SRM VALLIAMMAI ENGINEERING COLLEGE

(An Autonomous Institution)

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DEPARTMENT OF INFORMATION TECHNOLOGY

QUESTION BANK

I SEMESTER

1918109- MATHEMATICAL FOUNDATION FOR DATA SCIENCE

Regulation – 2019

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QUESTION BANK

SUBJECT CODE: 1918109

SUBJECT TITLE: MATHEMATICAL FOUNDATION FOR DATA SCIENCE

YEAR /SEMESTER: I Year / I Semester M.Tech.

Q.No.	Question	BT -Level	Competence
UNIT I MATRICES	Eigen values, Eigenvectors and Diagonalization , singular value decomposition , Positive definite Matrices-Minima, Maxima and saddle points, Test of Positive definiteness, semi- definite and indefinite Matrices		
PART-A			
1	Find the eigen values of A=$\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$	BTL-1	Remembering
2	Two eigen values of the matrix A= $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$ are 0 and 1. Find the third eigen value.	BTL-3	Applying
3	Find the eigen values of A^{-1} if the matrix A is $\begin{pmatrix} 2 & 5 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{pmatrix}$	BTL-3	Applying
4	Find the characteristic equation of A=$\begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$	BTL-1	Remembering
5	Find the eigen values of A^2 if A=$\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$	BTL-1	Remembering
6	If the eigen values of the matrix A of order 3X3 are 2,3 and 1, then find the determinant of A	BTL-2	Understanding
7	If the sum of 2 eigen values and the trace of a 3×3 matrix are equal, find the value of $\|A\|$	BTL-1	Remembering
8	Find the sum of the eigen values of 2A, if A= $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	BTL-3	Applying
9	The product of the 2 eigen values of A= $\begin{pmatrix} 6 & -2 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 14. Find the 3^{rd}eigen value.	BTL-3	Applying
10	Find the sum and product of the eigen values of A= $\begin{pmatrix} 2 & -2 & 2 \\ -2 & -1 & -1 \\ 2 & -1 & -1 \end{pmatrix}$	BTL-3	Applying
11	Find the sum and product of the eigen values of A= $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$	BTL-1	Remembering
12	Find the constants a and b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3,-2 be the eigen values of A	BTL-1	Remembering
13	Find the matrix corresponding to the quadratic form 2xy-2yz+2xz.	BTL-1	Remembering
14	Find the quadratic form corresponding to the matrix A= $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	BTL-4	Analyzing
15	Write the 3 X 3 matrix corresponding to the function $x_1^2 + x_2^2 + x_3^2 - 2x_1 - 2x_1x_2 + 2x_2x_3$	BTL-2	Understanding
16	If the eigen values of the matrix A=$\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ are 2,-2 then find the eigenvalues of A^T.	BTL-2	Understanding
17	If A= $\begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$ Find the eigen values of A^2.	BTL-2	Understanding

18	If 1,1,5 are the eigen values of A= $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$	BTL-2	Understanding
19	What are the tests available to test the positive definiteness of a matrix	BTL-4	Analyzing
20	Decide for the positive definiteness of $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 2 \end{pmatrix}$ using eigen values	BTL-5	Evaluating
21	Decide for the positive definiteness of $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ using sub matrices	BTL-5	Evaluating
22	Write down the condition for minimum of the function $ax^2 + 2bxy + cy^2$	BTL-6	Creating
23	Test for minima or maxima for the function $2x^2 + 4xy + y^2$	BTL-6	Creating
24	Define positive definite matrix	BTL-2	Understanding
25	State Singular value decomposition theorem	BTL-2	Understanding
PART-B			
1	Find the eigen values and eigen vectors of the matrix A= $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$	BTL-5	Evaluating
2	Find the eigen values and eigen vectors of the matrix A= $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$	BTL-5	Evaluating
3	Find the eigen values and eigen vectors of the matrix A= $\begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$	BTL-5	Evaluating
4	Obtain the eigen values and eigen values and eigen vectors of the matrix A= $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$.	BTL-6	Creating
5	Obtain the eigen values and eigen vectors of the matrix A= $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	BTL-2	Understanding
6	Reduce the matrix A= $\begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$ to diagonal form	BTL-4	Analyzing
7	Diagonalize the matrix A= $\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 5 \end{pmatrix}$ by means of an orthogonal transformation.	BTL-2	Understanding
8	Diagonalize the matrix A= $\begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$ by means of an orthogonal transformation.	BTL-4	Analyzing
9	Diagonalize the matrix A= $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ by means of an orthogonal transformation.	BTL-5	Evaluating
10	Obtain the singular value decomposition of A= $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$	BTL-4	Analyzing
11	Get the singular value decomposition of $\begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}$	BTL-1	Remembering
12	Find the singular value decomposition of A= $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$	BTL-4	Analyzing

13	Evaluate the singular value decomposition of $\begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix}$	BTL-5	Evaluating
14	Discuss the positive semi definiteness of the matrix $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$ using all the tests	BTL-2	Understanding
15	Test whether A^TA is positive definite if A=$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$	BTL-2	Understanding
16	Discuss the positive definiteness of the matrix $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ using all the tests	BTL-2	Understanding
17	Check the matrix for positive definite A=$\begin{pmatrix} 5 & 6 \\ 6 & 7 \end{pmatrix}$	BTL-1	Remembering
18	Test whether the quadratic (i) $f = x^2 + 4xy + 2y^2$ has a saddle point. Also write f as a difference of two squares (ii) $f = 2x^2 + 4xy + y^2$has a saddle point. Also write f as a difference of two squares	BTL-1	Remembering
	Part C		
1.	Reduce the matrix A=$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ to diagonal form using orthogonal transformation.	BTL-6	Creating
2.	Diagonalize the matrix A=$\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$ by means of an orthogonal transformation.	BTL-6	Creating
3.	The Eigen vectors of a $3X3$ real symmetric matrix A corresponding to the eigen values 1,2,4 are $(1,\ 0,\ 0)^T, (0,\ 1,\ 1)^T, (0,\ 1,\ -1)^T$ respectively Create the matrix A.	BTL-6	Creating
4	Prepare the singular value decomposition of $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$	BTL-6	Creating
5	Obtain the singular value decomposition of $\begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$	BTL-6	Creating

UNIT II LINEAR ALGEBRA

Geometry of linear equations, real vector spaces (Rn, matrices etc.) and subspaces, linear independence, basis and dimensions, linear transformations, orthogonality, Orthogonal basis, Gram Schmidt Process s projections and least square applications

Q.No.	Question	BTLevel	Domain
	PART – A		
1.	Draw the row picture and column picture for the system $2x - y = 1, x + y = 5$	BTL -1	Remembering
2.	Draw the row picture and column picture for the system $x + y = 4, 2x - 2y = 4$	BTL -4	Analyzing
3.	Draw the row picture and column picture for the system $2x - y = 0, -x + 2y = 3.$	BTL -2	Understanding
4.	Draw the row picture and column picture for the system $x - 2y = 0, x + y = 6.$	BTL -3	Applying
5.	Write down the column form of the system equations $2x + y + z = 5, 4x - 6y = -2, -2x + 7y + 2z = 9$	BTL -2	Understanding
6.	Define Vector Space	BTL -4	Analyzing
7.	State and prove cancellation law for vector addition	BTL -6	Creating
8.	Define Subspace of a vector space	BTL -3	Applying
9.	What are the possible subspaces of R^2	BTL -3	Applying
10.	Is {(1,4,–6),(1,5,8),(2,1,1),(0,1,0)}is a linearly independent subset of R^3?Justify your answer	BTL -5	Evaluating

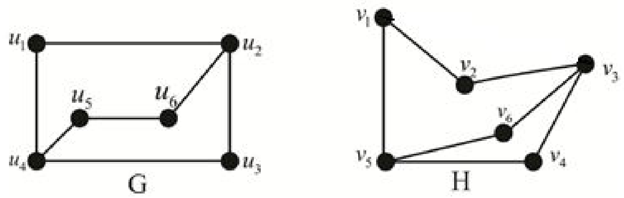
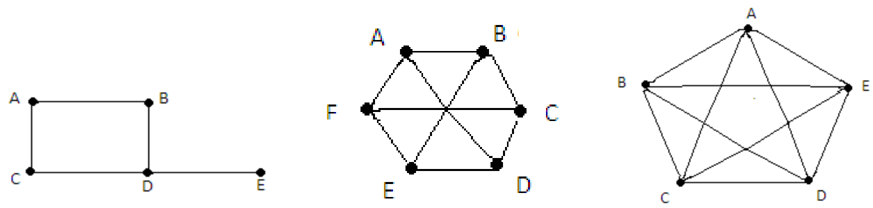
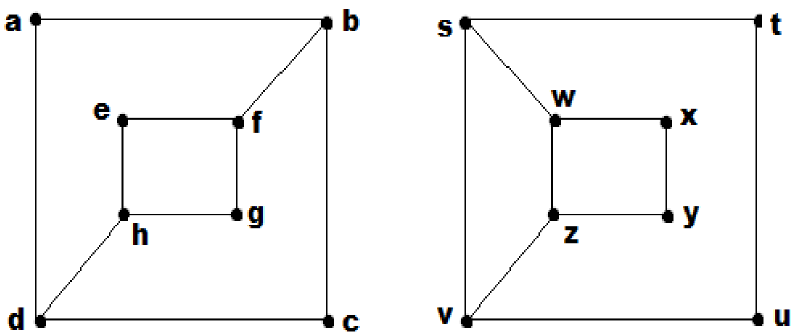
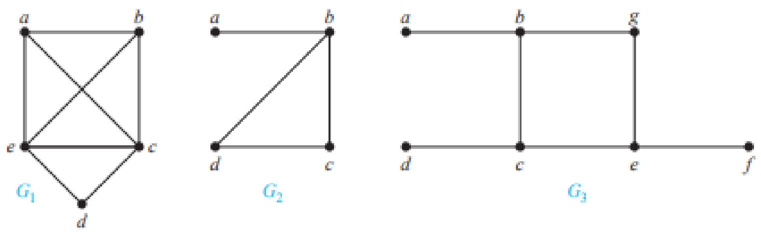
11.	Write the vectors $v=(1,-2,5)$ as a linear combination of the vectors $x=(1,1,1), y=(1,2,3)$ and $z=(2,-1,1)$	BTL -2	Understanding
12.	Show that the vectors $\{(1,1,0),(1,0,1)$ and $(0,1,1)\}$ generate F^3	BTL -2	Understanding
13.	Define Null space	BTL -2	Understanding
14.	Evaluate which of the following sets are bases for R^3: (i)$\{(1,0,-1),(2,5,1),(0,-4,3)\}(ii)\{(-1,3,1),(2,-4,-3),(-3,8,2)\}$	BTL -2	Understanding
15.	Illustrate that the transformation $T:R^2\to R^2$ defined by $T(a_1,a_2)=(2a_1+a_2,a_2)$ is linear	BTL -2	Understanding
16.	Define linear dependent set and Independent set of a vector space.	BTL -2	Understanding
17.	Determine whether the vectors $v_1 = (1, -2, 3)$, $v_2 = (5, 6, -1)$, $v_3 = (3, 2, 1)$ form a linearly dependent or linearly independent set in R^3	BTL -3	Applying
18.	Find the norm and distance between the vectors (1,0,1) and (-1,1,0)	BTL -3	Applying
19.	Check whether the vectors (2,2,-1) and (-1,2,2) are orthogonal to each other. Also find the lengths.	BTL -4	Analyzing
20.	State dimension theorem	BTL -2	Understanding
21.	Define linear transformation	BTL -3	Applying
22.	State the properties of linear transformation	BTL -3	Applying
23.	Find the values of a if the vectors (2, a) and (6, 4) are orthogonal vectors in R^2.	BTL -3	Applying
24.	Find k so that u=(1,2,k,3) and v=(3,k,7,-5) in R^4 are orthogonal.	BTL -3	Applying
25.	Define orthogonal basis.	BTL -2	Understanding
PART – B			
1.	Determine whether the set of all 2×2 matrices of the form $\begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix}$ a, b in R with respect to standard matrix addition and scalar multiplication is a vector space or not? If not list all the axioms that fail to hold.	BTL -3	Applying
2.	Find a combination $x_1w_1 + x_2w_2 + x_3w_3$ that gives a zero vector, where $w_1 = \begin{pmatrix}1\\2\\3\end{pmatrix}, w_2 = \begin{pmatrix}4\\5\\6\end{pmatrix}, w_3 = \begin{pmatrix}7\\8\\9\end{pmatrix}$	BTL -4	Analyzing
3.	Show that the set of all positive real numbers, with $x + y = xy$, $cx = x^c$ is a vector space. What is the zero vector	BTL -3	Applying
4.	Let v denote the set of ordered pairs of real numbers. If (a_1, a_2) and (b_1, b_2) are elements of V and $c\in$ R, define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2)$ and $c(a_1, a_2) = (ca_1, a_2)$. Is V a vector space over R with these operations? Justify your answer.	BTL -3	Applying
5. (a)	Determine whether the set of vectors u={6,2,3,4}, v={0,5-3,1} and w=(0,0,7,-2} are linearly independent.	BTL -4	Analyzing
5. (b)	Illustrate that the vectors $\{(1,1,0),(1,0,1),(0,1,1)\}$ generate R^3	BTL -3	Applying
6.	Determine whether the set of vectors $X_1 = (1,1,2)$, $X_2 = (1,0,1)$,and $X_3 = (2,1,3)$ span R^3	BTL -4	Analyzing
7.	Identify whether the set $\{x^3+2x^2,-x^2+3x+1,x^3-x^2+2x-1\}$ in $P_3(R)$ is linearly independent or not	BTL -1	Remembering
8. (a)	Determine whether the set of vectors s={(1,3,-4,2),(2,2,-4,0),(1,-3,2,-4), (-1,0,1,0)} are linearly independent.	BTL -4	Analyzing
8. (b)	Show that the vectors u=(1,2,3), v=(0,1,2) and w=(0,0,1) generate R^3	BTL -4	Analyzing
9.	Decide whether or not the set $S=\{x^3+3x-2,2x^2+5x-3,-x^2-4x+4\}$ is a basis for $P_2(R)$	BTL -3	Applying
10.	Let$T:R^3\to R^3$ be a linear transformation defined by $(x,y,z)=(x+2y-z,y+z, x+y-2z)$. Evaluate a basis and dimension of null space N(T) and range space R(T). Also verify dimension theorem	BTL -3	Applying
11.	Let $T:R^3\to R^2$ be defined by $(x,,)=(2x-y,3z)$ verify whether T is linear or not. Find N(T)and R(T) and hence verify the dimension theorem	BTL -3	Applying
12.(a)	Prove that any intersection of subspaces of a vector space V is a subspace of V	BTL -3	Applying

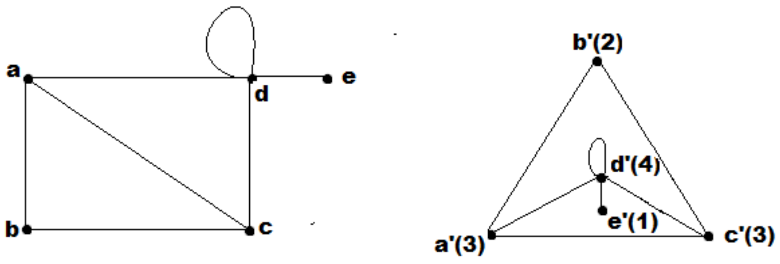
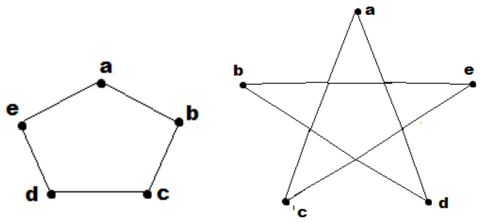
12.(b)	Prove that the union of two subspaces is not necessarily a subspace	BTL -3	Applying
13.	Let V=$P_2(R)$.) and $< f(x), g(x) > = \int_{-1}^{1} f(t)g(t)dt$, β be the standard ordered basis, using Gram- Schmidt process obtain orthonormal basis for $P_2(R)$.	BTL -4	Analyzing
14.	Let the vector space P_2 have the inner product $< p, q > = \int_0^1 p(x)q(x)dx$, Apply Gram- Schmidt process to transform the basis S={1,x,x^2} into an orthonormal basis.	BTL -4	Analyzing
15.	Apply the Gram-Schmidt process to the vectors u_1=(1,0,1), u_2=(1,0,-1), u_3=(0,3,4) to obtain an orthonormal basis for R^3.	BTL -3	Applying
16.	Apply the Gram-Schmidt process to the vectors u_1=(1,1,1), u_2=(0,1,1), u_3=(0,0,1) to obtain an orthonormal basis for R^3.	BTL -3	Applying
17.	Solve the following system of equations in the least square sense $2x_1 + 2x_2 - 2x_3 = 1, 2x_1 + 2x_2 - 2x_3 = 3, -2x_1 - 2x_2 + 6x_3 = 2$	BTL -2	Understanding
18.	Solve the following system of equations in the least square sense $x_1 + x_2 + 3x_3 = 1$; $x_1 + x_2 + 3x_3 = 2$	BTL -3	Applying
	Part C		
1.	Discuss various singular cases in the system of linear equations in three dimensions	BTL -3	Applying
2.	Let $x = (2, 1 + i, i)$ and $y = (2 - i, 2, 1 + 2i)$ be vectors in C^3. Compute $< x, y >$. $\||x|, |y|$ and $|x + y|$. Then verify both the Cauchy Schwarz inequality and the triangle inequality.	BTL -5	Evaluating
3.	Apply Gram Schmidt process to the given vector space V=R^3 and the subset S= {(1,0,1), (0,1,1) , (1,3,3)} and x = (1,1,2) of the inner product space V to obtain an orthogonal bases, normalize the vectors to obtain orthonormal basis.	BTL -6	Creating
4.	Solve the following system of equations in the least square sense $x_1 + x_2 + x_3 = 1$; $x_1 + x_2 + x_3 = 2$; $x_1 + x_2 + x_3 = 3$.	BTL -6	Creating
5.	Solve the following system of equations in the least square sense x + 2y - z = 1 ; 2x + 3y + z = 2 ; 4x + 7y - z = 4.	BTL -6	Creating

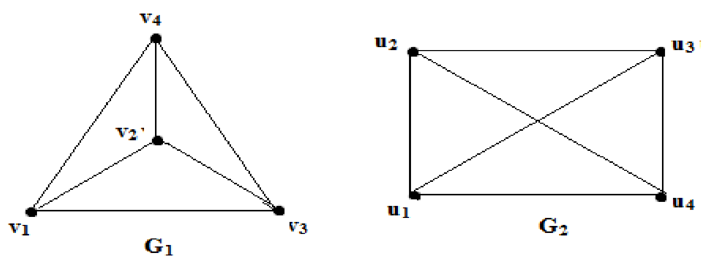
UNIT-III GRAPH THEORY AND COMBINATORICS

Graph Theory: Isomorphism, Planar graphs, graph coloring, Hamilton circuits and Euler cycles. Permutations and Combinations with and without repetition. Specialized techniques to solve combinatorial enumeration problems

Q.No	Question	BTLevel	Domain
	PART – A		
1.	Define isomorphism of two graphs	BTL -1	Remembering
2.	Define planar graph	BTL -1	Remembering
3.	What are the graph invariants of graph isomorphism?	BTL -1	Remembering
4.	Define Euler path and Euler Circuit	BTL -1	Remembering
5.	Define Hamilton path and circuit	BTL -2	Understanding
6.	Define planar graph	BTL -1	Remembering
7.	Write down Euler formula for planar graphs	BTL -3	Applying
8.	Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?	BTL -3	Applying
9.	Show that K_5 is non planar	BTL -2	Understanding
10.	Define graph coloring	BTL -3	Applying
11.	Define chromatic number of a graph	BTL -4	Analyzing
12.	State Ore's theorem and Dirac's theorem	BTL -3	Applying
13.	Show that in any group any group 8 people at least two have birthdays which falls on same day of the week in any given year	BTL -1	Remembering
14.	How many permutations are there in the word MISSISSIPPI?	BTL -2	Understanding
15.	State the Pigeonhole principle.	BTL -3	Applying
16.	How many cards must be selected from a deck of 52 cards to guarantee that atleast 3 cards of the same suit are chosen?	BTL -2	Understanding

17.	How many different bit strings are there of length seven?	BTL -3	Applying
18.	How many permutations are there in the word MALAYALAM?	BTL -4	Analyzing
19.	How many different committees of three students can be formed from a group of four students?	BTL -4	Analyzing
20.	How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?	BTL -4	Analyzing
21.	How many 16-bit strings are there containing exactly 5 zeros?	BTL -4	Analyzing
22.	How many 7 digit numbers can be formed using the digits 1,2,0,2,4,2 and 4?	BTL -4	Analyzing
23.	How many permutations of $\{a, b, c, d, e, f, g\}$ starting with a?	BTL -4	Analyzing
24.	In how many ways can all the letters in MATHEMATICAL be arranged?	BTL -4	Analyzing
25.	Give an example of a graph which is Eulerian but not Hamiltonian	BTL -4	Analyzing
	PART – B		
1.(a)	Examine whether the following pair of graphs are isomorphic or not. Justify your answer. 	BTL -2	Understanding
1.(b)	Find an Euler path or an Euler Circuit if it exists, in each of the three graphs given below. 	BTL -3	Analyzing
2.(a)	Show that the following graphs G and H are not isomorphic. 	BTL -2	Understanding
2.(b)	Which of the simple graphs in Figure have a Hamilton circuit or, if not, a Hamilton path? 	BTL -3	Applying
3.	Show that the following graphs are isomorphic.	BTL -4	Analyzing

			
4. (a)	Using adjacency matrix examine whether the following pairs of graphs $G\ and\ G^1$ given below are isomorphism or not. 	BTL -4	Analyzing
4.(b)	What is the chromatic number of the complete bipartite graph Km,n, where m and n are positive integers. Also illustrate the coloring of $K_{3,4}$	BTL -3	Analyzing
5.	The adjacency matrices of two pairs of graph as given below. Examine the isomorphism of G and H by finding a permutation matrix. $$A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$	BTL -4	Applying
6. (a)	Define Isomorphism between the two graphs. Are the simple graphs with the following adjacency matrices isomorphic? $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$	BTL -3	Analyzing
6.(b)	What is the chromatic number of Kn? . Also illustrate the coloring of K_5	BTL -3	Analyzing
7. (a)	From a club consisting of six men and seven women, in how many ways we select a committee of (1) 3 men and 4 women? (2) 4 persons which has at least one woman? (3) 4 persons that has at most one man? (4) 4 persons that has both sexes?	BTL -3	Analyzing
7. (b)	If we select ten points in the interior of an equilateral triangle of side 1 , show that there must be at least two points whose distance apart less than 1/3	BTL -2	Understanding
8.	There are three piles of identical red, blue and green balls, where each piles contains at least 10 balls. In how many ways can 10 balls be selected (1) If there is no restriction? (2) If at least 1 red ball must be selected?	BTL -1	Remembering

	(3) If at least 1 red, at least 2 blue and at least 3green balls must be selected? (4) If at most 1 red ball is selected?		
9. (a)	How many permutations can be made out of the letters of the word "BASIC"? How many of those (1) Begin with B? (2) End with C? B and C occupy the end places?	BTL -3	Applying
9.(b)	How many bits of string of length 10 contain (i). Exactly four 1's (ii) At most four 1's (iii) At least four 1's (iv) An equal number of 0's and 1's	BTL -3	Applying
10.(a)	Find the number of integers between 1 to 100 that are not divisible by any of the integers 2,3,5 or 7.	BTL -4	Applying
10.(b)	Prove that in a group of six people at least three must be mutual friends or at least three must be mutual strangers.	BTL -2	Understanding
11.(a)	Find the number of integers between 1 to 250 that are not divisible by any of the integers 2,3,5 and 7	BTL -4	Applying
11.(b)	Determine the number of positive integer n, $1 \leq n \leq 2000$ that are not divisible by 2, 3 or 5 but are divisible by 7.	BTL -2	Understanding
12.(a)	A Committee of 5 is to be selected from 6 boys and 5 girls. Determine the number of ways of selecting the committee if it is to consist of atleast 1 boy and 1 girl.	BTL -5	Evaluating
12.(b)	Triangle ACE is equilateral with AC=1. If five points are selected from the interior of the triangle, there are atleast two whose distance apart is less than ½.	BTL -3	Applying
13.	A survey of 550 television watchers produced the following information: 285 watch football game, 195 watch hockey game, 115 watch baseball game, 45 watch football and baseball games, 70 watch football and hockey games, 50 watch hockey and baseball games , 100 do not watch any of the three games. Then (a) How many people in the survey watch all three games? (b) How many people watch exactly one of the three games?	BTL -3	Applying
14.(a)	40 computer programmers interviewed for a job. 25 knew JAVA, 28 knew ORACLE and 7 knew neither language. How many knew both languages?	BTL -3	Applying
14.(b)	A survey of 500 from a school produced the following information. 200 play volleyball, 120 play hockey,60 play both volley ball and hockey. How many are not playing either volley ball or Hockey.	BTL -3	Applying
15.	Define isomorphism. Establish an isomorphism for the following the  graphs.	BTL -4	Analyzing
16.(a)	A team of 11 players is to be chosen from 15 members. In how many ways can this be done if (i)One particular player is always included. (ii)Two such players have always to be included.	BTL -3	Applying
16.(b)	A box contains 6 white balls and 5 red balls. Find the number of ways 4 balls can be drawn from the box if (i)They can be any colour. (ii)Two must be white and two red.	BTL -3	Applying

	(iii)They must all be the same colour.		
17 .(a)	Give an example of a graph which is (i) Eulerian but not Hamiltonian (ii) Hamiltonian but not Eulerian (iii) Both Eulerian and Hamiltonian (iv) Not Eulerian and not Hamiltonian	BTL -4	Analyzing
17 .(b)	Write the adjacency matrix of the digraph $G = \left\{ \begin{matrix} (v_1, v_3), (v_1, v_2), (v_2, v_4), \\ (v_3, v_1), (v_2, v_3), (v_3, v_4), \\ (v_4, v_1), (v_4, v_2), (v_4, v_3) \end{matrix} \right\}$. Also draw the graph.	BTL -3	Applying
18.	In a survey of 100 students, it was found that 30 studied Mathematics, 54 studied statistics,25 students operation research, 1 studied all the three subjects. 20 studied Mathematics and statistics, 3 studied mathematics and operation Research and 15 studied statistics and operation research. (i) How many students studied none of these subjects? (ii) How many students studied only Mathematics?	BTL -4	Analyzing
	PART C		
1.	How can the final exams at a university be scheduled so that no student has two exams at the same time?	**BTL -6**	Creating
2.	In the town of Königsberg in Prussia, there was a river containing two islands. The islands were connected to the banks of the river by seven bridges (as seen below). The bridges were very beautiful, and on their days off, townspeople would spend time walking over the bridges. As time passed, a question arose: was it possible to plan a walk so that you cross each bridge once and only once?	**BTL -5**	Evaluating
3.	In a survey of 120 passengers, an Airline found that 52 enjoyed wine with their meals, 75 enjoyed mixed drinks and 62 enjoyed iced tea. 35 enjoyed any given pair these beverages and 20 passengers enjoyed all of them. Find the no. of passengers who enjoyed (i) Only tea (ii) Only one of the three (iii) Exactly two of the three beverages None of the drinks	**BTL -4**	Applying
4.	A total of 1232 students have taken a course in Spanish, 879 have taken a course in French and 114 have taken a course in Russian. Further 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian and 14 have taken courses in both French and Russian. If 2092 students have taken at least one Spanish, French and Russian, how students have taken a course in all three languages?	**BTL -3**	Applying
5.	Suppose 10 new radio stations are to be set up in a currently unpopulated (by radio stations) region. The radio stations that are close enough to each other to cause interference are recorded in the table below. What is the fewest number of frequencies the stations could use?	BTL -2	Understanding

	A	B	C	D	E	F	G	H	I	J
A			x			x	x			x
B			x	x						
C	x					x	x			x
D		x			x	x		x		
E				x					x	
F	x		x	x			x			x

G	x		x			x				x
H				x					x	
I					x			x		x
J	x		x			x	x		x	

UNIT IV OPTIMIZATION TECHNIQUES

Introduction- Formulation of optimization problems, classification of optimization problems, overview of analytical solution for unconstrained optimization problems, constrained optimization, convex set, convex functions, convex optimization problem, Kuhn- Tucker conditions.

Q.No	**Question**	**BTLevel**	**Domain**
	PART - A		
1.	Define optimization	BTL -1	Remembering
2.	What is an objective function?	BTL -1	Remembering
3.	Define constraints in an optimization problem	BTL -1	Remembering
4.	Write down the algorithms in solving single variable optimization problems	BTL -1	Remembering
5.	Describe Direct search method in solving single variable optimization problems	BTL -2	Understanding
6.	Describe gradient based search method in solving single variable optimization problems	BTL -3	Applying
7.	A company manufactures two products A and B. It takes 30 minutes to process one unit of product A and 15 minutes for one unit of product B. The maximum machine time available is 35 hours per week. One unit of product A requires 2 kg of raw material, while product B requires 3 kg raw material per unit. The available raw material is limited to 180 kg per week. The products A and B have unlimited market potential and sell for Rs.200 and Rs. 500 per unit respectively. If the manufacturing costs for products A and Bare $2x^2, 3y^2$ respectively Formulate it as an optimization problem	BTL -3	Applying
8.	60 m^3 of a granular product are to be transported across a river in a ferry. The transportation cost across the river in Rs 100 per trip, irrespective of the amount transported, but there is restriction on the number of trips, which should not be more than 40. The cost of the container depends upon its dimensions as given below: Cost of bottom= Rs.10per square meter Cost of front and back sides = Rs. 10 per square meter And cost of the ends = Rs. 20 per square metre Thus if a small container is used, cost of container will be less, but the number of trips will be more , and if a large container is used, but the number of trips will be less. Find the dimensions of the container so that the sum of the container cost and transportation cost is minimized	BTL -3	Applying
9.	Define Local optimal point	BTL -1	Remembering
10.	Define global optimal point	BTL -1	Remembering
11.	Define inflection point	BTL -1	Remembering
12.	Write down the condition for a point x=a to be a minimum point	BTL -2	Understanding
13.	Estimate the inflection point of $f(x) = x^3$	BTL -2	Understanding
14.	Define concave and convex function theory	BTL -1	Remembering
15.	Check whether the function $f(x) = 10 - x^2$ is concave or convex	BTL -5	Evaluating
16.	Check whether the function $f(x) = 2x^3 - 3x^2$is concave or convex	BTL -5	Evaluating
17.	State Kuhn-Tucker condition	BTL -5	Evaluating
18.	Evaluate the optimum point for $f(x) = x^3 - 10x - 2x^2 + 10$	BTL -5	Evaluating
19.	Identify the optimum point for $f(x) = e^x - x^3$	BTL -3	Analyzing
20.	List the uses of the classical optimization	BTL -1	Remembering
21.	What are the types of optimization problems	BTL -1	Remembering
22.	Find the optimal value for $f(x) = x^2 - 4x + 2$	BTL -5	Evaluating

23.	What are the five steps involved in optimization problems	BTL -1	Remembering
24.	Check whether the function $f(x) = 5 - 3x^2$ is concave or convex	BTL -5	Evaluating
25.	Check whether the function $f(x) = x^4 + x^3$ is concave or convex	BTL -5	Evaluating
	PART -B		
1.	Perform a test for an optimization for the function $f(x) = x^{\ 3}$	BTL -5	Evaluating
2.	Perform a test for an optimization for the function $f(x) = x^4$	BTL -3	Applying
3.	Solve : Minimize $f(x) = x^2 + \frac{54}{x}$ at (0,5) using exhaustive search method	BTL -5	Evaluating
4.	Check whether the function (i) $f(x) = x^4 + 6x^2 + 12x$ (ii) $f(x) = x^4 + x^2$ is concave or convex or neither	BTL -3	Applying
5.	Use golden section search method in order to minimize the function $f(x) = x^2 - 6x + 15$ in the interval (0,4)	BTL -3	Applying
6.	Use golden section search method in order to minimize the function $f(x) = 4x^3 + x^2 - 7x + 14$ in the interval (0,1)	BTL -5	Evaluating
7.	Maximize $f(x) = 20x - 3x^2 - x^4$ using one- dimensional search procedure	BTL -3	Applying
8.	Maximize $f(x) = 6x - x^2$ using one- dimensional search procedure. Take the initial upper bound and lower bounds as 4.8 and 0 and error tolerance $\in = 0.04$	BTL -3	Applying
9.	Minimize $f(x) = x^4 - x^2 - 4x$ using one- dimensional search procedure. Take the initial upper bound and lower bounds as 2 and 0 and error tolerance $\in = 0.01$	BTL -5	Evaluating
10	Use the one dimensional search procedure to interactively solve, approximately the following equation : Minimize $f(x) = x^3 + 2x - 2x^2 - .025x^4$ Use an error intolerance $\in = 0.04$ and the initial lower and upper bounds as 0 and 2.4	BTL -5	Evaluating
11.	Minimize f(x)=x(x-1.5) using bounding phase method	BTL -5	Evaluating
12.	Minimize $f(x) = x^2 + \frac{54}{x}$ using bisection method at (2,5) and $\in = 10^{-3}$	BTL - 4	Applying
13.	Use the golden section search method to find the value of x that minimizes $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ in the range [0,2].Locate this value of x to within a range of 0.3	BTL -4	Applying
14.	Discuss about the Exhaustive search method in solving single variate optimization problem	BTL -3	Analyzing
15.	Minimize $f(x) = x^2 + \frac{54}{x}$ using interval halving method at (0,5) and $\in = 10^{-3}$	BTL -4	Applying
16.	Minimize $f(x) = x^2 + \frac{54}{x}$ using Golden section search method at (0,5)	BTL -4	Applying
17.	Solve using bisection method $(x) = x^2 + 2x - 200$ and $\in = 0.2$	BTL -4	Applying
18.	Minimize $f(x) = x^2 + \frac{16}{x}$ using interval halving method at (0,5) to an accuracy of 0.1	BTL -4	Applying
	PART C		
1.	Solve : Minimize $f(x) = x^2 + \frac{54}{x}$ using bounding phase method	BTL -5	Evaluating
2.	Use three iterations of the golden section search method in order to maximize the function$f(x) = x^2 - 3x - 20$. Use $x^{(0)} = 0$ and an initial $\Delta = 1$	BTL -6	Creating
3.	Maximize $Z = 12x - 3x^4 - 2x^6$ using bisection search method	BTL -5	Evaluating
4.	Discuss about the golden section method in solving single variate optimization problem	BTL -6	Creating
5.	Maximize $f(x) = 12x - 3x^4 - 2x^6$ using one dimensional search method	BTL -5	Evaluating

UNIT V

Search methods – Overview of single variable search methods, search methods for Multivariable unconstrained problems - Optimality criteria, unidirectional search – direct search methods- evolutionary search method, Hook-Jeeves pattern search method, gradient based methods –Cauchy's steepest descent method, Newton's method.

Q.No	Question	BT Level	Domain
	PART - A		
1.	Define stationary point	BTL -5	Evaluating
2.	State the condition for a minimum point for the function $f(x_1, x_2, \dots, x_n)$	BTL -2	Understanding
3.	State the condition for a maximum point for the function $f(x_1, x_2, \dots, x_n)$	BTL -2	Understanding
4.	State the condition for inflection point for the function $f(x_1, x_2, \dots, x_n)$	BTL -2	Understanding
5.	Define single variable search methods	BTL -2	Understanding
6.	Define Multivariable unconstrained problems search methods	BTL -2	Understanding
7.	Define unidirectional search in multivariable optimization problem	BTL -2	Understanding
8.	What is multivariable optimization	BTL -2	Understanding
9.	Determine whether the function $f(x) = x_1x_2 - x_1^2 - x_2^2$ is convex, concave or neither	BTL -2	Understanding
10.	Discuss whether the function $f(x) = 3x_1 + 4x_2 + 2x_1^2 + x_2^2 - 2x_1x_2$ is convex, concave or neither	BTL -2	Understanding
11.	Find out whether the function $f(x) = x_1^2 + 3x_1x_2 + x_2^2$ is convex, concave or neither	BTL -2	Understanding
12.	Find out whether the function $f(x) = x_1^2 - 2x_1x_2 + x_2^2$ is convex, concave or neither	BTL -2	Understanding
13.	Define univariate search method	BTL -4	Applying
14.	How do you solve unconstrained optimization problems?	BTL -4	Applying
15.	Define Newton's method	BTL -4	Applying
16.	What are the gradient based methods available to solve multi variable unconstrained problems	BTL -3	Analyzing
17.	State the methods to solve multivariate optimization problem	BTL -2	Understanding
18.	Define direct search method in solving unconstrained optimization problems	BTL -2	Understanding
19.	Define indirect search method in solving unconstrained optimization problems	BTL -1	Remembering
20.	Define random jumping method	BTL -2	Understanding
21.	Define random walk method	BTL -2	Understanding
22.	What are the Direct search methods methods available to solve multi variable unconstrained problems	BTL -1	Remembering
23.	What are the random search methods methods available to solve multi variable unconstrained problems	BTL -1	Remembering
24.	Define unidirectional search method	BTL -2	Understanding
25.	Define optimality criteria	BTL -2	Understanding
	PART- B		
1.	Discuss about random search methods	BTL -2	Understanding
2.	Minimize $f(x_1, x_2) = {x_1}^3 + {x_2}^3 + 2{x_1}^2 + 4{x_2}^2$ in (-2,2) using Random jumping method.	BTL -4	Applying
3.	Discuss Hooke-Jeeves pattern search method	BTL -2	Understanding
4.	Solve the following two-variable unconstrained non-linear problem using search procedure Maximize $f(x_1, x_2) = 2x_1x_2 + x_2 - x_1^2 - 2x_2^2$	BTL -3	Analyzing
5.	Starting from the initial trial solution$(x_1, x_2) = (0,0)$, interactively apply two iterations of the gradient search procedure to the following two-variable unconstrained problem. Also determine the exact solution by solving $\nabla f(x) = 0$, Maximize $f(x) = 8x_1 - x_1^2 - 12x_2 - 2x_2^2 + 2x_1x_2$	BTL -3	Analyzing
6.	Solve the following two-variable unconstrained non-linear problem using search procedure Maximize $f(x_1, x_2) = x_1 - x_2 + 2x_1x_2 + 2x_1^2 + x_2^2$	BTL -4	Applying

7.	Minimize $(x_1, x_2) = x_1^3 + x_2^2 + 2x_1^2 + 4x_2^2$ using univariate search method with the initial point as (2,2)	BTL -1	Remembering
8.	Apply Newton's method to Minimize : $f(x_1, x_2) = x_1^2 + x_2^2 + 14x_1 + 14x_2 + 100$ with initial point as $\begin{pmatrix}0\\0\end{pmatrix}$	BTL -2	Understanding
9.	Apply Random search method to Minimize: $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$	BTL -4	Applying
10.	Apply Newton's method to solve: $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$	BTL -2	Understanding
11.	Apply Cauchy's steepest decent method to solve: Minimize : $f(x_1, x_2) = x_1^2 - x_1x_2 + x_2^2$, the error not exceed by 0.05 for function approximation $X_1 = \left(1, \frac{1}{2}\right)$	BTL -2	Understanding
12.	Minimize: $f(x) = (1 - x_1)^2 + (2 - x_2)^2$using simplex search method with initial point as $\begin{pmatrix}0\\0\end{pmatrix}$and scaling factor $\alpha = 2$	BTL -2	Understanding
13.	Discuss Cauchy's steepest decent method	BTL -2	Understanding
14.	Apply Hooke-Jeeves pattern search method to solve: Minimize : $(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$	BTL -6	Creating
15.	Minimize $f(x_1, x_2) = {x_1}^3 + {x_2}^3 + 2{x_1}^2 + 4{x_2}^2$ in (-2,2) using Random walk method.	BTL -6	Creating
16.	Discuss the procedure of Gradient search in Multivariable unconstrained problems	BTL -2	Understanding
17.	Apply Newton's method to solve: $f(x_1, x_2) = x_1^3 + 2(x_1 - x_2)^2 - 3x_1$ start from $x^{(0)} = \begin{pmatrix}0.5\\0.5\end{pmatrix}$, minima at $\begin{pmatrix}1\\1\end{pmatrix}$	BTL -6	Creating
18.	Discuss the step by step process involved in simplex search method.	BTL -2	Understanding
	PART- C		
1.	Solve the following two-variable unconstrained non-linear problem using search procedure Maximize$f(x_1, x_2) = 2x_1x_2 + 2x_2 - x_1^2 - 2x_2^2$, $\left(\frac{1}{2}, \frac{1}{2}\right)$ may be taken as the starting trial solution. Draw the path of the trial solution by solving the system of linear equations by setting $\nabla f(x) = 0$	BTL -5	Evaluating
2.	Solve $f(x) = 4x_1^2 + x_2^2 - 2x_1x_2$ by Newton's method	BTL -6	Creating
3.	Discuss briefly about the direct search methods in solving the multivariate unconstrained optimization problem	BTL -5	Evaluating
4.	Apply Cauchy's steepest decent method to solve Minimize : $4x_1^2 + 3x_1x_2 + 2.5x_2^2 - 5.5x_1 - 4x_2$	BTL -5	Evaluating
5.	Minimize $(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ using univariate search method with the starting point as (0,0)	BTL -6	Creating