

# **SRM VALLIAMMAI ENGINEERING COLLEGE**

(An Autonomous Institution)

ESTD. 1999 - Approved by AICTE - Accredited by NBA  
'A' Grade Accreditation by NAAC - Affiliated to Anna University  
ISO 9001:2015 Certified Institution

## **DEPARTMENT OF MATHEMATICS**

### **QUESTION BANK**



### **I SEMESTER**

**(COMMON TO ALL BRANCHES)**

**1918102 - ENGINEERING MATHEMATICS-I**

**Regulation – 2019**

**Academic Year 2021- 2022**

*Prepared by*

**Ms.B.Aarthi, Assistant Professor / Mathematics**

**Ms. B. Vasuki, Assistant Professor / Mathematics**

**Mr.D.Captain Prabakaran, Assistant Professor/ Mathematics**

## QUESTION BANK

**SUBJECT : 1918102 – Engineering Mathematics - I**

**YEAR /SEMESTER: I Year / I Semester B.E./ B.Tech.**

**(Common to all Branches)**

| <b>UNIT I MATRICES</b>  |   |           |               |
|---|---|-----------|---------------|
| System of Equations – Consistency and inconsistency - Eigen values and Eigenvectors of a real matrix – Characteristic equation – Properties of Eigen values and Eigenvectors – Statement and application Cayley-Hamilton theorem– Reduction of a quadratic form to canonical form by orthogonal transformation. |   |           |               |
| Q.N<br>o.   | Question  | BT -Level | Competence    |
| <b>PART-A</b>   |   |           |               |
| 1   | Find the characteristic equation of $A = \begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$  | BTL-1     | Remembering   |
| 2   | Find the eigen values of $A^2$ if $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$  | BTL-1     | Remembering   |
| 3   | If the eigen values of the matrix A of order 3X3 are 2,3 and 1, then find the determinant of A  | BTL-2     | Understanding |
| 4   | If the sum of 2 eigen values and the trace of a 3×3 matrix are equal, find the value of  A  | BTL-1     | Remembering   |
| 5   | Prove that sum of eigen values of a matrix is equal to its trace.   | BTL-3     | Applying      |
| 6   | Prove that the eigen values of $A^{-1}$ are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \dots, \frac{1}{\lambda_n}$   | BTL-3     | Applying      |
| 7   | Find the sum of the eigen values of 2A, if $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$  | BTL-3     | Applying      |
| 8   | The product of the 2 eigen values of $A = \begin{pmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 14. Find the 3 <sup>rd</sup> eigen value.                   | BTL-1     | Remembering   |
| 9   | Find the sum and product of the eigen values of $A = \begin{pmatrix} 2 & -2 & 2 \\ -2 & -1 & -1 \\ 2 & -1 & -1 \end{pmatrix}$   | BTL-1     | Remembering   |
| 10  | Find the sum and product of the eigen values of $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$   | BTL-1     | Remembering   |
| 11  | Find the constants a and b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3,-2 be the eigen values of A  | BTL-4     | Analyzing     |
| 12  | State Cayley-Hamilton theorem.  | BTL-2     | Understanding |
| 13  | Write any 2 applications of Cayley Hamilton theorem   | BTL-2     | Understanding |
| 14  | Use Cayley Hamilton theorem to find $A^{-1}$ if $A = \begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}$   | BTL-2     | Understanding |
| 15  | If $A = \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$ find $A^3$ using Cayley Hamilton theorem  | BTL-2     | Understanding |
| 16  | Find the matrix corresponding to the quadratic form $2xy - 2yz + 2xz$ .   | BTL-4     | Analyzing     |
| 17  | Find the quadratic form corresponding to the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$   | BTL-5     | Evaluating    |
| 18  | Find the matrix corresponding to the quadratic form $x^2 + y^2 + z^2$   | BTL-5     | Evaluating    |
| 19  | Define Index, Signature and Rank.   |           |               |
| 20  | Find the symmetric matrix A, whose eigen values are 1 and 3 with corresponding eigen vectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ | BTL-6     | Creating      |
| <b>PART-B</b>   |   |           |               |

|               |  |       |               |
|---------------|--|-------|---------------|
| 1             | Test for the consistency of the following system of equations and solve them, if consistent $3x + y + z = 8, -x + y - 2z = -5, x + y + z = 6, -2x + 2y - 3z = -7$ .  | BTL-5 | Evaluating    |
| 2             | Examine the consistency of the equations $x + y + z = 3, 2x - y + 3z = 4, 5x - y + 7z = 11$ .  | BTL-5 | Evaluating    |
| 3             | Investigate for the value of $\lambda, \mu$ the system of equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ have (i) Unique solution, (ii) Infinitely many solution, (iii) No solution | BTL-6 | Creating      |
| 4             | Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$   | BTL-2 | Understanding |
| 5             | Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$   | BTL-2 | Understanding |
| 6             | Obtain the eigen values and eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ .   | BTL-4 | Analyzing     |
| 7             | Obtain the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$  | BTL-5 | Evaluating    |
| 8             | Find the Characteristic equation of the matrix $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence find $A^4$ .  | BTL-4 | Analyzing     |
| 9             | Verify Cayley-Hamilton theorem and hence find $A^{-1}$ of $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$  | BTL-1 | Remembering   |
| 10            | Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ and also find $A^{-1}$ .   | BTL-4 | Analyzing     |
| 11            | Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal reduction.   | BTL-5 | Evaluating    |
| 12            | Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form by an orthogonal reduction.   | BTL-2 | Understanding |
| 13            | Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ into canonical form by an orthogonal reduction.   | BTL-1 | Remembering   |
| 14            | Reduce the quadratic form $2x_1^2 + 5x_2^2 + 3x_3^2 + 4x_1x_2$ into canonical form by an orthogonal reduction.   | BTL-1 | Remembering   |
| <b>Part C</b> |  |       |               |
| 1.            | Diagonalize the matrix $A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$ by means of an orthogonal transformation.   | BTL-6 | Creating      |
| 2.            | Determine the nature of the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ by reducing it into canonical form by orthogonal transformation  | BTL-6 | Creating      |
| 3.            | Determine the nature of the quadratic form $2xy - 2yz + 2xz$ by reducing it into canonical form by orthogonal transformation   | BTL-6 | Creating      |
| 4.            | The Eigen vectors of a $3 \times 3$ real symmetric matrix A corresponding to the eigen values 1,2,4 are $(1, 0, 0)^T, (0, 1, 1)^T, (0, 1, -1)^T$ respectively. Find the matrix A.                          | BTL-6 | Creating      |

### UNIT II DIFFERENTIAL CALCULUS

Limit of a function - Continuity – Differentiability - Differentiation rules – Roll’s Theorem and Mean Value Theorem – Taylor’s Series - Maxima and Minima of functions of one variable.

| Q.N<br>o.       | Question  | BTLevel | Domain        |
|-----------------|---|---------|---------------|
| <b>PART – A</b> |   |         |               |
| 1.              | Show that $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ .   | BTL -1  | Remembering   |
| 2.              | Check whether $\lim_{x \rightarrow -3} \frac{3x+9}{ x+3 }$ exist  | BTL -4  | Analyzing     |
| 3.              | Find $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ .  | BTL -2  | Understanding |
| 4.              | Use the squeeze theorem to show that $\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x} = 0$   | BTL -3  | Applying      |
| 5.              | Predict the values of a and b so that the function f given by<br>$f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax + b & \text{if } 3 < x < 5 \\ 7 & \text{if } x \geq 5 \end{cases}$ is continuous at x=3 and x=5.   | BTL -2  | Understanding |
| 6.              | If the function $f(x) = \begin{cases} \frac{x^2-16}{x-4} & \text{if } x \neq 4 \\ c & \text{if } x = 4 \end{cases}$ is continuous, what is the value of c?  | BTL -6  | Creating      |
| 7.              | Point out $\frac{dy}{dx}$ , if $y = \ln \cos(\ln x) $ .   | BTL -4  | Analyzing     |
| 8.              | Calculate $\frac{d}{dx}((x)^{\sqrt{x}})$  | BTL -3  | Applying      |
| 9.              | Compute $\frac{d}{dx}((x)^{\sin x})$  | BTL -3  | Applying      |
| 10.             | Evaluate $\frac{d}{dx}((\sin x)^{\ln x})$   | BTL -5  | Evaluating    |
| 11.             | Where the function $f(x) =  x $ is differentiable?  | BTL -2  | Understanding |
| 12.             | Estimate $\frac{d}{dx}((\sin x)^{\cos x})$  | BTL -2  | Understanding |
| 13.             | Estimate $y'$ if $x^3 + y^3 = 6xy$  | BTL -2  | Understanding |
| 14.             | Using Rolle's theorem find the value of c for the function<br>$f(x) = \sqrt{1-x^2}, -1 \leq x \leq 1$   | BTL -2  | Understanding |
| 15.             | Verify Lagrange's law for the function $f(x) = \frac{1}{x}, [1,2]$  | BTL -2  | Understanding |
| 16.             | Using Rolle's theorem find the value of c for the function<br>$f(x) = (x-a)(b-x), a \leq x \leq b, a \neq b$  | BTL -3  | Applying      |
| 17.             | Verify Lagrange's law for the function $f(x) = x^3, [-2,2]$   | BTL -4  | Analyzing     |
| 18.             | Find the Taylor's series expansion of the function $f(x) = \sin x$ about the point $x = \frac{\pi}{2}$  | BTL -2  | Understanding |
| 19.             | Find the critical numbers of the function $f(x) = 2x^3 - 3x^2 - 36x$  | BTL -3  | Applying      |
| 20.             | Find the critical points of $y = 5x^2 - 6x$   | BTL -3  | Applying      |
| <b>PART – B</b> |   |         |               |
| 1.(a)           | Point out the domain where the function f is continuous Also find the number at which the function f is discontinuous when<br>$f(x) = \begin{cases} 1 + x^2 & \text{if } x \leq 0 \\ 2 - x & \text{if } 0 < x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$ | BTL -4  | Analyzing     |
| 1. (b)          | Find $\frac{dy}{dx}$ if $y = x^2 e^{2x} (x^2 + 1)^4$ .  | BTL -3  | Applying      |
| 2.(a)           | Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection and local maxima and minima.   | BTL -1  | Remembering   |
| 2.(b)           | Obtain $y''$ if $x^4 + y^4 = 16$  | BTL -2  | Understanding |

|               |  |        |               |
|---------------|--|--------|---------------|
| 3. (a)        | For what value of the constant “c” is the function “f” continuous on $(-\infty, \infty)$ , $f(x) = \begin{cases} cx^2 + 2x; & x < 2 \\ x^3 - cx; & x \geq 2 \end{cases}$ | BTL -4 | Analyzing     |
| 3.(b)         | Find $y'$ for $\cos(xy) = 1 + \sin y$ .  | BTL -3 | Applying      |
| 4. (a)        | Find the absolute maximum and minimum of $f(x) = x - 2 \tan^{-1} x$ in $[0,4]$ .   | BTL -1 | Remembering   |
| 4.(b)         | Verify Lagrange’s law for the following $f(x) = 2x^3 + x^2 - x - 1, x \in [0,2]$ .   | BTL -4 | Analyzing     |
| 5. (a)        | Find $y'$ if $x = a \left( \cos\theta + \log \tan \frac{\theta}{2} \right), y = a \sin\theta$ .  | BTL -1 | Remembering   |
| 5.(b)         | Verify Rolle’s theorem for the following $f(x) = 2x^3 - 5x^2 - 4x + 3, x \in \left[ \frac{1}{2}, 3 \right]$ .  | BTL -4 | Analyzing     |
| 6. (a)        | Verify Lagrange’s law for the following $f(x) = 2x^2 - 4x - 3, x \in [1,4]$ .  | BTL -1 | Remembering   |
| 6.(b)         | Use second derivative test to examine the relative maxima for $f(x) = x(12 - 2x)^2$  | BTL -4 | Analyzing     |
| 7.            | Find the tangent line to the equation $x^3 + y^3 = 6xy$ at the point (3,3) and at what point the tangent line is horizontal in the first quadrant                        | BTL -3 | Applying      |
| 8.            | Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing. Also find the local maximum and local minimum of $f(x)$ .             | BTL -3 | Applying      |
| 9. (a)        | Verify mean value theorem for the following $f(x) = x^3 - 5x^2 - 3x, x \in [1,3]$ .  | BTL -4 | Analyzing     |
| 9.(b)         | Find the Taylor’s series expansion of $f(x) = \frac{1}{1+x}$ about $x=0$ .   | BTL -3 | Applying      |
| 10.(a)        | Find $\frac{dy}{dx}$ , when $y = \frac{a \cos x + b \sin x}{b \cos x - a \sin x}$ .  | BTL -4 | Analyzing     |
| 10.(b)        | Verify Rolle’s theorem for the following function $f(x) = \sin x, 0 \leq x \leq \pi$   | BTL -3 | Applying      |
| 11.           | If $f(x) = 2x^3 + 3x^2 - 36x$ , find the intervals on which it is increasing or decreasing, local maximum and minimum values of $f(x)$ .                                 | BTL -4 | Analyzing     |
| 12.(a)        | Find the Taylor’s series expansion of $f(x) = \tan^{-1} x$ about $x=0$ .   | BTL -3 | Applying      |
| 12.(b)        | Find $y''$ if $x^4 + y^4 = 16$ .   | BTL -2 | Understanding |
| 13.           | Examine the local extreme of $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$ . Also discuss the concavity and find the inflection points   | BTL -3 | Applying      |
| 14.(a)        | Find $\frac{dy}{dx}$ for the following functions $e^x + e^y = e^{x+y}$ .   | BTL -3 | Applying      |
| 14.(b)        | Verify Rolle’s theorem for the following $f(x) = x(x - 1)(x - 2), x \in [0,2]$ .   | BTL -3 | Applying      |
| <b>Part C</b> |  |        |               |
| 1.            | Find the point on the parabola $y^2 = 2x$ that is close to the point (1,4)   | BTL -3 | Applying      |
| 2.            | Find the equation of tangent at a point (a, b) to the curve $xy = c^2$ .   | BTL -5 | Evaluating    |
| 3.(a)         | Apply Rolle’s theorem to find points on curve $y = -1 + \cos x$ , where the tangents is parallel to x- axis in $0 \leq x \leq 2\pi$                                      | BTL -6 | Creating      |
| 3(b)          | At what points on the curve $x^2 - y^2 = 2$ , the slopes of tangents are equal to 2.   | BTL -5 | Evaluating    |
| 4.            | A cylindrical hole 4mm in diameter and 12mm deep in a metal block is re bored to increase the diameter to 4.12mm. Estimate the amount of metal removed.                  | BTL -6 | Creating      |

### UNIT-III FUNCTIONS OF SEVERAL VARIABLES

Partial derivatives – Total derivative -Jacobians and properties - Taylor’s series for functions of two variables – Maxima and minima of functions of two variables – Lagrange’s method of undetermined multipliers

| Q.No | Question | BTLevel | Domain |
|------|----------|---------|--------|
|------|----------|---------|--------|

### PART – A

|     |   |        |               |
|-----|---|--------|---------------|
| 1.  | If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ , then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ .      | BTL -1 | Remembering   |
| 2.  | Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3ax^2y$  | BTL -1 | Remembering   |
| 3.  | If $x^y + y^x = 1$ , then find $\frac{dy}{dx}$ .  | BTL -1 | Remembering   |
| 4.  | Find the value of $\frac{du}{dt}$ , given $u = \log(x + y + z)$ , $x = e^t$ ,<br>$y = \sin t$ , $z = \cos t$  | BTL -2 | Understanding |
| 5.  | Find the value of $\frac{du}{dt}$ , given $u = x^2 + y^2$ , $x = at^2$ , $y = 2at$ .  | BTL -1 | Remembering   |
| 6.  | If $u = x^3y^2 + x^2y^3$ where $x = at^2$ and $y = 2at$ , then find $\frac{du}{dt}$ .   | BTL -3 | Applying      |
| 7.  | Find $\frac{du}{dt}$ if $u = \sin\left(\frac{x}{y}\right)$ , where $x = e^t$ , $y = t^2$  | BTL -3 | Applying      |
| 8.  | Find $\frac{du}{dt}$ if $u = \frac{x}{y}$ , where $x = e^t$ , $y = \log t$ .  | BTL -2 | Understanding |
| 9.  | Find $\frac{\partial r}{\partial x}$ , if $x = r \cos \theta$ & $y = r \sin \theta$ .   | BTL -3 | Applying      |
| 10. | Find the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$ , if $x = r \cos \theta$ & $y = r \sin \theta$ ,<br>$u = 2xy$ , $v = x^2 - y^2$ without actual substitution. | BTL -4 | Analyzing     |
| 11. | If $u = \frac{y^2}{2x}$ and $v = \frac{x^2+y^2}{2x}$ , find $\frac{\partial(u,v)}{\partial(x,y)}$ .   | BTL -3 | Applying      |
| 12. | If $x = uv$ , $y = \frac{u}{v}$ . Find $\frac{\partial(x,y)}{\partial(u,v)}$ .  | BTL -1 | Remembering   |
| 13. | If $x = u^2 - v^2$ , $y = 2uv$ find the Jacobian of $x, y$ with respect to $u$ and $v$  | BTL -2 | Understanding |
| 14. | If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$ , find $\frac{\partial(u,v)}{\partial(x,y)}$  | BTL -3 | Applying      |
| 15. | Find the Taylor series expansion of $x^y$ near the point $(1, 1)$ up to first term  | BTL -2 | Understanding |
| 16. | Expand $xy + 2x - 3y + 2$ in powers of $(x - 1)$ & $(y + 2)$ , using Taylor's theorem up to first degree form   | BTL -3 | Applying      |
| 17. | Find the Stationary points of<br>$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ .  | BTL -4 | Analyzing     |
| 18. | Find the Stationary points of $x^2 - xy + y^2 - 2x + y$ .   | BTL -4 | Analyzing     |
| 19. | State the Sufficient condition for $f(x, y)$ to be extremum at a point  | BTL -4 | Analyzing     |
| 20. | Find the minimum point of $f(x, y) = x^2 + y^2 + 6x + 12$ .   | BTL -4 | Analyzing     |

**PART - B**

|       |   |        |               |
|-------|---|--------|---------------|
| 1.(a) | Find $\frac{du}{dx}$ if $u = \log(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$ ,   | BTL -2 | Understanding |
| 1.(b) | If $u = \frac{yz}{x}$ , $v = \frac{zx}{y}$ and $w = \frac{xy}{z}$ , find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ .  | BTL -3 | Analyzing     |
| 2.(a) | Find the Jacobian of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ of the transformation<br>$x = r \sin \theta \cos \phi$ , $y = r \sin \theta \sin \phi$ , $z = r \cos \theta$ | BTL -2 | Understanding |
| 2.(b) | Expand $x^3y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ using Taylor's series up to third degree terms   | BTL -3 | Applying      |
| 3.(a) | If $u = f(x - y, y - z, z - x)$ , then show that<br>$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$                             | BTL -4 | Analyzing     |
| 3.(b) | If $x + y + z = u$ , $y + z = uv$ , $z = uvw$ , prove that<br>$\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2v$  | BTL -2 | Understanding |
| 4.(a) | Find $\frac{du}{dx}$ if $u = \cos(x^2 + y^2)$ and $a^2x^2 + b^2y^2 = c^2$   | BTL -4 | Analyzing     |
| 4.(b) | A flat circular plate is heated so that the temperature at any point $(x, y)$ is<br>$u(x, y) = x^2 + 2y^2 - x$ . Find the coldest point on the plate                                | BTL -3 | Analyzing     |
| 5.    | Find the shortest distance from the origin to the hyperbola<br>$x^2 + 8xy + 7y^2 = 225$   | BTL -4 | Applying      |
| 6.(a) | If $u = x + y + z$ , $u^2v = y + z$ and $u^3w = z$ Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1}{u^5}$  | BTL -3 | Analyzing     |
| 6.(b) | If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ , then  | BTL -3 | Analyzing     |

|        |  |        |               |
|--------|--|--------|---------------|
|        | find $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z}$ |        |               |
| 7. (a) | Find the extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ .   | BTL -3 | Analyzing     |
| 7. (b) | Expand $e^x \log(1 + y)$ in powers of $x$ & $y$ up to terms of third-degree using Taylor's series  | BTL -2 | Understanding |
| 8.     | Discuss the maxima and minima of $f(x, y) = x^3 y^2 (1 - x - y)$ .   | BTL -1 | Remembering   |
| 9. (a) | Expand $\tan^{-1} \frac{y}{x}$ in the neighborhood of (1, 1) as Taylor's series up to second degree terms.                               | BTL -3 | Applying      |
| 9.(b)  | Find the Maximum value of $x^m y^n z^p$ when $x + y + z = a$ .   | BTL -3 | Applying      |
| 10.(a) | Find the Taylors series expansion of $e^x \sin y$ at the point $(-1, \frac{\pi}{4})$ up to the second-degree terms                       | BTL -4 | Applying      |
| 10.(b) | Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$ .  | BTL -2 | Understanding |
| 11.(a) | Expand $e^{xy}$ in powers of $(x - 1)$ and $(y - 1)$ upto second degree terms by Taylor's series   | BTL -4 | Applying      |
| 11.(b) | Examine $f(x, y) = x^3 + 3xy^2 - 15x^2 + 72x$ for extreme values   | BTL -2 | Understanding |
| 12.(a) | Expand Taylor's series of $x^3 + y^3 + xy^2$ in powers of $(x - 1)$ and $(y - 2)$ up to the second-degree terms.                         | BTL -5 | Evaluating    |
| 12.(b) | If $u = xyz, v = x^2 + y^2 + z^2$ and $w = x + y + z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$                                | BTL -3 | Applying      |
| 13.    | Find the shortest and longest distances from the point (1,2,-1) to the sphere $x^2 + y^2 + z^2 = 24$                                     | BTL -3 | Applying      |
| 14.    | Find the dimension of the rectangular box without a top of maximum capacity, whose surface area is 108 sq. cm.                           | BTL -3 | Applying      |

### Part C

|    |  |               |            |
|----|--|---------------|------------|
| 1. | Divide the number 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum                                    | <b>BTL -6</b> | Creating   |
| 2. | The temperature at any point $(x, y, z)$ in space is given by $T = kxyz^2$ , where $k$ is constant. Find the height temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$ | <b>BTL -5</b> | Evaluating |
| 3. | A rectangular box open at the top is to have a volume 32cc. Find the dimensions of the box that requires the least for its construction.   | <b>BTL -4</b> | Applying   |
| 4. | Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .                        | <b>BTL -3</b> | Applying   |

### UNIT IV INTEGRAL CALCULUS

Definite and Indefinite integrals - Substitution rule - Techniques of Integration - Integration by parts, Trigonometric integrals, Trigonometric substitutions, Integration by partial fraction, - Improper integrals.

| Q.No            | Question   | BTLevel | Domain      |
|-----------------|--|---------|-------------|
| <b>PART - A</b> |  |         |             |
| 1.              | Prove that the following integral by interpreting each in terms of areas $\int_a^b x dx = \frac{b^2 - a^2}{2}$ | BTL -1  | Remembering |
| 2.              | State fundamental theorem of calculus  | BTL -1  | Remembering |
| 3.              | Evaluate $\int_0^1 \sqrt{1 - x^2} dx$ in terms of areas.   | BTL -5  | Evaluating  |
| 4.              | If $f$ is continuous and $\int_0^4 f(x) dx = 10$ , find $\int_0^2 f(2x) dx$                                    | BTL -5  | Evaluating  |
| 5.              | Evaluate the integral $\int_a^b x dx$ by using Riemann sum method  | BTL -5  | Evaluating  |
| 6.              | Calculate $\int \frac{x^3}{\sqrt{4+x^2}} dx$   | BTL -3  | Applying    |

|                |   |        |               |
|----------------|---|--------|---------------|
| 7.             | Calculate $\int \sqrt{1+x^2} x^5 dx$  | BTL -3 | Applying      |
| 8.             | Find $\int \sqrt{2x+1} dx$  | BTL -3 | Applying      |
| 9.             | Find $\int \frac{x}{\sqrt{1-4x^2}} dx$  | BTL -3 | Applying      |
| 10.            | Evaluate $\int_0^1 \tan^{-1} x dx$  | BTL -5 | Evaluating    |
| 11.            | Calculate $\int \frac{(\ln x)^2}{x} dx$   | BTL -3 | Applying      |
| 12.            | Calculate $\int (\log x)^2 dx$  | BTL -3 | Applying      |
| 13.            | Evaluate $\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$   | BTL -2 | Understanding |
| 14.            | Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x} dx$   | BTL -5 | Evaluating    |
| 15.            | Evaluate $\int_4^{\infty} \frac{1}{\sqrt{x}} dx$ and determine whether it is convergent or divergent.                       | BTL -5 | Evaluating    |
| 16.            | Evaluate $\int_0^1 e^{-x^2} dx$   | BTL -5 | Evaluating    |
| 17.            | Estimate $\int_1^3 \sqrt{x^2+3} dx$   | BTL -5 | Evaluating    |
| 18.            | Evaluate the improper integral $\int_2^3 \frac{dx}{\sqrt{3-x}}$ , if possible.  | BTL -5 | Evaluating    |
| 19.            | Find $\int_2^5 \frac{dx}{\sqrt{x-2}}$   | BTL -3 | Analyzing     |
| 20.            | Prove that $\int_1^{\infty} \frac{1}{x} dx$ is divergent.   | BTL -1 | Remembering   |
| <b>PART -B</b> |   |        |               |
| 1.(a)          | Evaluate $\int \frac{(\ln x)^2}{x^2} dx$  | BTL -5 | Evaluating    |
| 1. (b)         | Calculate $\int \frac{1}{\sqrt{a^2-x^2}} dx$ , by using trigonometric substitution.   | BTL -3 | Applying      |
| 2.             | Find $\int x^3 \sqrt{9-x^2} dx$ by trigonometric substitution.  | BTL -3 | Applying      |
| 3.             | Evaluate $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3 \cos x + 2} dx$ .  | BTL -5 | Evaluating    |
| 4.             | Evaluate $\int e^{ax} \cos bx dx$ using integration by parts  | BTL -3 | Applying      |
| 5.             | Evaluate $\int e^{ax} \sin bx dx, a > 0$ using integration by parts.  | BTL -3 | Applying      |
| 6. (a)         | Prove that $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx (n \neq 1)$ .                                | BTL -5 | Evaluating    |
| 6.(b)          | Find $\int \frac{\sec^2 x}{\tan^2 x + 3 \tan x + 2} dx$   | BTL -3 | Applying      |
| 7.             | Evaluate $\int x \tan^{-1} x dx$  | BTL -3 | Applying      |
| 8.(a)          | Calculate by partial fraction $\int \frac{x^2+1}{(x-3)(x-2)^2} dx$  | BTL -3 | Applying      |
| 8.(b)          | Evaluate $\int \frac{\tan x}{\sec x + \cos x} dx$   | BTL -5 | Evaluating    |
| 9. (a)         | Evaluate $\int \frac{x e^{2x}}{(1+2x)^2} dx$  | BTL -5 | Evaluating    |
| 9.(b)          | Calculate using partial fraction $\int \frac{10}{(x-1)(x^2+9)} dx$  | BTL -3 | Applying      |
| 10.(a)         | Evaluate $\int \sin^6 x \cos^3 x dx$ .  | BTL -5 | Evaluating    |
| 10.(b)         | For what values of p is $\int_1^{\infty} \frac{1}{x^p} dx$ convergent?  | BTL -5 | Evaluating    |
| 11.(a)         | Evaluate $\int_0^{\pi/2} \sin^7 x \cos^5 x dx$  | BTL -5 | Evaluating    |
| 11.(b)         | Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x dx$   | BTL -6 | Creating      |
| 12.            | Prove the reduction formula<br>$\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$ . | BTL -1 | Remembering   |
| 13.            | Prove the reduction formula   | BTL -1 | Remembering   |



|               |  |        |             |
|---------------|--|--------|-------------|
|               | $\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$  |        |             |
| 14.           | Prove that $\int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$ ( $n \neq 1$ )  | BTL -1 | Remembering |
| <b>Part C</b> |  |        |             |
| 1.            | The region enclosed by the circle $x^2 + y^2 = a^2$ is divided into two segments by the line $x = h$ . Find the area of the smaller segment  | BTL -5 | Evaluating  |
| 2.            | Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum  | BTL -6 | Creating    |
| 3.            | Let A be the area of the region that lies under the graph of $f(x) = x^3$ between $x=0$ and $x=1$ (i) Using right end points, Find an expression for A as a limit<br>(ii) Estimate the area by taking the sample points to be midpoints and using four subintervals. | BTL -5 | Evaluating  |
| 4.            | Let A be the area of the region that lies under the graph of $f(x) = 1 + x^2$ between $x = -1$ and $x = 2$ . Estimate the area by taking the sample points to be mid-point (i) using three subintervals and (ii) using six subintervals.                             | BTL -6 | Creating    |

### UNIT V MULTIPLE INTEGRALS

Double integrals in Cartesian and polar coordinates – Change of order of integration – Area enclosed by plane curves  
Change of variables in double integrals (Polar coordinates) – Triple integrals – Volume of solids

| Q.No            | Question  | BT Level | Domain        |
|-----------------|---|----------|---------------|
| <b>PART - A</b> |   |          |               |
| 1.              | Evaluate $\int_2^3 \int_1^2 \frac{dx dy}{xy}$   | BTL -5   | Evaluating    |
| 2.              | Evaluate $\int_0^\pi \int_0^{\sin \theta} r dr d\theta$   | BTL -2   | Understanding |
| 3.              | Find the area bounded by the lines $x = 0, y = 1$ and $y = x$   | BTL -2   | Understanding |
| 4.              | Evaluate $\int_0^\pi \int_0^a r dr d\theta$   | BTL -2   | Understanding |
| 5.              | Evaluate $\int_0^5 \int_0^2 (x^2 + y^2) dx dy$  | BTL -2   | Understanding |
| 6.              | Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} dy dx$   | BTL -2   | Understanding |
| 7.              | Evaluate $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y} dx dy$  | BTL -2   | Understanding |
| 8.              | Evaluate $\int_0^\pi \int_0^5 r^4 \sin \theta dr d\theta$   | BTL -2   | Understanding |
| 9.              | Evaluate $\int_0^2 \int_0^x \frac{dx dy}{x^2 + y^2}$  | BTL -2   | Understanding |
| 10.             | Evaluate $\iint dx dy$ over the region bounded by $x = 0, x = 2, y = 0$ and $y = 2$   | BTL -2   | Understanding |
| 11.             | Change the order of integration $\int_0^1 \int_{y^2}^y f(x, y) dx dy$   | BTL -2   | Understanding |
| 12.             | Change the order of integration $\int_0^\infty \int_x^\infty f(x, y) dx dy$   | BTL -2   | Understanding |
| 13.             | Find the limits of integration in the double integral $\iint_R f(x, y) dx dy$ where R is in the first quadrant and bounded $x=1, y=0, y^2 = 4x$ | BTL -4   | Applying      |
| 14.             | Evaluate $\iiint (x + y + z) dx dy dz$ over the region bounded by $x = 0, x = 1, y = 0$ and $y = 1, z = 0, z = 1$                               | BTL -4   | Applying      |
| 15.             | Write down the double integral to find the area of the circles $r = 2 \sin \theta, r = 4 \sin \theta$   | BTL -4   | Applying      |
| 16.             | Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy(x + y) dy dx$   | BTL -3   | Analyzing     |
| 17.             | Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2) dy dx$  | BTL -2   | Understanding |
| 18.             | Evaluate $\int_1^3 \int_3^4 \int_1^4 xyz dz dy dx$  | BTL -2   | Understanding |
| 19.             | Evaluate $\int_0^1 dx \int_0^2 dy \int_0^3 (x + y + z) dz$  | BTL -1   | Remembering   |
| 20.             | Evaluate $\int_a^b \int_c^d \int_f^g e^{x+y+z} dz dy dx$  | BTL -2   | Understanding |

| <b>PART B</b> |   |        |               |
|---------------|---|--------|---------------|
| <b>1.(a)</b>  | Evaluate $\iint xy dx dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$  | BTL -4 | Applying      |
| <b>1. (b)</b> | Change the order of integration $\int_0^2 \int_0^{\sqrt{4-y^2}} xy dx dy$ and hence evaluate it   | BTL -2 | Understanding |
| <b>2.</b>     | Using double integral find the area of the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .  | BTL -4 | Applying      |
| <b>3.</b>     | Change the order of integration $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy dx dy$ and hence evaluate it   | BTL -2 | Understanding |
| <b>4.</b>     | Change the order of integration $\int_0^1 \int_y^{2-y} xy dx dy$ and hence evaluate it  | BTL -3 | Analyzing     |
| <b>5.</b>     | By changing in to polar Co – ordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ . Hence find the value of $\int_0^\infty e^{-x^2} dx$ . | BTL -3 | Analyzing     |
| <b>6.</b>     | Change the order of integration $\int_0^\infty \int_0^\infty \frac{e^{-y}}{y} dy dx$ and hence evaluate it  | BTL -4 | Applying      |
| <b>7.</b>     | Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ .                                     | BTL -1 | Remembering   |
| <b>8.</b>     | By change the order of integration and evaluate $\int_0^2 \int_{x^2}^{2-x} xy dy dx$  | BTL -2 | Understanding |
| <b>9. (a)</b> | Find the area included between the curves $y^2 = 4x$ and $x^2 = 4y$   | BTL -3 | Analyzing     |
| <b>9.(b)</b>  | Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z dz dy dx$  | BTL -2 | Understanding |
| <b>10.(a)</b> | Change the integral into polar coordinates $\int_0^a \int_0^x \frac{x^3}{\sqrt{x^2+y^2}} dx dy$ and hence evaluate it                                       | BTL -2 | Understanding |
| <b>10.(b)</b> | Find the area common to the cardioids $r = a(1 + \cos \theta)$ and $r = a(1 - \cos \theta)$   | BTL -2 | Understanding |
| <b>11.</b>    | Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$   | BTL -2 | Understanding |
| <b>12.</b>    | Find the value of $\iiint xyz dx dy dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \leq a^2$ .  | BTL -2 | Understanding |
| <b>13.</b>    | Find the volume bounded by the cylinder $x^2 + y^2 = 1$ and the planes $x + y + z = 3, z = 0$   | BTL -5 | Evaluating    |
| <b>14.(a)</b> | Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) dx dy dz$  | BTL -6 | Creating      |
| <b>14.(b)</b> | Find the area which is inside the circle $r = 3a \cos \theta$ and outside the cardioid $r = a(1 + \cos \theta)$ .   | BTL -2 | Understanding |
| <b>Part C</b> |   |        |               |
| <b>1.</b>     | Find the area bounded by parabola $y = x^2$ and straight line $2x - y + 3 = 0$ .  | BTL -5 | Evaluating    |
| <b>2.</b>     | Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  | BTL -6 | Creating      |
| <b>3.</b>     | Find the volume of finite region of space (tetra-hadron) bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 3y + 4z = 12$                                | BTL -5 | Evaluating    |
| <b>4.</b>     | Find the volume of sphere bounded by $x^2 + y^2 + z^2 = a^2$ .  | BTL -6 | Creating      |