

**SRM VALLIAMMAI ENGINEERING COLLEGE**  
(An Autonomous Institution)

S.R.M. Nagar, Kattankulathur - 603203

**DEPARTMENT OF MATHEMATICS**

**QUESTION BANK**



**I SEMESTER**

**(COMMON TO ALL BRANCHES)**

**1918102 - ENGINEERING MATHEMATICS-I**

**Regulation – 2019**

**Academic Year – 2022 - 2023**

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SRM Nagar, Kattankulathur – 603203.



## DEPARTMENT OF MATHEMATICS

**SUBJECT : 1918102-Engineering Mathematics –I**

**SEM / YEAR: I / I year B.E./ B.Tech.**

**(Common to all Branches)**

<b>UNIT I - MATRICES</b>			
System of Equations – Consistency and inconsistency - Eigen values and Eigenvectors of a real matrix – Characteristic equation – Properties of Eigen values and Eigenvectors – Statement and application Cayley-Hamilton theorem– Reduction of a quadratic form to canonical form by orthogonal transformation.			
Q.No.	Question	BT Level	Competence
<b>PART – A</b>			
1.	Examine the consistency of the equations $x + y = 3$ , $2x + 3y = 7$ .	BTL-5	Evaluating
2.	Examine the consistency of the equations $x + y = 2$ , $x + 2y = 3$ .	BTL-5	Evaluating
3.	Find the characteristic equation of $A = \begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$	BTL-1	Remembering
4.	If the eigen values of the matrix A of order 3X3 are 2,3 and 1, then find the determinant of A	BTL-2	Understanding
5.	Prove that sum of eigen values of a matrix is equal to its trace.	BTL-3	Applying
6.	Find the sum of the eigen values of 2A, if $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	BTL-3	Applying
7.	Find the sum and product of the eigen values of $A = \begin{pmatrix} 2 & -2 & 2 \\ -2 & -1 & -1 \\ 2 & -1 & -1 \end{pmatrix}$	BTL-1	Remembering
8.	Find the constants a and b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3,-2 be the eigen values of A	BTL-4	Analyzing
9.	Write any 2 applications of Cayley Hamilton theorem	BTL-2	Understanding
10.	If $A = \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$ find $A^3$ using Cayley Hamilton theorem	BTL-2	Understanding
11.	Find the quadratic form corresponding to the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	BTL-5	Evaluating
12.	Define Index, Signature and Rank.	BTL-2	Understanding
13.	Find the eigen values of $A^{-1}$ if $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$	BTL-1	Remembering
14.	If the sum of 2 eigen values and the trace of a 3x3 matrix are	BTL-1	Remembering

	equal, find the value of $ A $		
15.	Prove that the eigen values of $A^{-1}$ are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \dots, \frac{1}{\lambda_n}$	BTL-3	Applying
16.	The product of the 2 eigen values of $A = \begin{pmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 14. Find the 3 <sup>rd</sup> eigen value.	BTL-1	Remembering
17.	Find the sum and product of the eigen values of $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$	BTL-1	Remembering
18.	State Cayley-Hamilton theorem.	BTL-2	Understanding
19.	Use Cayley Hamilton theorem to find $A^{-1}$ if $A = \begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}$	BTL-2	Understanding
20.	Find the matrix corresponding to the quadratic form $2xy - 2yz + 2xz$ .	BTL-4	Analyzing
21.	Find the matrix corresponding to the quadratic form $x^2 + y^2 + z^2$	BTL-5	Evaluating
22.	Two eigen values of $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are equal to unity each. Find the 3 <sup>rd</sup> eigen value.	BTL-1	Remembering
23.	For the given matrix A of order 3, $ A =32$ , and two eigen values are 8 & 2. Find the sum of the Eigenvalues.	BTL -3	Applying
24.	Find the Rank, Index, Signature and Nature of given matrix $A = \begin{pmatrix} 3 & -1 & 4 \\ 0 & 5 & 2 \\ 0 & 0 & -1 \end{pmatrix}$	BTL-1	Remembering
25.	If the characteristic equation of a matrix is $\lambda^2 - 3\lambda - 10 = 0$ , then find the the eigen values of the matrix $10A^{-1} - 2I$ .	BTL-4	Analyzing

**PART – B**

1.	Investigate for the value of $\lambda, \mu$ the system of equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ have (i) Unique solution, (ii) Infinitely many solution, (iii) No solution	BTL-6	Creating
2.(a)	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$	BTL-2	Understanding
2.(b)	Examine the consistency of the equations $x + y + z = 3, 2x - y + 3z = 4, 5x - y + 7z = 11$ .	BTL-5	Evaluating
3.	Test for the consistency of the following system of equations and solve them, if consistent $3x + y + z = 8, -x + y - 2z = -5, x + y + z = 6, -2x + 2y - 3z = -7$	BTL-5	Evaluating
4.(a)	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$	BTL-2	Understanding
4.(b)	Find the Characteristic equation of the matrix	BTL-4	Analyzing

	$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence find $A^4$ .		
5.	Verify Cayley-Hamilton theorem and hence find $A^{-1}$ of $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$	BTL-1	Remembering
6.(a)	Obtain the eigen values and eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ .	BTL-4	Analyzing
6.(b)	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$	BTL-2	Understanding
7.	For the value of $\mu$ the system of equations $x + y + 3z = 0$ , $4x + 3y + \mu z = 0$ , $2x + y + 2z = 0$ have a (i) Trivial solution, (ii) Non-trivial solution.	BTL-6	Creating
8.(a)	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$	BTL-2	Understanding
8.(b)	Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -2 \\ -1 & 1 & 2 \end{pmatrix}$ .	BTL -4	Analyzing
9.			
10.	Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ and also find $A^{-1}$ .	BTL-4	Analyzing
11.	Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal reduction.	BTL-5	Evaluating
12.	Use Cayley-Hamilton theorem to find the value of the matrix given by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ if the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$	BTL -3	Applying
13.	Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form by an orthogonal reduction.	BTL-2	Understanding
14.(a)	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$	BTL-2	Understanding
14.(b)	Using Cayley-Hamilton theorem evaluate the matrix $A^4 + A^3 - 18A^2 - 39A + 2I$ given the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$	BTL -3	Applying

15.	Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ into canonical form by an orthogonal reduction.	BTL-1	Remembering
16.	Determine the nature of the quadratic form $2xy - 2yz + 2xz$ by reducing it into canonical form by orthogonal transformation	BTL -4	Analyzing
17.	Reduce the quadratic form $2x_1^2 + 5x_2^2 + 3x_3^2 + 4x_1x_2$ into canonical form by an orthogonal reduction.	BTL-1	Remembering
18.	Diagonalize the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ using an orthogonal transformation.	BTL -3	Applying

**PART – C**

1.	Verify Cayley-Hamilton theorem and hence find $A^{-1}$ & $A^4$ of $A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$	BTL-6	Creating
2.	Diagonalize the matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ using an orthogonal transformation.	BTL-6	Creating
3.	Determine the nature of the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 - 4x_1x_2 + 6x_2x_3 - 10x_1x_3$ by reducing it into canonical form by orthogonal transformation and hence find rank,index,signature,nature.	BTL-6	Creating
4.	Determine the nature of the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ by reducing it into canonical form by orthogonal transformation and hence find rank,index,signature,nature.	BTL-6	Creating
5.	The Eigen vectors of a $3 \times 3$ real symmetric matrix A corresponding to the eigen values 1,2,4 are $(1, 0, 0)^T, (0, 1, 1)^T, (0, 1, -1)^T$ respectively. Find the matrix A.	BTL-6	Creating

**UNIT II DIFFERENTIAL CALCULUS**

Limit of a function - Continuity – Differentiability - Differentiation rules – Roll's Theorem and Mean Value Theorem – Taylor's Series - Maxima and Minima of functions of one variable

Q.No.	Question	BT Level	Competence
<b>PART – A</b>			
1.	Find the domain of a function $y = \sqrt{x + 4}$	BTL -1	Remembering
2.	Find $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ .	BTL -1	Remembering
3.	Evaluate $\lim_{t \rightarrow 1} \frac{t^4 - 1}{t^3 - 1}$	BTL -1	Remembering
4.	Check whether $\lim_{x \rightarrow -3} \frac{3x+9}{ x+3 }$ exist	BTL -2	Understanding
5.	Show that $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ using Squeeze theorem	BTL -3	Applying
6.	Predict the values of a and b so that the function f given by	BTL -3	Applying

	$f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax + b & \text{if } 3 < x < 5 \text{ is continuous at } x=3 \text{ and } x=5. \\ 7 & \text{if } x \geq 5 \end{cases}$		
7.	If the function $f(x) = \begin{cases} \frac{x^2-16}{x-4} & \text{if } x \neq 4 \\ c & \text{if } x = 4 \end{cases}$ is continuous, what is the value of c?	BTL -2	Understanding
8.	If $f(x) = xe^x$ find $f'(x)$	BTL -2	Understanding
9.	Point out $\frac{dy}{dx}$ , if $y = \ln \cos(\ln x) $ .	BTL -3	Applying
10.	Calculate $\frac{d}{dx}((x)^{\sqrt{x}})$	BTL -2	Understanding
11.	Compute $\frac{d}{dx}((x)^{\sin x})$	BTL -4	Analyzing
12.	Evaluate $\frac{d}{dx}((\sin x)^{\ln x})$	BTL -3	Applying
13.	Estimate $\frac{d}{dx}((\sin x)^{\cos x})$	BTL -4	Analyzing
14.	Evaluate $\frac{d}{dx}(\sinh x)$	BTL -4	Analyzing
15.	Estimate $\frac{d}{dx}(\cosh^{-1} x)$	BTL -3	Applying
16.	Estimate $y'$ if $x^3 + y^3 = 6xy$	BTL -2	Understanding
17.	Using Rolle's theorem find the value of c for the function $f(x) = \sqrt{1-x^2}, -1 \leq x \leq 1$	BTL -4	Analyzing
18.	Verify Lagrange's law for the function $f(x) = \frac{1}{x}, [1,2]$	BTL -1	Remembering
19.	Using Rolle's theorem find the value of c for the function $f(x) = (x-a)(b-x), a \leq x \leq b, a \neq b$	BTL -4	Analyzing
20.	Verify Lagrange's law for the function $f(x) = x^3, [-2,2]$	BTL -2	Understanding
21.	Find the Taylor's series expansion of the function $f(x) = \sin x$ about the point $x = \frac{\pi}{2}$	BTL -4	Analyzing
22.	Find the critical points of $y = 5x^3 - 6x$	BTL -2	Understanding
23.	Find the critical numbers of the function $f(x) = 2x^3 - 3x^2 - 36x$	BTL -4	Analyzing
24.	Find the critical points of $y = 5x^2 - 6x$	BTL -3	Applying
25.	Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so where?	BTL -4	Analyzing
<b>PART - B</b>			
1.(a)	Point out the domain where the function f is continuous Also find the number at which the function f is discontinuous when $f(x) = \begin{cases} 1+x^2 & \text{if } x \leq 0 \\ 2-x & \text{if } 0 < x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$	BTL -4	Analyzing
1.(b)	Find $\frac{dy}{dx}$ if $y = x^2 e^{2x} (x^2 + 1)^4$ .	BTL -3	Applying
2.(a)	For what value of the constant "c" is the function "f" continuous on $(-\infty, \infty)$ , $f(x) = \begin{cases} cx^2 + 2x; & x < 2 \\ x^3 - cx; & x \geq 2 \end{cases}$	BTL -4	Analyzing
2.(b)	Obtain $y''$ if $x^4 + y^4 = 16$	BTL -5	Evaluating
3.(a)	Find $y'$ for $\cos(xy) = 1 + \sin y$ .	BTL -3	Applying

3.(b)	Find the derivative of $f(x) = \cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$	BTL -5	Evaluating
4.(a)	Find the values of a and b that make f continuous on $(-\infty, \infty)$ $f(x) = \begin{cases} \frac{x^3-8}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$	BTL -3	Applying
4.(b)	Find $y'$ if $x = a\left(\cos\theta + \log \tan\frac{\theta}{2}\right)$ , $y = a \sin\theta$ .	BTL -3	Applying
5. (a)	For what value of the constant b, is the function f continuous on $(-\infty, \infty)$ if $f(x) = \begin{cases} bx^2 + 2x & \text{if } x < 2 \\ x^3 - bx & \text{if } x \geq 2 \end{cases}$	BTL -3	Applying
5.(b)	Find $\frac{dy}{dx}$ , when $y = \tan^{-1}\left(\frac{a \cos x + b \sin x}{b \cos x - a \sin x}\right)$	BTL -5	Evaluating
6.(a)	Find $\frac{dy}{dx}$ for the following functions $e^x + e^y = e^{x+y}$ .	BTL -3	Applying
6.(b)	Verify Rolle's theorem for the following $f(x) = 2x^3 - 5x^2 - 4x + 3, x \in \left[\frac{1}{2}, 3\right]$ .	BTL -3	Applying
7.	For the function $f(x) = 2+2x^2 - x^4$ , find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity and the inflection points.	BTL -1	Remembering
8.(a)	Find the absolute maximum and minimum of $f(x) = x - 2 \tan^{-1} x$ in $[0,4]$ .	BTL -4	Analyzing
8.(b)	Verify Lagrange's law for the following $f(x) = 2x^3 + x^2 - x - 1, x \in [0,2]$ .	BTL -3	Applying
9.	Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing. Also find the local maximum, local minimum, concavity and the inflection points of $f(x)$ .	BTL -5	Evaluating
10.(a)	Verify Lagrange's law for the following $f(x) = 2x^2 - 4x - 3, x \in [1,4]$ .	BTL -6	Creating
10.(b)	Find the tangent line to the equation $x^3 + y^3 = 6xy$ at the point (3,3) and at what point the tangent line is horizontal in the first quadrant	BTL -3	Applying
11.	If $f(x) = 2x^3 + 3x^2 - 36x$ , find the intervals on which it is increasing or decreasing, local maximum, local minimum values, concavity and the inflection points of $f(x)$ .	BTL -4	Analyzing
12.(a)	Verify mean value theorem for the following $f(x) = x^3 - 5x^2 - 3x, x \in [1,3]$ .	BTL -3	Applying
12.(b)	Find the Taylor's series expansion of $f(x) = \frac{1}{1+x}$ about $x=0$ .	BTL -3	Applying
13.	Discuss the curve $y = x^4 - 4x^3$ find the intervals on which it is increasing or decreasing, local maximum, local minimum values, concavity and the inflection points of $f(x)$ .	BTL -4	Analyzing
14.(a)	Verify Rolle's theorem for the following function	BTL -3	Applying



	$f(x) = \sin x, 0 \leq x \leq \pi$		
14.(b)	Find the Taylor's series expansion of $f(x) = \tan^{-1}x$ about $x = 0$ .	BTL -3	Applying
15.	Use second derivative test to examine the relative maxima for $f(x) = x(12 - 2x)^2$	BTL -4	Analyzing
16.(a)	Examine the local extreme of $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$ . Also discuss the concavity and find the inflection points	BTL -4	Analyzing
16.(b)	Verify Rolle's theorem for the following $f(x) = x(x - 1)(x - 2), x \in [0,2]$ .	BTL -3	Applying
17.	Find the local maximum and minimum values of $f(x) = \sqrt{x} - \sqrt[4]{x}$ using both first and second derivatives tests.	BTL -4	Analyzing
18.(a)	Of all the right circular cones of given slant length $l$ , find the dimensions and volume of the cone of maximum volume.	BTL -3	Applying
18.(b)	Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$	BTL -3	Applying
<b>PART - C</b>			
1.	Find the point on the parabola $y^2 = 2x$ that is close to the point (1,4)	BTL -4	Analyzing
2.	Find the equation of tangent at a point (a, b) to the curve $xy = c^2$ .	BTL -4	Analyzing
3.(a)	Apply Rolle's theorem to find points on curve $y = -1 + \cos x$ , where the tangents is parallel to x- axis in $0 \leq x \leq 2\pi$	BTL -2	Understanding
3.(b)	At what points on the curve $x^2 - y^2 = 2$ , the slopes of tangents are equal to 2.	BTL -5	Evaluating
4.	A cylindrical hole 4mm in diameter and 12mm deep in a metal block is rebored to increase the diameter to 4.12mm. Estimate the amount of metal removed.	BTL -2	Understanding
5.	A normal window consists of a rectangle surmounted by a semi-circle. Given the perimeter of the window to be k, find its height and breadth, if the quantity of light admitted is to be maximum.	BTL -3	Applying

### UNIT III -FUNCTIONS OF SEVERAL VARIABLES

Partial derivatives – Total derivative -Jacobians and properties - Taylor's series for functions of two variables – Maxima and minima of functions of two variables – Lagrange's method of undetermined multipliers

Q.No.	Question	BT Level	Competence
<b>PART - A</b>			
1.	If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ , then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ .	BTL -1	Remembering
2.	Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3ax^2y$	BTL -1	Remembering
3.	If $x^y + y^x = 1$ , then find $\frac{dy}{dx}$ .	BTL -1	Remembering



4.	Find the value of $\frac{du}{dt}$ , given $u = \log(x + y + z)$ , $x = e^t$ , $y = \sin t, z = \cos t$	BTL -2	Understanding
5.	Find the value of $\frac{du}{dt}$ , given $u = x^2 + y^2, x = at^2, y = 2at$ .	BTL -3	Applying
6.	If $u = x^3y^2 + x^2y^3$ where $x = at^2$ and $y = 2at$ , then find $\frac{du}{dt}$ .	BTL -3	Applying
7.	Find $\frac{du}{dt}$ if $u = \sin\left(\frac{x}{y}\right)$ , where $x = e^t, y = t^2$	BTL -2	Understanding
8.	Find $\frac{du}{dt}$ if $u = \frac{x}{y}$ , where $x = e^t, y = \log t$ .	BTL -2	Understanding
9.	Find $\frac{\partial r}{\partial x}$ , if $x = r \cos \theta$ & $y = r \sin \theta$ .	BTL -3	Applying
10.	Find the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$ , if $x = r \cos \theta$ & $y = r \sin \theta$ , $u = 2xy, v = x^2 - y^2$ without actual substitution.	BTL -2	Understanding
11.	If $u = \frac{y^2}{2x}$ and $v = \frac{x^2+y^2}{2x}$ , find $\frac{\partial(u,v)}{\partial(x,y)}$ .	BTL -4	Analyzing
12.	If $x = uv, y = \frac{u}{v}$ . Find $\frac{\partial(x,y)}{\partial(u,v)}$ .	BTL -3	Applying
13.	If $x = u^2 - v^2, y = 2uv$ find the Jacobian of $x, y$ with respect to $u$ and $v$	BTL -4	Analyzing
14.	If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$ , find $\frac{\partial(u,v)}{\partial(x,y)}$	BTL -2	Understanding
15.	Find the Taylor series expansion of $x^y$ near the point $(1, 1)$ up to first term	BTL -4	Analyzing
16.	Expand $xy + 2x - 3y + 2$ in powers of $(x - 1)$ & $(y + 2)$ , using Taylor's theorem up to first degree form	BTL -1	Remembering
17.	Find the Stationary points of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ .	BTL -4	Analyzing
18.	Find the Stationary points of $x^2 - xy + y^2 - 2x + y$ .	BTL -2	Understanding
19.	State the necessary and sufficient condition for $f(x, y)$ to have a relative maximum at a point $(a, b)$ .	BTL -4	Analyzing
20.	Find the minimum point of $f(x, y) = x^2 + y^2 + 6x + 12$ .	BTL -3	Applying
21.	If $u = x+y, y = uv$ then find $\frac{\partial(x,y)}{\partial(u,v)}$	BTL -2	Understanding
22.	Find the nature of the stationary point $(1,1)$ of the function $f(x,y)$ if $f_{yy} = 6x^3y, f_{xx} = 6y^3x, f_{xy} = 9x^2y$	BTL -4	Analyzing
23.	Find the minimum value of $f = x^2 + y^2$ subject to the constraint $x = 1$ .	BTL -3	Applying
24.	If $u = x^2+1, v = y^2 - 2$ then find $\frac{\partial(u,v)}{\partial(x,y)}$ .	BTL -2	Understanding
25.	Find the stationary points of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$	BTL -4	Analyzing
<b>PART - B</b>			
1.	If $u = \log(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$ , then prove that $u_{xx} + u_{yy} = 0$	BTL -3	Applying
2.(a)	Find $\frac{du}{dx}$ if $u = \cos(x^2 + y^2)$ and $a^2x^2 + b^2y^2 = c^2$	BTL -5	Evaluating
2.(b)	Find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ if $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$	BTL -5	Evaluating
3.	Find the shortest distance from the origin to the hyperbola	BTL -3	Applying

	$x^2 + 8xy + 7y^2 = 225$		
4.(a)	A flat circular plate is heated so that the temperature at any point $(x, y)$ is $u(x, y) = x^2 + 2y^2 - x$ . Find the coldest point on the plate	BTL -3	Applying
4.(b)	If $u = \frac{yz}{x}, v = \frac{zx}{y}$ and $w = \frac{xy}{z}$ , find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ .	BTL -3	Applying
5.	If $u = f(x - y, y - z, z - x)$ , then show $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	BTL -3	Applying
6.(a)	If $x + y + z = u, y + z = uv, z = uvw$ , prove $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$	BTL -3	Applying
6.(b)	Find the extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$	BTL -3	Applying
7.	Expand $x^3y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ using Taylor's series up to third degree terms	BTL -1	Remembering
8.(a)	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$	BTL -4	Analyzing
8.(b)	If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 = \frac{9}{(x+y+z)^2}$	BTL -3	Applying
9.	Discuss the extreme values of $f(x, y) = x^3y^2(1 - x - y)$ .	BTL -5	Evaluating
10.(a)	Expand Taylor's series of $x^3 + y^3 + xy^2$ in powers of $(x - 1)$ and $(y - 2)$ up to the second-degree terms	BTL -6	Creating
10.(b)	If $u = xyz, v = x^2 + y^2 + z^2$ and $w = x + y + z$ , find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	BTL -3	Applying
11.	Find the Taylor's series expansion of $e^x \sin y$ at the point $(-1, \frac{\pi}{4})$ up to third degree terms	BTL -4	Analyzing
12.(a)	Expand $\tan^{-1} \frac{y}{x}$ in the neighborhood of $(1, 1)$ as Taylor's series up to second degree terms	BTL -3	Applying
12.(b)	Find the Maximum value of $x^m y^n z^p$ when $x + y + z = a$ .	BTL -3	Applying
13.	Find the Taylor's series expansion of $e^x \sin y$ at the point $(1, \frac{\pi}{4})$ up to third degree terms	BTL -4	Analyzing
14.(a)	Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$ .	BTL -3	Applying
14.(b)	Expand $e^{xy}$ in powers of $(x - 1)$ and $(y - 1)$ up to second degree terms by Taylor's series	BTL -3	Applying
15.	Find the shortest and longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$	BTL -4	Analyzing
16.(a)	Examine $f(x, y) = x^3 + 3xy^2 - 15x^2 + 72x$ for extreme values	BTL -4	Analyzing
16.(b)	Expand $e^x \log(1 + y)$ in powers of $x$ & $y$ up to terms of third-	BTL -3	Applying

	degree using Taylor's series		
17.	Find the dimension of the rectangular box without a top of maximum capacity, whose surface area is 108 sq. cm.	BTL -4	Analyzing
18.(a)	If $u = x + y + z, u^2v = y + z, u^3w = z$ , prove $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1}{u^5}$	BTL -3	Applying
18.(b)	Find the minimum value of $x^2yz^3$ subject to $2x + y + 3z = a$ .	BTL -3	Applying
<b>PART – C</b>			
1.	Divide the number 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum	BTL -4	Analyzing
2.	The temperature at any point $(x, y, z)$ in space is given by $T = kxyz^2$ , where k is constant. Find the height temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$	BTL -4	Analyzing
3.	A rectangular box open at the top is to have a volume 32cc. Find the dimensions of the box that requires the least for its construction.	BTL -2	Understanding
4.	Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .	BTL -2	Understanding
5.	In a plane triangle find the maximum value of $\cos A \cdot \cos B \cdot \cos C$	BTL -3	Applying

#### UNIT IV -IV INTEGRAL CALCULUS

Definite and Indefinite integrals - Substitution rule - Techniques of Integration - Integration by parts, Trigonometric integrals, Trigonometric substitutions, Integration by partial fraction, - Improper integrals

Q.No.	Question	BT Level	Competence
<b>PART – A</b>			
1.	Prove that the following integral by interpreting each in terms of areas $\int_a^b x dx = \frac{b^2 - a^2}{2}$	BTL -1	Remembering
2.	State fundamental theorem of calculus	BTL -1	Remembering
3.	Evaluate $\int_0^1 \sqrt{1 - x^2} dx$ in terms of area.	BTL -1	Remembering
4.	If $f$ is continuous and $\int_0^4 f(x) dx = 10$ , find $\int_0^2 f(2x) dx$	BTL -2	Understanding
5.	Evaluate $\int \sin^3 x dx$	BTL -3	Applying
6.	Calculate $\int \frac{x^3}{\sqrt{4+x^2}} dx$	BTL -3	Applying
7.	Calculate $\int \sqrt{1+x^2} x^5 dx$	BTL -2	Understanding
8.	Find $\int \sqrt{2x+1} dx$	BTL -2	Understanding
9.	Find $\int \frac{x}{\sqrt{1-4x^2}} dx$	BTL -3	Applying
10.	Evaluate $\int_0^1 \tan^{-1} x dx$	BTL -2	Understanding
11.	Calculate $\int \frac{(\ln x)^2}{x} dx$	BTL -4	Analyzing
12.	Calculate $\int (\log x)^2 dx$	BTL -3	Applying
13.	Evaluate $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$	BTL -4	Analyzing

14.	Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan x} dx$	BTL -2	Understanding
15.	Evaluate $\int_4^{\infty} \frac{1}{\sqrt{x}} dx$ and determine whether it is convergent or divergent.	BTL -4	Analyzing
16.	If $f(1) = 12$ , $f'$ is continuous and $\int_1^4 f'(x) dx = 17$ , then find the value of $f(4)$ .	BTL -1	Remembering
17.	Estimate $\int_1^3 \sqrt{x^2 + 3} dx$	BTL -4	Analyzing
18.	Evaluate the improper integral $\int_2^3 \frac{dx}{\sqrt{3-x}}$ , if possible.	BTL -2	Understanding
19.	Find $\int_2^5 \frac{dx}{\sqrt{x-2}}$	BTL -4	Analyzing
20.	Prove that $\int_1^{\infty} \frac{1}{x} dx$ is divergent.	BTL -3	Applying
21.	Find $\int \sqrt{1 + \sin 2x} dx$	BTL -2	Understanding
22.	Test the convergence of $\int_1^{\infty} \frac{1}{x^2} dx$	BTL -4	Analyzing
23.	Evaluate $\int (1 + \tan x)^n \sec^2 x dx$	BTL -3	Applying
24.	Evaluate $\int \frac{1}{x(\log x)^n} dx$	BTL -2	Understanding
25.	Evaluate $\int \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} d\theta$	BTL -4	Analyzing
<b>PART – B</b>			
1.	Find $\int x^3 \sqrt{9 - x^2} dx$ by trigonometric substitution.	BTL -3	Applying
2.(a)	Evaluate $\int \frac{(\ln x)^2}{x^2} dx$	BTL -5	Evaluating
2.(b)	Find $\int \frac{\sec^2 x}{\tan^2 x + 3 \tan x + 2} dx$	BTL -5	Evaluating
3.	Evaluate $\int e^{ax} \cos bx dx$ using integration by parts	BTL -3	Applying
4.(a)	Prove that $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$ ( $n \neq 1$ )	BTL -3	Applying
4.(b)	Evaluate $\int \frac{\tan x}{\sec x + \cos x} dx$	BTL -3	Applying
5.	Calculate using partial fraction $\int \frac{10}{(x-1)(x^2+9)} dx$	BTL -3	Applying
6.(a)	Evaluate $\int \frac{xe^{2x}}{(1+2x)^2} dx$	BTL -3	Applying
6.(b)	Evaluate $\int x \tan^{-1} x dx$	BTL -3	Applying
7.	Prove the reduction formula $\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$	BTL -1	Remembering
8.(a)	Calculate by partial fraction $\int \frac{x^2+1}{(x-3)(x-2)^2} dx$	BTL -4	Analyzing
8.(b)	Evaluate $\int \sin^6 x \cos^3 x dx$ .	BTL -3	Applying
9.	Prove that $\int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$ ( $n \neq 1$ )	BTL -5	Evaluating
10.(a)	Evaluate $\int_0^{\pi/2} \sin^7 x \cos^5 x dx$	BTL -6	Creating
10.(b)	For what values of $p$ is $\int_1^{\infty} \frac{1}{x^p} dx$ convergent?	BTL -3	Applying
11.	Prove the reduction formula	BTL -4	Analyzing

	$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$		
12.(a)	Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x dx$	BTL -3	Applying
12.(b)	Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} \cos^5 \theta d\theta$	BTL -3	Applying
13.	Evaluate $\int e^{ax} \sin bx dx$ using integration by parts.	BTL -4	Analyzing
14.(a)	Evaluate $\int_0^{\frac{\pi}{2}} \tan^5 x \sec^6 x dx$	BTL -3	Applying
14.(b)	Find $\int \frac{x^5}{\sqrt{1+x^3}} dx$	BTL -3	Applying
15.	Evaluate $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3 \cos x + 2} dx$	BTL -4	Analyzing
16.(a)	Evaluate $\int \sqrt{\frac{x^2-1}{x^8}} dx$	BTL -4	Analyzing
16.(b)	Find $\int_0^{\pi} \sin^4 3t dt$	BTL -3	Applying
17.	Evaluate $\int \frac{x^2 + x + 1}{(x-1)^2(x^2+2)} dx$	BTL -4	Analyzing
18.(a)	Evaluate $\int e^x \sin x dx$	BTL -3	Applying
18.(b)	Evaluate $\int e^{\tan^{-1} x} \frac{(1+x+x^2)}{1+x^2} dx$	BTL -3	Applying
<b>PART - C</b>			
1.	The region enclosed by the circle $x^2 + y^2 = a^2$ is divided into two segments by the line $x = h$ . Find the area of the smaller segment	BTL -4	Analyzing
2.	Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum	BTL -4	Analyzing
3.	Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , $a > b$ about the major axis.	BTL -2	Understanding
4.	Find the volume of a sphere of radius $a$ units.	BTL -2	Understanding
5.	Evaluate $\int \frac{3x-4}{2x^2-5x+3} dx$	BTL -3	Applying

### UNIT V -MULTIPLE INTEGRALS

Double integrals in Cartesian and polar coordinates – Change of order of integration – Area enclosed by plane curves Change of variables in double integrals (Polar coordinates) – Triple integrals – Volume of solids

Q.No.	Question	BT Level	Competence
<b>PART - A</b>			
1.	Evaluate $\int_2^3 \int_1^2 \frac{dx dy}{xy}$	BTL -1	Remembering
2.	Evaluate $\int_0^2 \int_0^1 4xy dx dy$	BTL -2	Understanding
3.	Evaluate $\int_0^{\pi} \int_0^{\sin \theta} r dr d\theta$	BTL -1	Remembering

4.	Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(\theta + \varphi) d\theta d\varphi$	BTL -3	Applying
5.	Find the area bounded by the lines $x = 0, y = 1$ and $y = x$	BTL -1	Remembering
6.	Evaluate $\int_0^{\pi} \int_0^a r dr d\theta$	BTL -2	Understanding
7.	Evaluate $\int_0^5 \int_0^2 (x^2 + y^2) dx dy$	BTL -3	Applying
8.	Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dy dx$	BTL -3	Applying
9.	Evaluate $\int_1^{\ln 8} \int_0^{\ln y} e^{x+y} dx dy$	BTL -2	Understanding
10.	Evaluate $\int_0^{\pi} \int_0^5 r^4 \sin \theta dr d\theta$	BTL -2	Understanding
11.	Evaluate $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{xdy}{1+x^2+y^2}$	BTL -3	Applying
12.	Evaluate $\int_0^2 \int_0^x \frac{xdy}{x^2+y^2}$	BTL -3	Applying
13.	Evaluate $\int \int dx dy$ over the region bounded by $x = 0, x = 2, y = 0$ and $y = 2$	BTL -2	Understanding
14.	Change the order of integration $\int_0^1 \int_{y^2}^y f(x, y) dx dy$	BTL -4	Analyzing
15.	Change the order of integration $\int_0^{\infty} \int_x^{\infty} f(x, y) dx dy$	BTL -3	Applying
16.	Find the limits of integration in the double integral $\iint_R f(x, y) dx dy$ where R is in the first quadrant and bounded $x=1, y=0, y^2 = 4x$	BTL -4	Analyzing
17.	Find the limits of integration in the double integral $\iint_R f(x, y) dx dy$ where R is in the first quadrant and bounded $x = 0, y = 0, x + y = 1$	BTL -3	Applying
18.	Find the limits of integration in the double integral $\iint_R f(x, y) dx dy$ where R is in the first quadrant and bounded $x = 0, y = 0, \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .	BTL -3	Applying
19.	Evaluate $\int \int \int (x + y + z) dx dy dz$ over the region bounded by $x = 0, x = 1, y = 0$ and $y = 1, z = 0, z = 1$	BTL -2	Understanding
20.	Write down the double integral to find the area of the circles $r = 2\sin\theta, r = 4\sin\theta$	BTL -4	Analyzing
21.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy(x+y) dy dx$	BTL -1	Remembering
22.	Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2) dy dx$	BTL -4	Analyzing
23.	Evaluate $\int_1^3 \int_3^4 \int_1^4 xyz dz dy dx$	BTL -2	Understanding
24.	Evaluate $\int_0^1 dx \int_0^2 dy \int_0^3 (x + y + z) dz$	BTL -4	Analyzing
25.	Evaluate $\int_a^b \int_c^d \int_f^g e^{x+y+z} dz dy dx$	BTL -3	Applying
<b>PART – B</b>			
1.	Evaluate $\iint xy dx dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$	BTL -4	Applying

2.(a)	Change the order of integration $\int_0^2 \int_0^{\sqrt{4-y^2}} xy \, dx dy$ and hence evaluate it	BTL -2	Understanding
2.(b)	Using double integral find the area of the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .	BTL -4	Applying
3.	Change the order of integration $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy \, dx dy$ and hence evaluate it	BTL -2	Understanding
4.(a)	Change the order of integration $\int_0^1 \int_y^{2-y} xy \, dx dy$ and hence evaluate it	BTL -2	Understanding
4.(b)	Evaluate $\iint xy \, dx dy$ over the positive quadrant bounded by the line $2x + 3y = 6$	BTL -3	Analyzing
5.	Change the order of integration $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y \, dx dy$ and hence evaluate it	BTL -3	Analyzing
6.(a)	Change the order of integration $\int_0^\infty \int_0^\infty \frac{e^{-y}}{y} \, dy dx$ and hence evaluate it	BTL -4	Applying
6.(b)	Find the area bounded by parabolas $y = 4 - x$ and $y^2 = x$ by double integration.	BTL -1	Remembering
7.	Change the order of integration $\int_0^a \int_y^a \frac{x \, dx \, dy}{\sqrt{x^2+y^2}}$ and hence evaluate it	BTL -2	Understanding
8.(a)	By changing in to polar Co – ordinates, evaluate $\int_0^a \int_0^x \frac{x^3}{\sqrt{x^2+y^2}} \, dx dy$ .	BTL -3	Analyzing
8.(b)	By change the order of integration and evaluate $\int_0^2 \int_{x^2}^{2-x} xy \, dy dx$	BTL -2	Understanding
9.	By changing in to polar Co – ordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx dy$ . Hence find the value of $\int_0^\infty e^{-x^2} \, dx$ .	BTL -2	Understanding
10.(a)	Find the area included between the curves $y^2 = 4x$ and $x^2 = 4y$	BTL -2	Understanding
10.(b)	Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz dy dx$	BTL -2	Understanding
11.	Change the integral into polar coordinates $\int_0^a \int_0^x \frac{x^3}{\sqrt{x^2+y^2}} \, dx dy$ and hence evaluate it	BTL -2	Understanding
12.(a)	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$	BTL -5	Evaluating
12.(b)	Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) \, dx dy dz$	BTL -6	Creating
13.	Find the area common to the cardioids $r = a(1 + \cos \theta)$ and $r = a(1 - \cos \theta)$	BTL -2	Understanding
14.(a)	Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-y^2-z^2}} x \, dx dy dz$	BTL -4	Analyzing



14.(b)	Evaluate $\iint (x^2 + y^2) dx dy$ over the area bounded by the parabola $y^2 = 4x$ and its latus rectum.	BTL -3	Applying
15.	Evaluate $\iiint \frac{dx dy dz}{x^2+y^2+z^2}$ , taken throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$ .	BTL -3	Applying
16.(a)	Find the value of $\iiint xyz dx dy dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \leq a^2$ .	BTL -4	Analyzing
16.(b)	Find the area which is inside the circle $r = 3a \cos\theta$ and outside the cardioid $r = a(1 + \cos\theta)$ .	BTL -4	Analyzing
17.	Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ .	BTL -3	Applying
18.(a)	Find the volume bounded by the cylinder $x^2 + y^2 = 1$ and the planes $x + y + z = 3, z = 0$	BTL -4	Analyzing
18.(b)	Find the area between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ lying in the first quadrant, by double integration.	BTL -3	Applying
<b>PART - C</b>			
1.	Find the area bounded by parabola $y = x^2$ and straight line $2x - y + 3 = 0$ .	BTL -4	Analyzing
2.	Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	BTL -4	Analyzing
3.	Find the volume of finite region of space (tetra-hadron) bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 3y + 4z = 12$	BTL -2	Understanding
4.	Find the volume of sphere bounded by $x^2 + y^2 + z^2 = a^2$ .	BTL -2	Understanding
5.	Change the order of integration $\int_0^1 \int_{y^2}^y \frac{y dx dy}{x^2 + y^2}$ and hence evaluate it	BTL -3	Applying