{t_f} \left(x_1^2(t) + u^2(t)\right) dt.$	**(13)**	**BTL 4**	Analyze
5.	Discuss about the important stages in obtaining optimal control for the Constrained system	**(13)**	**BTL 6**	Create
6.	Derive equation for Pontryagin Minimum Principle.	**(13)**	**BTL 6**	Create
7.	Explain Optimal Control Using Dynamic Programming.	**(13)**	**BTL 1**	Remember
8.	Discuss about Optimal Control of Discrete-Time	**(13)**	**BTL 3**	Apply

	Systems.			
9.	(i) Draw the block diagram optimal contrl of Continuos-Time systems. **(5)** (ii) Discuss about Optimal Control of Continuou Time Systems .**(8)**		**BTL 4**	Analyze
10.	Derive Hamilton-Jacobi-Bellman Equation.	**(13)**	**BTL 3**	Apply
11.	(i) Write the necessity of H_J_B in LQR system. (ii) Obtain LQR System Using H-J-B Equation.**(8)**	**(5)**	**BTL 4**	Analyze
12.	Find the extremal of the functional $J = \int_{-2}^{0} \left[12tx(t) + \dot{x}^2(t)\right] dt$ to satisfy the boundary conditions $x(-2) = 3$, and $x(0) = 0$.	**(13)**	**BTL 2**	Understand
13.	Find the extremal for the following functional $J = \int_{1}^{2} \frac{\dot{x}^2(t)}{2t^3} dt$ With $x(1) = 1$ and $x(2) = 10$.	**(13)**	**BTL 2**	Understand
14.	Find the optimal control u*(t) of the plant $\dot{x}_1(t) = x_2(t); \quad x_1(0) = 3, \quad x_1(2) = 0$ $\dot{x}_2(t) = -2x_1(t) + 5u(t); \quad x_2(0) = 5, \quad x_2(2) = 0$ which minimizes the performance index $J = \frac{1}{2}\int_{0}^{2} \left[x_1^2(t) + u^2(t)\right] dt.$	**(13)**	**BTL 4**	Analyze
	PART C			
1.	Given a first-order system $\dot{x}(t) = -2x(t) + u(t)$ and the performance index (PI) $J = \frac{1}{2}x^2(t_f) + \frac{1}{2}\int_{0}^{t_f} [x^2(t) + u^2(t)]dt$ find the optimal control.	**(15)**	**BTL 5**	Evaluate
2.	Find the closed-loop optimal control for the first-order system $\dot{x}(t) = -2x(t) + u(t)$ with the performance index $J = \int_{0}^{\infty} \left[x^2(t) + u^2(t)\right] dt.$ Hint: Assume that $J^* = fx^2(t).$	**(15)**	**BTL 6**	Evaluate
3.	A mechanical system is described by $\ddot{x}(t) = u(t)$ find the optimal control and the states by minimizing $J = \frac{1}{2}\int_{0}^{5} u^2(t)dt$ such that the boundary conditions are	**(15)**	**BTL 6**	Evaluate

	$x(t=0)=2; \quad x(t=5)=0; \quad \dot{x}(t=0)=2; \quad \dot{x}(t=5)=0.$			
4.	Prove the Pontryagin Minimum Principle based on the works of Athans and Falb, Lee and Markus, Machki and Strauss and some of the recent works Pinch and Hocking.	**(15)**	**BTL 6**	Evaluate