SRM VALLIAMMAI ENGINEERING COLLEGE

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DEPARTMENT OF MATHEMATICS

QUESTION BANK

1918301 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to AGRI, CIVIL, EEE, ECE, EIE, MECH, MDE)

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DEPARTMENT OF MATHEMATICS

QUESTION BANK

SUBJECT : 1918301 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

SEMESTER / YEAR: III / II (AGRI, CIVIL, EEE, ECE, EIE, MECH & ME)

UNIT I - PARTIAL DIFFERENTIAL EQUATIONS

Formation of partial differential equations - Solutions Lagrange's linear equation — Linear partial differential equations of second and higher order with constant coefficients of both homogeneous and non-homogeneous types

PART- A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Form a partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $z = ax^2 + by^2$.	BTL -6	Creating
2.	Eliminate the arbitrary function from $z = f(x^2 - y^2)$ and form the partial differential equation	BTL -6	Creating
3.	Construct the partial differential equation of all spheres whose centers lie on the x-axis.	BTL -3	Applying
4.	Form the partial differential equation by eliminating the arbitrary function f from $z = e^{ay} f(x + by)$.	BTL- 6	Creating
5.	Form the partial differential equation by eliminating the arbitrary constants a, b from the relation $z = (x^2 + a^2)(y^2 + b^2)$	BTL -6	Creating
6.	Form the PDE by eliminating the arbitrary function from $\varphi\left[z^2 - xy, \frac{x}{z}\right] = 0$	BTL -6	Creating
7.	Form the partial differential equation from $(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$	BTL -6	Creating
8.	Form the partial differential equation by eliminating the arbitrary function Φ from $\Phi(x^2 - y^2, z) = 0$	BTL -6	Creating
9.	Form the partial differential equation by eliminating arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = 1$	BTL -6	Creating
10.	Form the partial differential equation by eliminating the arbitrary function f from $z = f(x/y)$	BTL -3	Applying
11.	Form the partial differential equation by eliminating the arbitrary function f from $z = f(x\,y)$	BTL -3	Applying
12.	Form a partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $z = a(x + y) + b$.	BTL -3	Applying

13.	Solve $px^2 + qy^2 = z^2$	BTL -3	Applying
14.	Solve $(D^2 - 7DD' + 6D'^2)z = 0$	BTL -3	Applying
15.	Solve $(D^3 - D^2D' - 8DD'^2 + 12D'^3)z = 0$	BTL -3	Applying
16.	Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 0$	BTL -3	Applying
17.	Solve $(D^4 - D'^4)z = 0$	BTL -3	Applying
18.	Solve $(D + D' - 1)(D - 2D' + 3)z = 0$	BTL -3	Applying
19.	Solve $(D - D')^3 z = 0$	BTL -3	Applying
20.	Solve $(D - 1)(D - D' + 1)z = 0$	BTL -3	Applying
	PART – B		
1.(a)	Find the partial differential equation by eliminating the arbitrary constants a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	BTL -6	Creating
1.(b)	Find the partial differential equation by eliminating the arbitrary functions from z = f (x + 2y) + g (x -2y)	BTL -2	Understanding
2.	Form the partial differential equation by eliminating arbitrary function Φ from $\Phi(x^2 + y^2 + z^2, ax + by + cz) = 0$	BTL -6	Creating
3. (a)	Form the partial differential equation by eliminating arbitrary functions f and g from $z = xf(\frac{y}{x}) + yg(x)$	BTL -6	Creating
3.(b)	Solve p cot x + q cot y = cot z	BTL -3	Applying
4. (a)	Solve $x^2 p + y^2 q = z(x + y)$	BTL -3	Applying
4.(b)	Form the partial differential equation by eliminating arbitrary function f and g from the relation $z = x\, f(x + t) + g(x + t)$	BTL -6	Creating
5. (a)	Find the general solution of $(3z - 4y)p + (4x - 2z)q = 2y - 3x$	BTL -3	Applying
5. (b)	Solve $(D^2 - 5DD' + 6D'^2)z = y \sin x$		
6.	Solve $(y^2 + z^2)p - xyq + xz = 0$	BTL -2	Understanding
7. (a)	Solve $(D^2 + DD' - 6D'^2)z = y \cos x$	BTL -4	Analyzing
7. (b)	Find the general solution of $(mz - ny)p + (nx - lz)q = ly - mx$	BTL -2	Understanding
8. (a)	Find the general solution of $(D^2 + D'^2)z = x^2 y^2$	BTL -2	Understanding
8.(b)	Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$	BTL -2	Understanding
9. (a)	Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$	BTL -3	Applying

9.(b)	Solve $(D^3 - 2D^2D')z = 2e^{2x} + 3x^2y$		
10.(a)	Solve $(D^2 - 3DD' + 2D'^2)z = (2+4x)e^{x+2y}$	BTL -3	Applying
10.(b)	Solve $(D^2 - 3DD' + 2D'^2)z = sin(x+5y)$	BTL -3	Applying
11.	Solve the Lagrange's equation $(x+2z)p + (2xz-y)q = x^2 + y$	BTL -3	Applying
12.	Solve the partial differential equation (x -2z) p + (2z –y) q = x - y	BTL -3	Applying
13.	Solve $(D^2 - DD' - 20D'^2)z = e^{5x+y} + sin(4x-y)$.	BTL -3	Applying
14.	Solve $(D^2 + 2DD' + D'^2)z = x^2y + e^{x-y}$	BTL -3	Applying

PART-C

1.	Solve $(x^2 - yz)p + (y^2 - xz)q = (z^2 - xy)$	BTL -1	Remembering
2.	Solve $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{2x+4y}$	BTL -1	Remembering
3.	Find the general solution of $(z^2 - y^2 - 2yz)p + (xy + zx)q = (xy - zx)$	BTL -1	Remembering
4.	Find the general solution of $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$	BTL -2	Understanding

UNIT II FOURIER SERIES:

Dirichlet's conditions – General Fourier series – Odd and even functions – Half range sine series – Half range cosine series – Parseval's identity – Harmonic analysis.

PART –A

Q.No	Question	Bloom's Taxonomy Level	Domain
1.	State the Dirichlet's conditions for a function f(x) to be expanded as a Fourier series.	BTL -1	Remembering
2.	Find the Fourier constant b_n for xsinx in $(-\pi, \pi)$.	BTL -1	Remembering
3.	If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{cosnx}{n^2}$ $in\ 0 < x < 2\pi$ then deduce that value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$.	BTL -1	Remembering
4.	Does $f(x) = tanx$ possess a Fourier expansion?	BTL -2	Understanding
5.	Determine the value of a_n in the Fourier series expansion of	BTL -5	Evaluating

	$f(x) = x^3$ in $(-\pi, \pi)$.		
6.	Find the constant term in the expansion of $f(x) = x^2 + x$ as a Fourier series in the interval $(-\pi, \pi)$.	BTL -2	Understanding
7.	If f(x) is an odd function defined in $(-l, l)$. What are the values of a_0 *and* a_n?	BTL -2	Understanding
8.	If the function f(x) = x in the interval $0 < x < 2\pi$ then find the constant term of the Fourier series expansion of the function f.	BTL -2	Understanding
9.	Find the Fourier coefficient b_n for the function $f(x) = 2x - x^2$ defined in the interval $0 < x < 2$.	BTL -4	Analyzing
10.	Write a_0 , a_n in the expression $x + x^3$ as a Fourier series in $(-l, l)$	BTL -3	Applying
11.	Find the RMS value of f(x) = x in $(0, l)$	BTL -4	Analyzing
12.	Find the root mean square value of $f(x) = x^2$ in $(0,\pi)$	BTL -1	Remembering
13.	Find the RMS value of $f(x) = x(l - x)$ in $0 \le x \le l$	BTL -3	Applying
14.	Find the RMS value of $f(x) = x^2$ in $(0, l)$	BTL -1	Remembering
15.	Write down the Parseval's formula on Fourier coefficients	BTL -5	Evaluating
16.	Define the RMS value of a function f(x) over the interval (a, b)	BTL -6	Creating
17.	Without finding the values of a_0 , a_n and b_n of the Fourier series, for the function $f(x) = x^2$ in the interval $(0,2\pi)$ find the value of $\left\{\frac{{a_0}^2}{2} + \sum_{n=1}^{\infty}({a_n}^2 + {b_n}^2)\right\}$	BTL -4	Analyzing
18.	Find the R.M.S value of $f(x) = 1 - x$ in $0 < x < 1$.	BTL -1	Remembering
19.	State Parseval's identity for the half-range cosine expansion of f(x) in (0,1).	BTL -6	Creating
20.	What is meant by Harmonic Analysis?	BTL -3	Applying
	PART – B		
1.	Obtain the Fourier series for the function f(x) given by $f(x) = \begin{cases} 1 - x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$ Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$	BTL -1	Remembering
2.	Find the Fourier series of $f(x) = x(2 - x)$ in $0 < x < 3$.	BTL -1	Remembering

<table>
<tr><td>3.</td><td>Expand $f(x) = \begin{cases} 1+\frac{2x}{\pi}, -\pi < x < 0 \\ 1-\frac{2x}{\pi}, \quad 0 < x < \pi \end{cases}$ as a full range Fourier series in the interval $(-\pi, \pi)$.</td><td>BTL -1</td><td>Remembering</td></tr>
<tr><td>4.</td><td>Determine the Fourier series for the function $f(x) = x\sin x \; in$ $0 < x < 2\pi$.</td><td>BTL -5</td><td>Evaluating</td></tr>
<tr><td>5.</td><td>Find the Fourier series expansion of
$f(x) = \begin{cases} x \quad for \;\; 0 \le x \le \pi \\ 2\pi - x \;\; for \;\; \pi \le x \le 2\pi. \end{cases}$</td><td>BTL -5</td><td>Evaluating</td></tr>
<tr><td>6.(a)</td><td>Expand $f(x) = \begin{cases} x, \quad 0 < x < 1 \\ 2-x, \;\; 1 < x < 2 \end{cases}$ as a series of cosines in the interval (0,2).</td><td>BTL -1</td><td>Remembering</td></tr>
<tr><td>6.(b)</td><td>Find the half range sine series of $f(x) = 4x - x^2$ in the interval 0 < x < 4</td><td>BTL -2</td><td>Remembering</td></tr>
<tr><td>7.</td><td>Find the Fourier series of $f(x) = x + x^2$ in $(-\pi, \pi)$ with period 2π.
Hence deduce $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.</td><td>BTL -3</td><td>Applying</td></tr>
<tr><td>8.</td><td>Compute the first two harmonics of the Fourier series of f(x) from the table given
<table>
<tr><td>x</td><td>0</td><td>$\pi/3$</td><td>$2\pi/3$</td><td>π</td><td>$4\pi/3$</td><td>$5\pi/3$</td><td>2π</td></tr>
<tr><td>f(x)</td><td>1</td><td>1.4</td><td>1.9</td><td>1.7</td><td>1.5</td><td>1.2</td><td>1</td></tr>
</table></td><td>BTL -6</td><td>Creating</td></tr>
<tr><td>9.</td><td><table>
<tr><td>t sec x:</td><td>0</td><td>T/6</td><td>T/3</td><td>T/2</td><td>2T/3</td><td>5T/6</td><td>T</td></tr>
<tr><td>A amps y:</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>-0.88</td><td>-0.25</td><td>1.98</td></tr>
</table>
The table gives the time (t) in seconds as x and current (A) in amps as y, Obtain the first two harmonics from the given data.</td><td>BTL -3</td><td>Applying</td></tr>
<tr><td>10.</td><td>Find the half range Fourier cosine series of $f(x) = (\pi - x)^2$ in the interval $(0, \pi)$. Hence Find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots$</td><td>BTL -1</td><td>Remembering</td></tr>
<tr><td>11.</td><td>By using Cosine series show that $\frac{\pi^4}{96} = 1 + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$ for $f(x) = x \;\; in \;\; 0 < x < \pi$</td><td>BTL -4</td><td>Analyzing</td></tr>
<tr><td>12.</td><td>Find the Fourier cosine series up to third harmonic to represent the function given by the following data: $l = 6$
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y</td><td>4</td><td>8</td><td>15</td><td>7</td><td>6</td><td>2</td></tr>
</table></td><td>BTL -6</td><td>Creating</td></tr>
</table>

13.	Find the Fourier expansion of the following periodic function of period 4 $f(x) = \begin{cases} 2+x, & -2 \le x \le 0 \\ 2-x, & 0 \le x \le 2 \end{cases}$ Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\infty = \frac{\pi^2}{8}$.	BTL -2	Remembering
14.	Find the Fourier series as far as the second harmonic to represent the function $f(x)$ With period 6 , $2l = 6$, $l = 3$.	BTL -6	Creating

x	0	1	2	3	4	5
y	9	18	24	28	26	20

PART-C

1.	Obtain the Fourier series to represent the function $f(x) = \lvert x \rvert, -\pi < x < \pi$ and deduce $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.	BTL -2	Understanding
2.	Find the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$ and hence deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots\infty = \frac{\pi^4}{90}$	BTL -2	Remembering
3.	Calculate the first 3 harmonics of the Fourier of f(x) from the following data	BTL -6	Creating

x	0	30	60	90	120	150	180	210	240	270	300	330	360
f(x)	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2	1.8

4.	Find a Fourier series with period 3 to represent $f(x) = 2x - x^2$ in $(0,3)$.	BTL -1	Remembering

UNIT III -APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Classification of PDE – Solutions of one dimensional wave equation – One dimensional equation of heat conduction – Steady state solution of two dimensional equation of heat conduction in infinite plates(excluding insulated edges)

PART –A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Classify the PDE $u_{xx} + u_{xy} + u_{yy} = 0$	BTL-4	Analyzing
2.	Classify the PDE $Z_{xx} + 2Z_{xy} + (1-y^2)Z_{yy} + xZ_x + 3x^2yz - 2Z = 0$	BTL-4	Analyzing
3.	Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$ by method of separation of variables	BTL-3	Applying
4.	What are the various solutions of one-dimensional wave equation	BTL-1	Remembering
5.	In the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ what does C^2 stand for?	BTL-2	Understanding

6.	What is the basic difference between the solutions of one-dimensional wave equation and one-dimensional heat equation with respect to the time?	BTL-3	Applying
7.	Write down the initial conditions when a taut string of length $2l$ is fastened on both ends. The midpoint of the string is taken to a height b and released from the rest in that position	BTL-1	Remembering
8.	A slightly stretched string of length l has its ends fastened at x = 0 and x = l is initially in a position given by $y(x,0) = y_0 sin^3 \frac{\pi x}{l}$.If it is released from rest from this position, write the boundary conditions	BTL-2	Understanding
9.	A tightly stretched string with end points $x = 0$ & $x = l$ is initially at rest in equilibrium position. If it is set vibrating giving each point velocity $\lambda x(l - x)$. Write the initial and boundary conditions	BTL-2	Understanding
10.	If the ends of a string of length l are fixed at both sides. The midpoint of the string is displaced transversely through a height h and the string is released from rest, state the initial and boundary conditions	BTL-2	Understanding
11.	State the assumptions in deriving the one-dimensional heat equation	BTL-1	Remembering
12.	What are the possible solutions of one-dimensional heat flow equation?	BTL-1	Remembering
13.	In the one-dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ what is C^2 ?	BTL-2	Understanding
14.	The ends A and B of a rod of length 20 cm long have their temperature kept 30^0 C and 80^0 C until steady state prevails. Find the steady state temperature on the rod	BTL-2	Understanding
15.	An insulated rod of length 60 cm has its ends at A and B maintained at 20^0C and 80^0 C respectively. Find the steady state solution of the rod.	BTL-2	Understanding
16.	An insulated rod of length l cm has its ends at A and B maintained at 0^0C and 80^0 C respectively. Find the steady state solution of the rod.	BTL-2	Understanding
17.	Write down the three possible solutions of Laplace equation in two dimensions	BTL-1	Remembering
18.	Write down the governing equation of two-dimensional steady state heat equation.	BTL-1	Remembering
19.	A rectangular plate with insulated surface is 10cm wide. The two long edges and one short edge are kept at0^0 C, while the temperature at short edge x =0 is given by $u = \begin{cases} 20y, & 0 \le y \le 5 \\ 20(10-y), & 5 \le y \le 10 \end{cases}$ Write the boundary conditions to solve two-dimensional heat flow equation.	BTL-2	Understanding
20.	A plate is bounded by the lines x=0, y=0, x=l and y=l. Its faces are insulated. The edge coinciding with x-axis is kept at 100^0 C. The edge coinciding with y-axis at 50^0 C. The other 2 edges are kept at 0^0 C. write the boundary conditions that are needed for solving two-dimensional heat flow equation.	BTL-2	Understanding
	PART-B		

1.	A string is stretched and fastened to two points that are distinct string l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$from which it is released at time t=0. Find the displacement of any point on the string at a distance of x from one end at time t.	BTL-2	Understanding
2.	A tightly stretched string of length $2l$ is fastened at both ends. The Midpoint of the string is displaced by a distance b transversely and the string is released from rest in this position. Find an expression for the transverse displacement of the string at any time during the subsequent motion.	BTL-2	Understanding
3.	A slightly stretched string of length l has its ends fastened at $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = y_0 sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement y at any distance x from one end and at any time.	BTL-2	Understanding
4.	A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating string giving each point a velocity $3x(l - x)$. Find the displacement of the string.	BTL-3	Applying
5.	A tightly stretched string with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If it is vibrating string by giving to each of its points a velocity $v = \begin{cases} \frac{2cx}{l} if 0 \le x \le \frac{l}{2} \\ \frac{2c(l-x)}{l} if \frac{l}{2} \le x \le l \end{cases}$. Find the displacement of the string at any distance x from one end at any time t.	BTL-2	Understanding
6.	A tightly stretched string of length l is initially at rest in this equilibrium position and each of its points is given the velocity $v_0 \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x,t)$.	BTL-2	Understanding
7.	Solve $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions (i)u(0, t)=0 for all $t \ge 0$ (ii) $u(l,t) = 0 \; for \; all \; t \ge 0$ (iii)$u(x,0) = \begin{cases} x \;\; if \; 0 \le x \le \frac{l}{2} \\ l - x if \frac{l}{2} \le x \le l \end{cases}$.	BTL-3	Applying
8.	A rod 30 cm long has its ends A and B kept at 20^0 and 80^0 respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature function u(x, t) taking x = 0 at A.	BTL-2	Understanding
9.	A bar 10 cm long with insulated sides has its ends A and B maintained at temperature 50^0C and 100^0C respectively. Until steady state conditions prevail. The temperature at A is suddenly raised to 90^0C and at the same time lowered to 60^0C at B. Find the temperature distributed in the bar at time t.	BTL-2	Understanding
10.	A rod 40 cm long has its ends A and B kept at 0^0 and 80^0 respectively until steady state conditions prevail. The temperature at each end B is then suddenly reduced to 40^0C and kept so, while at A the temperature is	BTL-2	Understanding

	0^o c. Find the resulting temperature function u(x, t) taking x = 0 at A		
11.	A square metal plate is bounded by the lines x = 0, x = a, y = 0, y = ∞. The edges x = a, y = 0, y = ∞ are kept at 0^0 temperature while the temperature at the edge y = a is100^0 temperature. Find the steady state temperature distribution at in the plate.	BTL-2	Understanding
12.	A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge y=0 is given by $u = \begin{cases} 20x , & 0 \le x \le 5 \\ 20(10-x), & 5 \le x \le 10 \end{cases}$ and all the other three edges are kept at 0^0C. Find the steady state temperature at any point in the plate.	BTL-2	Understanding
13.	An infinitely long rectangular plate with insulated surface is 10cm wide. The two long edges and one short edge are kept at 0^0C, while the other short edge x=0 is kept at temperature $u = \begin{cases} 20y , & 0 \le y \le 5 \\ 20(10-y), & 5 \le y \le 10 \end{cases}$. Find the steady state temperature distribution in the plate.	BTL-2	Understanding
14.	A long rectangular plate with insulated surface is $l$cm . If the temperature along one short edge y=0 is u(x,0) =K(l x -x^2) degrees, for 0 < x < l , while the other 2 edges x=0 and x=l as well as the other short edge are kept at 0^0C, find the steady state temperature function $u(x, y)$.	BTL-2	Understanding
	PART –C		
1.	A tightly stretched string of length l with fixed end points initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity y_t(x,0) = $v_0 sin\left(\frac{3\pi x}{l}\right) cos\left(\frac{\pi x}{l}\right)$ where 0 < x < l. Find the displacement of the string at a point at a distance x from one end at any instant " t".	BTL -1	Remembering
2.	A string is tightly stretched between x = 0 and x = 20 is fastened at both ends. The midpoint of the string is taken to be a height and then released from rest in that position. Find the displacement of any point of the string x at any time t.	BTL -1	Remembering
3.	A bar 20 cm long with insulated sides has its ends A and B maintained at temperature 40^0C and 90^0C respectively, until steady state conditions prevail. The temperature at A is suddenly raised to 70^0C and at the same time lowered to 50^0C at B. Find the temperature distributed in the bar at time t.	BTL -3	Applying
4.	A infinite rectangular plate with insulated surface is bounded by the lines x = 0, x = a, y = 0 and y= ∞. The temperature along the edge y = 0 kept at 100^o C, while the temperature along the other three edges are at 0^o C. Find the steady state temperature at any point in the plate.	BTL -3	Applying

UNIT –IV FOURIER TRANSFORM

Fourier transform pair – Fourier sine and cosine transforms – Properties – Transforms of simple functions – Convolution theorem – Parseval's identity

PART –A			
Q.No.	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	State Fourier integral Theorem	BTL -1	Remembering
2.	Write Fourier transform pair.	BTL -1	Remembering
3.	Write Fourier Sine transform pair	BTL -3	Applying
4.	If the Fourier transform of $f(x)$ is $F(s) = F[f(x)]$,then show that $F[f(x-a)] = e^{ias}F(s)$.	BTL -3	Applying
5.	Find the Fourier Transform of $e^{-a\lvert x\rvert}$.	BTL -2	Understanding
6.	Find the Fourier Transform of $f(x)=\begin{cases} e^{ikx}, & if \quad a<x<b \\ 0, & if \quad x\le a \,\&\, x>b \end{cases}$.	BTL -2	Understanding
7.	State and Prove any one Modulation theorem on Fourier Transform	BTL -2	Understanding
8.	Find the Fourier sine Transform of e^{-ax}.	BTL -2	Understanding
9.	Define self-reciprocal with respect to Fourier Transform	BTL -1	Remembering
10.	Prove that $F[f(ax)] = \frac{1}{a}F\left(\frac{s}{a}\right)$ if a>0.	BTL -2	Understanding
11.	Find the Fourier cosine Transform of e^{-2x}.	BTL -2	Understanding
12.	Prove that $F_S[f(ax)] = \frac{1}{a}F_S\left(\frac{s}{a}\right)$	BTL -3	Applying
13.	Write down the Fourier cosine Transform pair of formulae.	BTL -1	Remembering
14.	If F(s) is the Fourier Transform of $f(x)$. Show that the Fourier Transform of $e^{iax}f(x) \, is \, F(s+a)$.	BTL -3	Applying
15.	Find the Fourier cosine transform of e^{-4x}.	BTL -3	Applying
16.	If $F(s) = F[f(x)]$, $then\ find\ F[xf(x)]$.	BTL -2	Understanding
17.	Find the Fourier cosine Transform of f(x) = 2x in 0 < x < 4	BTL -2	Understanding
18.	Define Convolution of two functions f(x)*g(x).	BTL -3	Applying
19.	State Convolution theorem for Fourier Transform	BTL -1	Remembering
20.	State Parseval's Identity for Fourier Transform	BTL -1	Remembering
PART-B			
1.	Find the Fourier Transform of $f(x) = \begin{cases} 1, \lvert x\rvert \le a \\ 0, \lvert x\rvert > a > 0 \end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{\text{sint}}{\text{t}}\right)$ dt. also using Parseval's Identity Prove that $\int_0^\infty \left(\frac{\text{sint}}{\text{t}}\right)^2 dt = \frac{\pi}{2}$	BTL -2	Understanding
2.(a)	Find $F_C\left[\frac{e^{-ax}}{x}\right]$ and hence find $F_C\left[\frac{e^{-ax}-e^{-bx}}{x}\right]$	BTL -2	Understanding

2.(b)	Find the function whose Fourier Sine Transform is $\frac{e^{-as}}{s}, a > 0$	BTL -1	Remembering
3.	Find the Fourier Transform of the function f(x) = $\begin{cases} 1-\lvert x\rvert, if \lvert x\rvert \leq 1 \\ 0, if \lvert x\rvert > 1 \end{cases}$ Hence deduce that (i) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ $(ii) \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$.	BTL -2	Understanding
4	Show that the function $e^{\frac{-x^2}{2}}$ is self-reciprocal under the Fourier Transform.	BTL -3	Understanding
5.	Find the Fourier Transform of $f(x) = \begin{cases} 1-x^2, if \lvert x\rvert \leq 1 \\ 0, if \lvert x\rvert > 1 \end{cases}$ Hence Show that $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)\cos\frac{s}{2} ds = \frac{3\pi}{16}$	BTL -1	Remembering
6.(a)	Find the Fourier Sine Transform of $f(x) = \begin{cases} x, 0 < x < 1 \\ 2-x, 1 < x < 2 \\ 0, x > 2 \end{cases}$	BTL -2	Understanding
6.(b).	Using Parseval's Identity evaluate the following integrals. $(i) \int_0^\infty \frac{dx}{(a^2+x^2)^2}, (ii) \int_0^\infty \frac{x^2 dx}{(a^2+x^2)^2} where\, a > 0.$	BTL -5	Evaluating
7.(a)	Find the Fourier sine transform of e^{-ax} (a>0). Hence find $F_s[xe^{-ax}]$ and $F_s\left[\frac{e^{-ax}}{x}\right]$	BTL -3	Applying
7.(b)	Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier Transform	BTL -5	Evaluating
8.(a)	Solve the integral equation $\int_0^\infty f(x) \sin tx\, dx = \begin{cases} 1\ if\ 0 \leq t \leq 1 \\ 2\ if 1 \leq t < 2 \\ 0\ if\ t \geq 2 \end{cases}$	BTL -4	Analyzing
8.(b)	Find the infinite Fourier sine transform of $\frac{1}{x}$.	BTL -2	Understanding
9.	Show that $x\, e^{-x^2/2}$ is self-reciprocal under the Fourier sine transform.	BTL -2	Understanding
10.	Find the Fourier Transform of $e^{-a\lvert x\rvert}$ and hence deduce that $(i) \int_0^\infty \frac{\cos xt}{a^2+t^2} dt = \pi/2a\, e^{-a\lvert x\rvert}$ $(ii) F[xe^{-a\lvert x\rvert}] = i\sqrt{\frac{2}{\pi}} \frac{2as}{(a^2+s^2)^2}$, here F stands for Fourier Transform.	BTL -4	Analyzing
11.(a)	Find the Fourier cosine & sine Transform of e^{-x}. Hence evaluate $(i) \int_0^\infty \frac{1}{(x^2+1)^2} dx\ and\ (ii) \int_0^\infty \frac{x^2}{(x^2+1)^2} dx.$	BTL -2	Understanding
11.(b)	Find the Fourier Sine Transform of the function $f(x) = \begin{cases} \sin x, 0 \leq x < a \\ 0, x > a \end{cases}$	BTL -2	Understanding
12.	Find the Fourier sine and cosine transforms of x^{n-1}. Hence deduce that $\frac{1}{\sqrt{x}}$ is self reciprocal under both the transforms.	BTL -3	Understanding
13.(a)	Find the Fourier Transform of $e^{-\lvert x\rvert}$ and hence find the Fourier Transform of $f(x) = e^{-\lvert x\rvert} \cos 2x$.	BTL -2	Understanding

13.(b)	Using Parseval's Identity evaluate $\int_0^\infty \frac{dx}{(x^2+25)(x^2+9)}$.	BTL -5	Evaluating
14.(a)	Using Fourier Sine transform prove that $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2(a+b)}$	BTL -3	Applying
14.(b)	Prove that $F_c[xf(x)] = \frac{d}{ds}[F_s\{f(x)\}]$ and $F_s[xf(x)] = -\frac{d}{ds}[F_c\{f(x)\}]$	BTL -3	Applying

PART-C

1.	Show that the Fourier Transform of $f(x) = \begin{cases} a-\lvert x\rvert, if \lvert x\rvert \le a \\ 0, if \lvert x\rvert > a > 0 \end{cases}$ is $\sqrt{\frac{2}{\pi}}\left(\frac{1-\cos as}{s^2}\right)$. Hence deduce that (i) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$, (ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$.	BTL -3	Applying
2.	Show that the Fourier transform of $f(x) = \begin{cases} a^2-x^2, \lvert x\rvert < a \\ 0, \lvert x\rvert > a > 0 \end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as - as\cos as}{s^3}\right)$. Hence deduce that (i) $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right) dt = \frac{\pi}{4}$, (ii) $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$.	BTL -3	Applying
3.	State and Prove (i) Convolution Theorem (ii) Parseval's identity for Fourier Transform.	BTL -3	Applying
4.	Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence find the Fourier cosine transform of $e^{\frac{-x^2}{2}}$	BTL -2	Understanding

UNIT –V: Z - TRANSFORMS AND DIFFERENCE EQUATIONS

Z- transforms – Elementary properties – Inverse Z – transform (using partial fraction and residues) – Convolution theorem – Solution of difference equations using Z – transform.

PART –A

Q.No.	Questions	Bloom's Taxonomy Level	Domain
1.	Define Z – Transform of the sequence $\{f(n)\}$.	BTL -1	Remembering
2.	Find $\boldsymbol{Z}(\boldsymbol{3^{n+2}})$	BTL -2	Understanding
3.	Find $Z\left[\frac{a^n}{n!}\right]$	BTL -2	Understanding
4.	Find $Z\left[\frac{1}{n!}\right]$	BTL -2	Understanding
5.	Find $Z\left[\frac{1}{n(n+1)}\right]$	BTL -2	Remembering
6.	State initial value theorem	BTL -1	Remembering

7.	State final value theorem	BTL -1	Remembering
8.	Find $Z[n^2]$.	BTL -2	Understanding
9.	Find inverse Z transform of $\frac{z}{(z-1)(z-2)}$	BTL -2	Understanding
10.	Find $Z\left[\frac{1}{(n+1)!}\right]$	BTL -2	Understanding
11.	Find $Z[e^t \sin 2t]$.	BTL -2	Understanding
12.	Prove that $Z[a^n f(n)] = f(\frac{z}{a})$	BTL -5	Evaluating
13.	Prove that $Z[a^n] = \frac{z}{z-a}$	BTL -5	Evaluating
14.	Find $z^{-1}[\frac{z}{(z-1)^2}]$	BTL -2	Analyzing
15.	Find Z transform of $\frac{1}{n}$	BTL -2	Understanding
16.	Find $z^{-1}[\frac{z}{(z+1)^2}]$	BTL -2	Understanding
17.	Solve $y_{n+1} + 2y_n = 0$ given that $y(0)=2$	BTL -3	Applying
18.	State Convolution theorem in Z – Transforms	BTL -1	Remembering
19.	If $z\{f(n)\} = \frac{z^2}{(z^2+1)}$, then find f(0), using initial value theorem.	BTL -6	Creating
20.	Prove that $Z[f(n+1)] = zF(z) - zf(0)$.	BTL -6	Creating
	PART –B		
1.(a)	Find the z transform of $f(n) = \frac{2n+3}{(n+1)(n+2)}$	BTL -2	Understanding
1.(b)	Find the inverse Z – Transform using partial fraction method of $\frac{z^2}{(z-3)(z-4)}$		
2.	Find the inverse Z – Transform of $\frac{z^2+z}{(z-1)(z^2+1)}$ by partial fraction method, and Cauchy Residue theorem.	BTL -2	Understanding
3.	Using convolution theorem find inverse Z transform of $\left[\frac{z^2}{(z-a)(z-b)}\right]$	BTL -3	Applying
4.	Using convolution theorem find the inverse Z – Transform of $\frac{12z^2}{(3z-1)(4z+1)}$	BTL -3	Analyzing
5.	Using convolution theorem find inverse Z transform of $\frac{8z^2}{(2z-1)(4z+1)}$.	BTL -3	Applying
6.	Using Cauchy Residue, find $Z^{-1}\left[\frac{4z^3}{(2z-1)^2(z-1)}\right]$.	BTL -3	Applying

7.	Using Z transform Solve $y_{n+2} - 3y_{n+1} + 2y_n = 0$ given that y(0) = 0,y(1) = 1	BTL -3	Analyzing
8.	Using Residue theorem and Partial fraction method find the inverse Z transform of U(z)=$\left[\frac{z^2}{(z+2)(z+4)}\right]$	BTL -3	Applying
9.	Using convolution theorem evaluate $Z^{-1}\left[\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right]$	BTL -3	Applying
10.	Using Z transform solve $y_{n+2} - 3y_{n+1} - 10y_n = 0$ with y(0) = 0,y(1) = 1	BTL -3	Applying
11.	Using Z transform solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ given that u(0) = 0,u(1) = 0	BTL -3	Applying
12.	Solve $y_{n+2} + y_n = 2$ given that y(0) = 0, y(1) = 0	BTL -1	Remembering
13.	Form the difference equation $y(k+3) - 3y(k+1) + 2y(k) = 0$ $with$ $y(0) = 4, y(1) = 0$ and $y(2) = 8$	BTL -3	Applying
14.	Solve the equation using Z – Transform $y_{n+2} - 5y_{n+1} + 6y_n = 36$ given that y(0) = y(1) = 0	BTL -3	Applying
	PART –C		
1.	Find$(i) Z[r^n \cos n\theta], (ii) Z[r^n \sin n\theta] iii) Z(e^{-at} \cos b t)$	BTL-2	Understanding
2.(a)	Find inverse Z -Transform of $\frac{z^3}{(z-1)^2(z-2)}$ by the method of Partial fraction	BTL -2	Understanding
2.(b)	Find the $Z^{-1}\left(\frac{10z}{z^2-3z+2}\right)$	BTL -2	Understanding
3.(a)	Using convolution theorem find $Z^{-1}\left[\frac{z^2}{(z-4)(z-5)}\right]$	BTL -3	Applying
3.(b)	Using Residue method find $Z^{-1}\left(\frac{z}{z^2-2z+2}\right)$	BTL -3	Applying
4.	Solve $U_{n+2} + 4\, U_{n+1} + 3\, U_n = 3^n$, given that $U_0 = 0$ and $U_1 = 1$.	BTL -6	Creating