

**SRM VALLIAMMAI ENGINEERING COLLEGE**  
(An Autonomous Institution)

S.R.M. Nagar, Kattankulathur - 603203

**DEPARTMENT OF MATHEMATICS**



**IV SEMESTER**

*B.E - COMPUTER SCIENCE ENGINEERING*  
*B.E – CYBER SECURITY*

**1918402 –PROBABILITY AND QUEUEING THEORY**

Regulation – 2019

**Academic Year – 2022 - 2023**

*Prepared by*

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# SRM VALLIAMMAI ENGINEERING COLLEGE

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## DEPARTMENT OF MATHEMATICS

S.No	QUESTIONS	BT Level	Competence																
<b>UNIT I RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS</b>																			
Random Variables - Discrete and continuous random variables – Moments – Moment generating functions - Binomial, Poisson, Geometric, Uniform, Exponential and Normal distribution																			
<b>Part - A ( 2 MARK QUESTIONS)</b>																			
1.	<p>The number of hardware failures of a computer system in a week of operations has the following p.d.f, Find the mean of the number of failures in a week.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>No.of failures</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Probability</td> <td>.18</td> <td>.28</td> <td>.25</td> <td>.18</td> <td>.06</td> <td>.04</td> <td>.01</td> </tr> </table>	No.of failures	0	1	2	3	4	5	6	Probability	.18	.28	.25	.18	.06	.04	.01	BTL-2	Understanding
No.of failures	0	1	2	3	4	5	6												
Probability	.18	.28	.25	.18	.06	.04	.01												
2.	<p>The number of hardware failures of a computer system in a week of operations has the following p.d.f, Calculate the value of K.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>No.of failures</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>Probability</td> <td>K</td> <td>2 K</td> <td>2 K</td> <td>K</td> <td>3 K</td> <td>K</td> <td>4 K</td> </tr> </table>	No.of failures	0	1	2	3	4	5	6	Probability	K	2 K	2 K	K	3 K	K	4 K	BTL-2	Understanding
No.of failures	0	1	2	3	4	5	6												
Probability	K	2 K	2 K	K	3 K	K	4 K												
3.	<p>Check whether the function given by <math>f(x) = \frac{x+2}{25}</math> for <math>x=1, 2,3,4,5</math> can serve as the probability distribution of a discrete random variable.</p>	BTL-2	Understanding																
4.	<p>If the random variable X takes the values 1,2,3 and 4 such that <math>2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)</math> , find the probability distribution of X</p>	BTL-1	Remembering																
5.	<p>The RV X has the following probability distribution:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> </tr> <tr> <td>P(x)</td> <td>0.4</td> <td>k</td> <td>0.2</td> <td>0.3</td> </tr> </table> <p>Find k and the mean value of X</p>	x	-2	-1	0	1	P(x)	0.4	k	0.2	0.3	BTL-2	Understanding						
x	-2	-1	0	1															
P(x)	0.4	k	0.2	0.3															
6.	<p>If <math>f(x) = K(x + x^2)</math> in <math>1 &lt; x &lt; 5</math> is a pdf of a continuous random variables. Find the value of K.</p>	BTL-1	Remembering																
7.	<p>The p.d.f of a continuous random variable X is <math>f(x) = k(1 + x), 2 &lt; x &lt; 5</math> Find k.</p>	BTL-1	Remembering																
8.	<p>For a continuous distribution <math>f(x) = k(x - x^2), 0 \leq x \leq 1</math>, where k is a constant. Find k.</p>	BTL-2	Understanding																
9.	<p>If <math>f(x) = kx^2, 0 &lt; x &lt; 3</math>, is to be a density function, find the value of k.</p>	BTL-2	Understanding																
10.	<p>A test engineer discovered that the cumulative distribution function of the life time of an equipment ( in years)is given by <math>F(x) = 1 - e^{-\frac{x}{5}}</math>, <math>x \geq 0</math>. What is the expected lifetime of the equipment?</p>	BTL-2	Understanding																
11.	<p>The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution.</p>	BTL3	Applying																
12.	<p>In 256 sets of 8 tosses of a coin, in how many sets one may expect heads and tails in equal number?</p>	BTL4	Analyzing																
13.	<p>If 3% of the electric bulbs manufactured by a company are defective, Find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective.</p>	BTL4	Analyzing																
14.	<p>Suppose that, on an average, in every three pages of a book there is one</p>	BTL3	Applying																

	typographical error. If the number of typographical errors on a single page of the book is a Poisson random variable. What is the probability if at least one error on a specific page of the book?		
15.	The no. of monthly breakdowns of a computer is a RV having Poisson distribution with mean 1.8. Find the probability that this computer will function for a month with only one breakdown.	BTL4	Analyzing
16.	If X is a Poisson variate such that $2P(X = 0) + P(X = 2) = 2P(X = 1)$ , find E(X)	BTL4	Analyzing
17.	The probability that a candidate can pass in an examination is 0.6. What is the probability that he will pass in third trial?	BTL4	Analyzing
18.	If X is a geometric variate, taking the values 1,2,3,..., find P(X is odd)	BTL3	Applying
19.	If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test on the fourth trial	BTL4	Analyzing
20.	If X has uniform distribution in (-3,3), find $P( x - 2  < 2)$	BTL-2	Understanding
21.	If the MGF of a continuous RV is $\frac{1}{t}(e^{5t} - e^{4t})$ what is the distribution of X? What are the mean and variance of X?	BTL4	Analyzing
22.	Suppose that the life of industrial lamp(in thousands of hours) is exponentially distributed with mean life of 3000 hours, Evaluate the probability that the lamp will last between 2000 and 3000 hours.	BTL5	Evaluating
23.	A continuous RV X has the density function $ce^{-\frac{x}{5}}, x > 0$ . Find c. Create E(x) and Var(X)	BTL6	Creating
24.	If X is a normal random variable with mean 3 and variance 9, find the probability that X lies between 2 and 5.	BTL3	Applying
25.	A normal distribution has mean $\mu = 20$ and standard deviation $\sigma = 10$ . Evaluate $(15 \leq X \leq 40)$ .	BTL5	Evaluating

**PART – B (13 MARK QUESTIONS)**

1.(a)	<p>A random variable X has the following probability distribution:</p> <table border="1" style="margin-left: 20px;"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>P(X)</td> <td>0</td> <td>k</td> <td>2k</td> <td>2k</td> <td>3k</td> <td>k<sup>2</sup></td> <td>2k<sup>2</sup></td> <td>7k<sup>2</sup>+k</td> </tr> </table> <p>Find (i) the value of k (ii) <math>P(1.5 &lt; X &lt; 4.5 / X &gt; 2)</math></p>	X	0	1	2	3	4	5	6	7	P(X)	0	k	2k	2k	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> +k	BTL-2	Understanding		
X	0	1	2	3	4	5	6	7															
P(X)	0	k	2k	2k	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> +k															
1.(b)	Find the MGF of Binomial distribution and hence find its mean and variance	BTL-1	Remembering																				
2.(a)	<p>The probability mass function of a discrete R. V X is given in the following table:</p> <table border="1" style="margin-left: 20px;"> <tr> <td>X</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>P(X=x)</td> <td>0.1</td> <td>K</td> <td>0.2</td> <td>2k</td> <td>0.3</td> <td>k</td> </tr> </table> <p>Find (1) Find the value of k, (2) <math>P(X &lt; 1)</math>, (3) <math>P(-1 &lt; X \leq 2)</math></p>	X	-2	-1	0	1	2	3	P(X=x)	0.1	K	0.2	2k	0.3	k	BTL-2	Understanding						
X	-2	-1	0	1	2	3																	
P(X=x)	0.1	K	0.2	2k	0.3	k																	
2.(b)	Obtain the MGF of Poisson distribution and hence find its mean and variance	BTL-1	Remembering																				
3.(a)	<p>The probability mass function of a discrete R. V X is given in the following table</p> <table border="1" style="margin-left: 20px;"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>P(X)</td> <td>a</td> <td>3a</td> <td>5a</td> <td>7a</td> <td>9a</td> <td>11a</td> <td>13a</td> <td>15a</td> <td>17a</td> </tr> </table> <p>Find (i) the value of a, (ii) <math>P(X &lt; 3)</math>, (iii) Mean of X, (iv) Variance of X.</p>	X	0	1	2	3	4	5	6	7	8	P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a	BTL-2	Understanding
X	0	1	2	3	4	5	6	7	8														
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a														
3.(b)	Deduce the MGF of a geometric distribution and hence find the mean and	BTL-1	Remembering																				

	variance																				
4.(a)	If the discrete random variable X has the probability function given by the table. <table border="1" style="margin-left: 20px;"> <tr> <td><math>x</math></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td><math>P(x)</math></td> <td><math>k/3</math></td> <td><math>k/6</math></td> <td><math>k/3</math></td> <td><math>k/6</math></td> </tr> </table> <p>Find the value of k and Cumulative distribution of X.</p>	$x$	1	2	3	4	$P(x)$	$k/3$	$k/6$	$k/3$	$k/6$	BTL-2	Understanding								
$x$	1	2	3	4																	
$P(x)$	$k/3$	$k/6$	$k/3$	$k/6$																	
4.(b)	Derive the MGF of Uniform distribution and hence deduce the mean and variance	BTL-1	Remembering																		
5.(a)	The probability mass function of a RV X is given by $P(X = r) = kr^3$ , $r = 1,2,3,4$ . Find (1) the value of k, (2) $P(\frac{1}{2} < X < \frac{5}{2} / X > 1)$	BTL3	Applying																		
5.(b)	Deduce the MGF of Exponential distribution and hence find its mean and variance	BTL-1	Remembering																		
6.(a)	The probability distribution of an infinite discrete distribution is given by $P[X = j] = \frac{1}{2^j}$ ( $j = 1,2,3,\dots$ ) Find (1) Mean of X, (2) $P[X \text{ is even}]$ , (3) $P(X \text{ is odd})$ (4) $P(X \text{ is divisible by } 3)$	BTL-2	Understanding																		
6.(b)	State and prove the memory less property of exponential distribution	BTL3	Applying																		
7.(a)	Find the mean and variance of the following probability distribution <table border="1" style="margin-left: 20px;"> <tr> <td><math>X_i</math></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td><math>P_i</math></td> <td>0.08</td> <td>0.12</td> <td>0.19</td> <td>0.24</td> <td>0.16</td> <td>0.10</td> <td>0.07</td> <td>0.04</td> </tr> </table>	$X_i$	1	2	3	4	5	6	7	8	$P_i$	0.08	0.12	0.19	0.24	0.16	0.10	0.07	0.04	BTL-2	Understanding
$X_i$	1	2	3	4	5	6	7	8													
$P_i$	0.08	0.12	0.19	0.24	0.16	0.10	0.07	0.04													
7.(b)	Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 engineering students (i) exactly 10, (ii) at least 15 are good in mathematics.	BTL3	Applying																		
8.	Obtain the MGF of a normal distribution and hence find its mean and variance	BTL-1	Remembering																		
9.(a)	If a random variable X has p.d.f $f(x) = \begin{cases} \frac{1}{4}, &  X  < 2 \\ 0, & \text{Otherwise} \end{cases}$ Find (a) $P(X < 1)$ (b) $P( X  > 1)$ (c) $P(2X + 3 > 5)$ .	BTL-2	Understanding																		
9.(b)	Out of 2000 families with 4 children each, Find how many family would you expect to have i) at least 1 boy ii) 2 boys.	BTL3	Applying																		
10.(a)	Find the MGF of the random variable X having the probability density function $f(x) = \begin{cases} \frac{x}{4} e^{-\frac{x}{2}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$ . Also find the mean and variance	BTL-2	Understanding																		
10.(b)	4 coins were tossed simultaneously. What is the probability of getting (i) 2 heads, (ii) at least 2 heads, (iii) at most 2 heads.	BTL4	Analyzing																		
11.(a)	A random variable X has c.d.f $F(x) = \begin{cases} 0, & \text{if } x < -1 \\ a(1+x), & \text{if } -1 < x < 1 \\ 1, & \text{if } x \geq 1 \end{cases}$ Find the value of a. Also $P(X > 1/4)$ and $P(-0.5 \leq X \leq 0)$ .	BTL-2	Understanding																		
11.(b)	The atoms of a radioactive element are randomly disintegrating. If every gram of this element, on average, emits 3.9 alpha particles per second, then what is the probability that during the next second the number of alpha particles emitted from 1 gram is (1) at most 6 (2) at least 2 and (3) at least and at most 5	BTL4	Analyzing																		

12.	<p>If <math>f(x) = \begin{cases} ax, &amp; 0 \leq x \leq 1 \\ a, &amp; 1 \leq x \leq 2 \\ 3a - ax, &amp; 2 \leq x \leq 3 \\ 0, &amp; \text{elsewhere} \end{cases}</math> is the p.d.f of X. Calculate</p> <p>(i) The value of a ,  (ii) The cumulative distribution function of X  (iii) If <math>X_1, X_2</math> and <math>X_3</math> are 3 independent observations of X. Find the probability that exactly one of these 3 is greater than 1.5?</p>	BTL-2	Understanding
13.(a)	<p>The Probability distribution function of a R.V. X is given by  <math>f(x) = \frac{4x(9 - x^2)}{81}, 0 \leq x \leq 3</math>. Find the mean, variance</p>	BTL-2	Understanding
13.(b)	<p>The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without breakdown (2) with only one breakdown and (3) with at least one breakdown.</p>	BTL3	Applying
14.(a)	<p>Messages arrive at a switch board in a Poisson manner at an average rate of 6 per hour. Find the probability that exactly 2 messages arrive within one hour, no messages arrives within one hour and at least 3 messages arrive within one hour</p>	BTL3	Applying
14.(b)	<p>Suppose that the life of an industrial lamp in 1,000 of hours is exponentially distributed with mean life of 3,000 hours. Find the probability that (i) The lamp last more than the mean life (ii) The lamp last between 2,000 and 3,000 hours (iii) The lamp last another 1,000 hours given that it has already lasted for 250 hours.</p>	BTL3	Applying
15.(a)	<p>The time (in hours) required to repair a machine is exponentially distributed with parameter <math>\lambda = 1/2</math>.  (a) What is the probability that the repair time exceeds 2 hours?  (b) What is the conditional probability that a repair time exceeds at least 10 hours that its distribution exceeds 9 hours?</p>	BTL4	Analyzing
15.(b)	<p>Let X be a Uniformly distributed R. V. over [-5, 5]. Evaluate (i) <math>P(X \leq 2)</math> (ii) <math>P( X  &gt; 2)</math> (iii) Cumulative distribution function of X (iv) <math>\text{Var}(X)</math></p>	BTL5	Evaluating
16.(a)	<p>Buses arrive at a specified stop at 15 minutes interval starting at 7am that is, 7:15, 7:30, 7:45, and so on, If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 77:30 am, evaluate the probability that he waits  (a) Less than 5 minutes for a bus and  (b) At least 12 minutes for a bus</p>	BTL5	Evaluating
16.(b)	<p>The marks obtained by a number of students for a certain subject is assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are taken at random from this set Find the probability that exactly 2 of them will have marks over 70?</p>	BTL4	Analyzing
17.	<p>In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and Standard Deviation of 60 hours. Find the number of bulbs likely to burn for (i) more than 2150 hours (ii) less than 1950 hours and (iii) more than 1920 hours burs less than 2160 hours.</p>	BTL3	Applying
18.(a)	<p>The mileage which car owners get with a certain kind of radial tire is a RV having an exponential distribution with mean 40,000 km. Find the</p>	BTL-2	Understanding

	probabilities that one of these will last (a) at least 20,000 km, and (b) at most 30,000km.		
18.(b)	The annual rainfall in inches in a certain region has a normal distribution with a mean of 40 and variance of 16. What is the probability that the rainfall in a given year is between 30 and 48 inches?	BTL3	Applying

**PART C(15 Mark Questions)**

1.	Out of 2000 families with 4 children each, Create how many family would you expect to have i) at least 1 boy ii) 2 boys and 2 girls iii) at most 2 girls iv) children of both genders	BTL6	Creating
2.	In a certain factory manufacturing razor blades, there is a small chance of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) No defective (ii) One defective (iii) Two defective blades Respectively in a consignment of 10,000 packet	BTL4	Analyzing
3.	Buses arrive at a specified stop at 15 minutes interval starting at 6 AM ie they arrive at 6 AM, 6.15AM, 6.30 AM and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 6 and 6.30 AM. Evaluate the probability that he waits (i) Less than 5 minutes for a bus. (ii) More than 10 minutes for a bus.	BTL5	Evaluating
4.	The daily consumption of milk in excess of 20,000 liters in a town is approximately exponentially distributed with parameter 1/3000. The town has a daily stock of 35,000L. What is the probability that of 2 days selected at random, the stock is insufficient for both days?	BTL3	Applying
5.	In an Engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, between 45% and 60% between 60% and 75% and above 75%respectively. In a particular year 10% of the students failed in the examination and 5% of the students get distinction. Find the percentage of students who have got first class and second class. Assume normal distribution of marks.	BTL-2	Understanding

**UNIT II TWO – DIMENSIONAL RANDOM VARIABLES**

**9L+3T**

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and linear regression – Transformation of random variables – Central limit theorem (for independent and identically distributed random variables)

**PART-A( 2 MARK QUESTIONS)**

1.	The joint probability distribution of X and Y is given by $p(x, y) = \frac{x+y}{21}$ , $x = 1,2,3; y = 1, 2$ . Find the marginal probability distributions of X	BTL-2	Understanding									
2.	The joint probability function (X,Y) is given by $P(x, y) = k(2x + 3y)$ , $x = 0,1,2 y = 1,2,3$ , Find the value of K.	BTL-2	Understanding									
3.	Find the probability distribution of X + Y from the bivariate distribution of (X,Y) given below: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>X \ Y</td> <td>1</td> <td>2</td> </tr> <tr> <td>1</td> <td>0.4</td> <td>0.2</td> </tr> <tr> <td>2</td> <td>0.3</td> <td>0.1</td> </tr> </table>	X \ Y	1	2	1	0.4	0.2	2	0.3	0.1	BTL-2	Understanding
X \ Y	1	2										
1	0.4	0.2										
2	0.3	0.1										
4.	Let X and Y have the joint p.m.f <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Y \ X</td> <td>0</td> <td>1</td> <td>2</td> </tr> </table>	Y \ X	0	1	2	BTL-2	Understanding					
Y \ X	0	1	2									

		0	0.1	0.4	0.1		
		1	0.2	0.2	0		
	Find $P(X+Y > 1)$						
5.	Find the marginal distributions of X and Y from the bivariate distribution of (X,Y) given below:					BTL-2	Understanding
		X \ Y	1	2			
		1	0.1	0.2			
		2	0.3	0.4			
6.	The joint probability distribution function of the random variable (X,Y) is given by $f(x,y) = k(x^3y - xy^3)$ , $0 \leq x \leq 2, 0 \leq y \leq 2$ . Derive the value of k					BTL3	Applying
7.	If the joint probability density function of a random variable X and Y is given by $f(x,y) = \begin{cases} \frac{x^3y^3}{16}, & 0 < x < 2, 0 < y < 2 \\ 0, & otherwise \end{cases}$ . Obtain the marginal density function of X.					BTL3	Applying
8.	The joint probability density of a two dimensional random variable (X,Y) is given by $f(x,y) = \begin{cases} kxe^{-y}; & 0 \leq x < 2, y > 0 \\ 0, & otherwise \end{cases}$ . Evaluate k.					BTL5	Evaluating
9.	The joint probability density function of a random variable (X,Y) is $f(x,y) = ke^{-(2x+3y)}$ , $x \geq 0, y \geq 0$ . Point out the value of k.					BTL4	Analyzing
10.	If the joint pdf of (X, Y) is $f(x,y) = \begin{cases} \frac{1}{4}, & 0 < x, y < 2 \\ 0, & otherwise \end{cases}$ . Find $P(X + Y \leq 1)$					BTL4	Analyzing
11.	Let X and Y be random variables with joint density function $f(x,y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1 \\ 0, & otherwise \end{cases}$ formulate the value of E(XY)					BTL4	Analyzing
12.	Let the joint density function of a random variable X and Y be given by $f(x,y) = 8xy$ , $0 < y \leq x \leq 1$ . Calculate the marginal probability function of X					BTL3	Applying
13.	What is the condition for two random variables are independent?					BTL-1	Remembering
14.	If the joint probability density function of X and Y is $f(x,y) = e^{-(x+y)}$ , $x, y \geq 0$ . Are X and Y independent					BTL4	Analyzing
15.	State any two properties of correlation coefficient					BTL-1	Remembering
16.	Write the angle between the regression lines					BTL-1	Remembering
17.	The regression equations are $x + 6y = 14$ and $2x + 3y = 1$ . Evaluate the correlation coefficient between X & Y .					BTL5	Evaluating
18.	If $\bar{X} = 970, \bar{Y} = 18, \sigma_x = 38, \sigma_y = 2$ and $r = 0.6$ , Devise the line of regression of X on Y.					BTL4	Analyzing
19.	In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible; Variance of X = 9; Regression equations are $8X - 10Y + 66 = 0$ and $40X - 18Y = 214$ . Find the mean values of X and Y?					BTL3	Applying
20.	The regression equations are $3x + 2y = 26$ and $6x + y = 31$ . Find the correlation coefficient.					BTL3	Applying
21.	State central limit theorem					BTL-1	Remembering
22.	Prove that $-1 \leq r_{xy} \leq 1$					BTL-2	Understanding

23.	The equations of two regression lines are $3x+2y=19$ and $3y+9x=46$ . Obtain the mean of X and Y.	BTL3	Applying
24.	The equations of two regression lines are $3x+2y=19$ and $3y+9x=46$ . Derive the correlation coefficient between X and Y.	BTL3	Applying
25.	If $X = R\cos\phi$ and $Y = R\sin\phi$ , how are the joint probability density function (X,Y) and $(R, \phi)$ are related?	BTL3	Applying

**PART B (13 Mark Questions)**

1.	<p>From the following table for bivariate distribution of (X, Y). Find            (i) <math>P(X \leq 1)</math>                      (ii) <math>P(Y \leq 3)</math>                      (iii) <math>P(X \leq 1, Y \leq 3)</math>            (iv) <math>P(X \leq 1/Y \leq 3)</math>              (v) <math>P(Y \leq 3/X \leq 1)</math>              (vi) <math>P(X + Y \leq 4)</math></p> <table border="1" style="margin-left: 20px;"> <tr> <td style="border: none;">Y</td> <td style="border: none;"></td> <td style="border: none;">1</td> <td style="border: none;">2</td> <td style="border: none;">3</td> <td style="border: none;">4</td> <td style="border: none;">5</td> <td style="border: none;">6</td> </tr> <tr> <td style="border: none;">X</td> <td style="border: none;"></td> <td style="border: none;"></td> <td style="border: none;"></td> <td style="border: none;"></td> <td style="border: none;"></td> <td style="border: none;"></td> <td style="border: none;"></td> </tr> <tr> <td style="border: none;"></td> <td style="border: none;">0</td> <td style="border: none;">0</td> <td style="border: none;">0</td> <td style="border: none;"><math>\frac{1}{32}</math></td> <td style="border: none;"><math>\frac{2}{32}</math></td> <td style="border: none;"><math>\frac{2}{32}</math></td> <td style="border: none;"><math>\frac{3}{32}</math></td> </tr> <tr> <td style="border: none;"></td> <td style="border: none;">1</td> <td style="border: none;"><math>\frac{1}{16}</math></td> <td style="border: none;"><math>\frac{1}{16}</math></td> <td style="border: none;"><math>\frac{1}{8}</math></td> <td style="border: none;"><math>\frac{1}{8}</math></td> <td style="border: none;"><math>\frac{1}{8}</math></td> <td style="border: none;"><math>\frac{1}{8}</math></td> </tr> <tr> <td style="border: none;"></td> <td style="border: none;">2</td> <td style="border: none;"><math>\frac{1}{32}</math></td> <td style="border: none;"><math>\frac{1}{32}</math></td> <td style="border: none;"><math>\frac{1}{64}</math></td> <td style="border: none;"><math>\frac{1}{64}</math></td> <td style="border: none;">0</td> <td style="border: none;"><math>\frac{2}{64}</math></td> </tr> </table>	Y		1	2	3	4	5	6	X									0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$		1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$		2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	BTL-2	Understanding
Y		1	2	3	4	5	6																																				
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	2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$																																				

2.(a)	The two dimensional random variable (X, Y) has the joint probability mass function $f(x, y) = \frac{x+2y}{27}, x = 0,1,2; y = 0,1,2$ . Find the marginal distributions of X and Y. Also find the conditional distribution of Y given $X = 1$ also find the conditional distribution of X given $Y = 1$ .	BTL3	Applying
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2.(b)	<p>The joint pdf a bivariate R.V(X, Y) is given by</p> $f(x, y) = \begin{cases} Kxy & , 0 < x < 1, 0 < y < 1 \\ 0 & , \text{otherwise} \end{cases}$ <p>Find K. (2) Find <math>P(X+Y &lt; 1)</math>. (3) Are X and Y independent R.V's.</p>	BTL3	Applying
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3.(a)	If the joint pdf of (X, Y) is given by $P(x, y) = K(2x+3y), x=0, 1, 2, 3, y = 1, 2, 3$ Find all the marginal probability distribution. Also find the probability distribution of $X+Y$ .	BTL3	Applying
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3.(b)	The joint pdf of the RV (X,Y) is given by $f(x, y) = kxye^{-(x^2+y^2)}, x > 0, y > 0$ . Find the value of k. Also prove that X and Y are independent	BTL4	Analyzing
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4.	<p>The following table represents the joint probability distribution of the discrete RV (X,Y). Find all the marginal and conditional distributions.</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="border: none;">Y</td> <td colspan="3" style="border: none;">X</td> </tr> <tr> <td style="border: none;"></td> <td style="border: none;">1</td> <td style="border: none;">2</td> <td style="border: none;">3</td> </tr> <tr> <td style="border: none;">1</td> <td style="border: none;">1/2</td> <td style="border: none;">1/6</td> <td style="border: none;">0</td> </tr> <tr> <td style="border: none;">2</td> <td style="border: none;">0</td> <td style="border: none;">1/9</td> <td style="border: none;">1/5</td> </tr> <tr> <td style="border: none;">3</td> <td style="border: none;">1/18</td> <td style="border: none;">1/4</td> <td style="border: none;">2/15</td> </tr> </table>	Y	X				1	2	3	1	1/2	1/6	0	2	0	1/9	1/5	3	1/18	1/4	2/15	BTL-2	Understanding
Y	X																						
	1	2	3																				
1	1/2	1/6	0																				
2	0	1/9	1/5																				
3	1/18	1/4	2/15																				

5.	<p>Find the marginal distribution of X and Y and also <math>P(P(X \leq 1, Y \leq 1), P(X \leq 1), P(Y \leq 1)</math>. Check whether X and Y are independent. The joint probability mass function of X and Y is</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="border: none;"></td> <td style="border: none;">Y</td> <td style="border: none;">0</td> <td style="border: none;">1</td> <td style="border: none;">2</td> </tr> <tr> <td style="border: none;">X</td> <td style="border: none;"></td> <td style="border: none;"></td> <td style="border: none;"></td> <td style="border: none;"></td> </tr> <tr> <td style="border: none;"></td> <td style="border: none;">0</td> <td style="border: none;">0.10</td> <td style="border: none;">0.04</td> <td style="border: none;">0.02</td> </tr> <tr> <td style="border: none;"></td> <td style="border: none;">1</td> <td style="border: none;">0.08</td> <td style="border: none;">0.20</td> <td style="border: none;">0.06</td> </tr> </table>		Y	0	1	2	X						0	0.10	0.04	0.02		1	0.08	0.20	0.06	BTL-2	Understanding
	Y	0	1	2																			
X																							
	0	0.10	0.04	0.02																			
	1	0.08	0.20	0.06																			



	2	0.06	0.14	.030																						
6.	A machine is used for a particular job in the forenoon and for a different job in the afternoon. The joint probability distribution of (X, Y), where X and Y represent the number of times the machine breakdown in the forenoon and in the afternoon respectively, is given in the following table. Examine if X and Y are independent RV's				BTL4	Analyzing																				
7.	If the joint pdf of a two-dimensional RV(X,Y) is given by $f(x,y) = \begin{cases} x^2 + \frac{xy}{3}; & 0 < x < 1, 0 < y < 2 \\ 0, & \text{elsewhere} \end{cases}$ Find (i) $P\left(X > \frac{1}{2}\right)$ (ii) $P\left(Y < \frac{1}{2}, X < \frac{1}{2}\right)$ (iii) $P\left(Y < \frac{1}{2} / X < \frac{1}{2}\right)$				BTL3	Applying																				
8.	The joint pdf of a two dimensional random variable (X, Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2, 0 \leq y \leq 1.$ Compute (i) $P\left(X > 1 / Y < \frac{1}{2}\right)$ (ii) $P\left(Y < \frac{1}{2} / X > 1\right)$ (iii) $P(X + Y) \leq 1.$				BTL3	Applying																				
9.	(b)The joint pdf of X and Y is given by $f(x,y) = \begin{cases} kx(x-y), & 0 < x < 2, -x < y < x \\ 0, & \text{otherwise} \end{cases}$ (i) Find K (ii) Find $f_x(x)$ and $f_y(y)$				BTL3	Applying																				
10.	Find the Coefficient of Correlation between industrial production and export using the following table				BTL-2	Understanding																				
	<table border="1"> <tr> <td>Production (X)</td> <td>14</td> <td>17</td> <td>23</td> <td>21</td> <td>25</td> </tr> <tr> <td>Export (Y)</td> <td>10</td> <td>12</td> <td>15</td> <td>20</td> <td>23</td> </tr> </table>					Production (X)	14	17	23	21	25	Export (Y)	10	12	15	20	23									
Production (X)	14	17	23	21	25																					
Export (Y)	10	12	15	20	23																					
11.	Find the correlation coefficient for the following heights of fathers X, their sons Y and also find the equations of regression lines. Hence find the height of son when the height of father is 71				BTL-2	Understanding																				
	<table border="1"> <tr> <td>X</td> <td>65</td> <td>66</td> <td>67</td> <td>67</td> <td>68</td> <td>69</td> <td>70</td> <td>72</td> </tr> <tr> <td>Y</td> <td>67</td> <td>68</td> <td>65</td> <td>68</td> <td>72</td> <td>72</td> <td>69</td> <td>71</td> </tr> </table>					X	65	66	67	67	68	69	70	72	Y	67	68	65	68	72	72	69	71			
X	65	66	67	67	68	69	70	72																		
Y	67	68	65	68	72	72	69	71																		
12.	Obtain the lines of regression				BTL-2	Understanding																				
	<table border="1"> <tr> <td>X</td> <td>50</td> <td>55</td> <td>50</td> <td>60</td> <td>65</td> <td>65</td> <td>65</td> <td>60</td> <td>60</td> </tr> <tr> <td>Y</td> <td>11</td> <td>14</td> <td>13</td> <td>16</td> <td>16</td> <td>15</td> <td>15</td> <td>14</td> <td>13</td> </tr> </table>					X	50	55	50	60	65	65	65	60	60	Y	11	14	13	16	16	15	15	14	13	
X	50	55	50	60	65	65	65	60	60																	
Y	11	14	13	16	16	15	15	14	13																	
13.	If $f(x,y) = \frac{6-x-y}{8}, 0 \leq x \leq 2, 2 \leq y \leq 4$ for a bivariate random variable (X,Y), Evaluate the correlation coefficient $\rho$ .				BTL5	Evaluating																				
14.	Two random variables X and Y have the joint density function $f(x,y) = x + y, 0 \leq x \leq 1, 0 \leq y \leq 1.$ Evaluate the Correlation coefficient between X and Y.				BTL5	Evaluating																				
15.(a)	20 dice are thrown. Find the approximate probability that the sum obtained is between 65 and 75 using central limit theorem				BTL3	Applying																				
15.(b)	The two regression lines are $4x-5y+33=0$ and $20x-9y=107$ . Find the mean of X and Y. Also find the correlation coefficient between them				BTL3	Applying																				
16.(a)	If $X_1, X_2, X_3, \dots, X_n$ are Poisson variates with mean 2, use central limit theorem to estimate $P(120 < S_n < 160)$ where $S_n = X_1 + X_2 + X_3 + \dots + X_n$ and $n=75$ .				BTL4	Analyzing																				
16.(b)	If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of $U = X-Y$ .				BTL4	Analyzing																				

17.(a)	If X and Y independent Random Variables with pdf $e^{-x}, x \geq 0$ and $e^{-y}, y \geq 0$ . Devise the density function of $U = \frac{X}{X+Y}$ and $V = X+Y$ . Are they independent?	BTL4	Analyzing
17.(b)	Two random variables X and Y have the following joint probability density function $f(x, y) = \begin{cases} x+y; & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$ . Find the probability density function of the random variable $U = XY$ .	BTL3	Applying
18.(a)	A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using central limit theorem find what probability that we can assert that the mean of the sample will not differ from $\mu$ more than 4?	BTL5	Evaluating
18.(b)	If X and Y follows an exponential distribution with parameters 2 and 3 respectively and are independent, Create the probability density function of $U = X+Y$	BTL6	Creating

**PART-C(15 Mark Questions)**

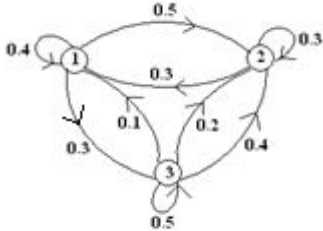
1.	Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 blue balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, Find the probability distribution of X and Y.	BTL3	Applying
2.	Out of the two lines of regression given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ , which one is the regression line of X on Y? Analyze the equations to find the means of X and Y. If the variance of X is 12, find the variance of Y.	BTL4	Analyzing
3.	From the following data, Find (i) The two regression equations (ii) The coefficient of correlation between the marks in Mathematics and Statistics (iii) The most likely marks in Statistics when marks in Mathematics are 30 Marks in Maths: 25 28 35 32 31 36 29 38 34 32 Marks in Statistics: 43 46 49 41 36 32 31 30 33 39	BTL-2	Understanding
4.	For a particular brand of TV picture tube, it is known that the mean operating life of the tubes is 1000 hours with a standard deviation of 250 hours, Devise the probability that the mean for a random sample of size 25 will be between 950 and 1050 hours?	BTL5	Evaluating
5.	The lifetime of a certain brand of an electric bulb may be considered a RV with mean 1200h and standard deviation 250h. Find the probability, using central limit theorem, that the average life time of 60 bulbs exceeds 1250 h.	BTL6	Creating

**UNIT III : RANDOM PROCESSES 9L+3T**

Classification – Stationary process – Markov process – Poisson process – Discrete parameter Markov chain – Chapman Kolmogorov equations – Limiting distributions.

**PART-A(2 Mark Questions)**

1.	What are the four types of a stochastic process?	BTL-1	Remembering
2.	Define Discrete Random sequence with example.	BTL-1	Remembering
3.	Define Discrete Random Process with example.	BTL-1	Remembering
4.	Define Continuous Random sequence with example.	BTL-1	Remembering
5.	Define Continuous Random Process with example.	BTL-1	Remembering
6.	Define wide sense stationary process.	BTL-1	Remembering
7.	Define Strict Sense Stationary Process.	BTL-1	Remembering
8.	Show that the random process $X(t) = A \cos(\omega_c t + \theta)$ is not stationary if it	BTL-2	Understanding

	is assumed that A and $\omega_c$ are constants and $\theta$ is a uniformly distributed variable on the interval $(0, \pi)$ .		
9.	A random process $X(t) = A \sin t + B \cos t$ where A and B are independent random variables with zero means and equal standard deviations. Find the mean of the process.	BTL-2	Understanding
10.	Consider the random process $X(t) = \cos(t + \phi)$ , where $\phi$ is uniform random variable in $(-\pi/2, \pi/2)$ . Check whether the process is stationary.	BTL3	Applying
11.	Consider the random process $X(t) = \cos(\omega_0 t + \theta)$ , where $\theta$ is uniform random variable in $(-\pi, \pi)$ . Check whether the process is stationary or not	BTL3	Applying
12.	Find the mean of a stationary random process whose auto correlation function is given by $R_{(Z)} = \frac{25Z^2 + 36}{6.25Z^2 + 4}$	BTL-2	Understanding
13.	Find the mean of a stationary random process whose auto correlation function is given by $R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2}$	BTL-2	Understanding
14.	A random process has the autocorrelation function $R_{xx}(\tau) = \frac{4\tau^2+6}{\tau^2+1}$ , find the mean square value of the problem	BTL-2	Understanding
15.	Compute the mean value of the random process whose auto correlation function is given by $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$	BTL-2	Understanding
16.	Define Poisson process	BTL-1	Remembering
17.	State and two properties of Poisson process.	BTL-1	Remembering
18.	Check whether the Poisson process $X(t)$ given by the probability law $P\{X(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$ , $n = 0, 1, 2, \dots$ is stationary or not.	BTL4	Analyzing
19.	A hospital receives on an average of 3 emergency calls in 10 minutes interval. What is the probability that there are 3 emergency calls in a 10 minute interval	BTL4	Analyzing
20.	Define Markov chain	BTL-1	Remembering
21.	State Chapman- Kolmogorov theorem	BTL-1	Remembering
22.	Consider the Markov chain with 2 states and transition probability matrix $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ . Find the stationary probabilities of the chain.	BTL4	Analyzing
23.	The one-step transition probability matrix of a Markov chain with states (0,1) is given by $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ . Evaluate whether it is irreducible Markov chain?	BTL5	Evaluating
24.	Obtain the transition matrix of the following transition diagram. 	BTL5	Evaluating

25.	Check whether the Markov chain with transition probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ is irreducible or not?	BTL4	Analyzing
<b>PART-B (13 Marks Questions)</b>			
1.	The process $\{X(t)\}$ whose probability distribution under certain conditions is given by $P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2 \\ \frac{at}{(1+at)}, & n = 0 \end{cases}$ Show that it is not stationary.	BTL-2	Understanding
2.(a)	Show that the random process $X(t) = A \sin(\omega t + \theta)$ is wide-sense stationary process where A and $\omega$ are constants and $\theta$ is uniformly distributed in $(0, 2\pi)$ .	BTL-2	Understanding
2.(b)	Find the mean and autocorrelation of the Poisson processes	BTL-1	Remembering
3.(a)	Given that the random process $X(t) = \cos(t + \varphi)$ where $\varphi$ is a random variable with density function $f(x) = \frac{1}{\pi}, \frac{-\pi}{2} < \varphi < \frac{\pi}{2}$ . Discuss whether the process is stationary or not	BTL3	Applying
3.(b)	Prove that the sum of two independent Poisson process is a Poisson process.	BTL-1	Remembering
4.(a)	Consider a random process $X(t) = B \cos(50t + \Phi)$ where B and $\Phi$ are independent random variables. B is a random variable with mean 0 and variance 1. $\Phi$ is uniformly distributed in the interval $[-\pi, \pi]$ . Determine the mean and auto correlation of the process.	BTL3	Applying
4.(b)	Prove that the difference of two independent Poisson process is not a Poisson process	BTL-1	Remembering
5.(a)	Show that the random process $X(t) = A \cos(\omega t + \theta)$ is wide sense stationary, if A and $\omega$ are constant and $\theta$ is a uniformly distributed random variable in $(0, 2\pi)$ .	BTL3	Applying
5.(b)	On the average a submarine on patrol sights 6 enemy ships is not stationary per hour. Assuming that the number of ships sighted in a given length of time is a Poisson variate. Find the probability of sighting 6 ships in the next half-an-hour, 4 ships in the next 2 hours and at least 1 ship in the next 15 minutes.	BTL4	Analyzing
6.(a)	Show that the random process $X(t) = A \cos(\omega t + \theta)$ is not stationary if A and $\omega$ are constants and $\theta$ is uniformly distributed random variable in $(0, \pi)$ .	BTL3	Applying
6.(b)	Prove that the inter arrival time of the Poisson process follows exponential distribution	BTL-1	Remembering
7.	Show that the random process $X(t) = A \cos \omega t + B \sin \omega t$ is wide sense stationary process if A and B are random variables such that $E(A) = E(B) = 0, E(A^2) = E(B^2)$ and $E(AB) = 0$	BTL-2	Understanding
8.	A salesman's territory consists of three regions A, B, C. He never sells in the same region on successive days. If he sells in region A, then the next day he sells in B. However, if he sells either B or C, then the next day he is twice as likely to sell in A as in the other region. Explain How often does he sell in each of the regions in the steady state?	BTL3	Applying

9.(a)	If the process $X(t) = P + Qt$ , where P and Q are independent random variables with $E(P) = p, E(Q) = q, Var(P) = \sigma_1^2, Var(Q) = \sigma_2^2$ , find $E(X(t)), R(t_1, t_2)$ . Is the process $\{X(t)\}$ stationary?	BTL-2	Understanding
9.(b)	The probability of a dry day following a rainy day is $1/3$ and that the probability of a rainy day following a dry day is $1/2$ . Given that May 1 <sup>st</sup> is a dry day. Obtain the probability that May 3 <sup>rd</sup> is a dry day also May 5 <sup>th</sup> is a dry day.	BTL3	Applying
10.(a)	Suppose that customers arrive at a bank according to a Poisson process with a mean rate of per minute; Discuss the probability that during a time interval of 2 minutes (a) exactly 4 customers arrive, and (b) more than 4 customers arrive.	BTL4	Analyzing
10.(b)	An engineer analyzing a series of digital signals generated by at testing system observes that only 1 out of 15 highly distorted signal with no recognizable signal whereas 20 out of 23 recognized signals follow recognizable signals with no highly distorted signals between. Given that only highly distorted signals are not recognizable, Create the fraction of signals that are highly distorted	BTL6	Creating
11.(a)	Consider the random process $Y(t) = X(t)\cos(\omega_0 t + \theta)$ , where $X(t)$ is wide sense stationary process, $\theta$ is a Uniformly distributed R.V. over $(-\pi, \pi)$ and $\omega_0$ is a constant. It is assumed that $X(t)$ and $\theta$ are independent. Show that $Y(t)$ is a wide sense stationary	BTL-2	Understanding
11.(b)	At an intersection, a working traffic light will be out of order the next day with probability 0.07, and an out of order traffic light will be working on the next day with probability 0.88. Find the state space and TPM. Also find $P(X_2=1)$ .	BTL4	Analyzing
12.	The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3, \dots$ having 3 states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P(0) = (0.7, 0.2, 0.1)$ . Evaluate i) $P(X_2 = 3)$ ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$	BTL-2	Understanding
13.	Consider the Markov chain $\{X_n, n= 0, 1, 2,3, \dots\}$ having 3 states space $S=\{1,2,3\}$ and one step TPM $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ and initial probability distribution $P(X_0=i)=1/3, i= 1,2,3$ . Compute (1) $P(X_3=2, X_2=1, X_1=2/X_0=1)$ (2) $P(X_3=2, X_2=1/X_1=2, X_0=1)$ (3) $P(X_2=2/X_0=2)$ (4) Invariant Probabilities of the Markov Chain.	BTL-2	Understanding
14.(a)	Let $\{X_n : n = 1,2,3, \dots\}$ be a Markov chain on the space $S = \{1,2,3\}$ with one step t.p.m $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$	BTL-2	Understanding

	1. Sketch the transition diagram, 2. Is the chain irreducible? Explain. 3. Is the chain ergodic? Explain.		
14.(b)	If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, Evaluate the probability that the interval between 2 consecutive arrivals is (a) more than 1 minute, (b) between 1 minute and 2 minutes and (c) 4 minutes or less	BTL5	Evaluating
15.	Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition probability matrix and classify the states	BTL6	Creating
16.	Consider a Markov chain chain $\{X_n, n= 0, 1, 2, \dots\}$ having states space $S=\{ 1,2\}$ and one step TPM $P = \begin{bmatrix} 4 & 6 \\ 10 & 10 \\ 8 & 2 \\ 10 & 10 \end{bmatrix}$ . (1) Draw a transition diagram, (2) Is the chain irreducible? (3) Is the state -1 ergodic? Explain. (4) Is the chain ergodic? Explain	BTL4	Analyzing
17.	Classify the states of the Markov chain for the one-step transition probability matrix $P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$ with state space $S = \{1,2,3\}$	BTL5	Evaluating
18.	Consider a Markov chain on $(0, 1, 2)$ having the transition matrix given by $P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ . Show that the chain is irreducible. Devise the period the stationary distribution	BTL4	Analyzing
<b>Part C: 15 - MARK QUESTIONS</b>			
1.	On a given day, a retired English professor, Dr. Charles Fish amuses himself with only one of the following activities reading (i), gardening (ii) or working on his book about a river valley (iii) for $1 \leq i \leq 3$ , let $X_n = i$ , if Dr. Fish devotes day $n$ to activity $i$ . Suppose that $\{X_n : n=1,2,\dots\}$ is a Markov chain, and depending on which of these activities on the next day is given by the t. p. m $P = \begin{bmatrix} 0.30 & 0.25 & 0.45 \\ 0.40 & 0.10 & 0.50 \\ 0.25 & 0.40 & 0.35 \end{bmatrix}$ Find the proportion of days Dr. Fish devotes to each activity.	BTL-2	Understanding
2.	A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if 6 appeared. Find (i) the probability that he takes a train on the third day (ii) the probability that he drives to work in the long run.	BTL3	Applying
3.	A machine goes out of order whenever a component fails. The failure of this part follows a Poisson process with mean rate of 1 per week. Find the probability that 2 weeks have a elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not	BTL4	Analyzing

	due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks		
4.	A fair die is tossed repeatedly. If $X_n$ denotes the maximum of the numbers occurring in the first $n$ tosses, Evaluate the transition probability matrix $P$ of the Markov chain $\{X_n\}$ . Find also $P\{X_2=6\}$ and $P^2$ .	BTL5	Evaluating
5.	A student's study habits are as follows: If he studies one night, he is 70% sure not to study next night. On the other hand, if he does not study one night, he is 60% sure not to study the next nights as well. In the long run how often does he study?	BTL6	Creating
<b>UNIT IV : QUEUEING MODELS</b>			
Markovian queues – Birth and death processes – Single and multiple server queueing models – Little's formula – Queues with finite waiting rooms – Queues with impatient customers: Balking and renegeing.			
1.	State the characteristics of a Queueing model.	BTL1	Remembering
2.	Write Kendall's notation for Queueing Model.	BTL2	Understanding
3.	What are the service disciplines available in the queueing model?	BTL1	Remembering
4.	Write the formulae for $P_0$ and $P_n$ in a Poisson queue system in the steady – state	BTL3	Applying
5.	Define steady state and transient state queueing systems		
6.	For (M/M/1): ( $\infty$ /FIFO) model, Write the Little's formula.	BTL2	Understanding
7.	Find the probability of at least 10 customers in the system (M/M/1): ( $\infty$ /FIFO) queue system, if $\lambda=6$ per hour and $\mu=8$ per hour?		
8.	Find the probability that a customer has to wait more than 15 min to get his service completed in a (M/M/1): ( $\infty$ /FIFO) queue system, if $\lambda=6$ per hour and $\mu=10$ per hour?	BTL2	Understanding
9.	For a (M/M/1): ( $\infty$ /FIFO) queue system, if $\lambda=4$ per hour and $\mu=6$ per hour, find the average queue length.	BTL4	Analyzing
10.	If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 minutes before the picture starts and it takes exactly 1.5 minutes to reach the correct seat after purchasing the ticket. What he expect to be seated for the start of the picture?	BTL3	Applying
11.	If the inter arrival time and service time in a public telephone booth with a single phone follow exponential distribution with means of 10 and 8 minutes respectively. Find the average number of callers in the booth at any time.	BTL2	Understanding
12.	If the arrival and departure rates in a M/M/1 queue are $\frac{1}{2}$ per minute and $\frac{2}{3}$ per minute respectively, find the average waiting time of a customer in the queue.	BTL2	Understanding
13.	If there are 2 servers in an infinite capacity Poisson queue system with $\lambda=10$ per hour and $\mu=15$ per hour, Examine the percentage of idle time for each server?	BTL4	Analyzing
14.	Find the traffic intensity for an (M/M/C): ( $\infty$ /FIFO) queue with $\lambda=10$ per hour and $\mu=15$ per hour and 2 servers.	BTL2	Understanding
15.	Write the effective arrival rate for M/M/1: K/FIFO queueing model	BTL1	Remembering
16.	Give the formula for average waiting time of a customer in the queue for (M/M/1): (K/FIFO).	BTL1	Remembering
17.	If $\lambda=3$ per hour, $\mu=4$ per hour and maximum capacity $K=7$ in a (M/M/1) (K/FIFO) system, Find the average number of customers in the system.	BTL6	Creating
18.	A drive in banking service is modeled as an M/M/1 queueing system with	BTL4	Analyzing

	customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99 percent of the time. How fast should the service rate be?		
19.	Describe the formula for $W_s$ and $W_q$ for the M/M/1/N queueing system.	BTL1	Remembering
20.	For (M/M/C): (N/FIFO) model, Write the formula for (a) average number of customers in the queue. (b) Average waiting time in the system.	BTL4	Analyzing
21.	Consider an M/M/C queueing system. Find the probability that an arriving customer is forced to join the queue.	BTL5	Evaluating
22.	Draw the transition diagram for M/M/1 queueing model.	BTL1	Remembering
23.	A self-service store employs one cashier at its counter. Nine customers arrive on an average every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, Find the average time a customer spends in the system.	BTL1	Remembering
24.	In a 3 server infinite capacity Poisson queue model if $\frac{\lambda}{\mu C} = \frac{2}{3}$ , Calculate $P_0$ .	BTL1	Remembering
25.	State the characteristics of a Queueing model.	BTL1	Remembering
<b>PART-B (13 Mark Questions)</b>			
1.	Derive the average number of customers in the system, in queue and average waiting time of a customer in the system and in queue for an M/M/1: ∞/FIFO queueing system	BTL1	Remembering
2.	Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min. between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min. a) Find the average number of persons waiting in the system b) What is the probability that a person arriving at the booth will have to wait in the queue? c) What is the probability that it will take him more than 10 min altogether to wait for the phone and complete his call? d) Estimate the fraction of the day when the phone will be in use e) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for atleast 3 min. for phone .By how much the flow of arrivals should increase in order to justify a second booth?	BTL2	Understanding
3.	Customers arrive at a one-man barber shop according to a Poisson with a mean inter arrival time of 20 min Customers spend an average of 15 min in the barber's chair 1) What is the expected number of customers in the barber shop ?In the Queue? 2) What is the probability that a customer will not have to wait for a hair cut? 3) How much can a customer expect to spend in the barbershop? 4) What are the average time customers spend in the queue? 5) What is the probability that the waiting time in the system is greater than 30 min? 6) What is the probability that there are more than 3 customers in the system?	BTL2	Understanding
4.	Customers arrive at a one-man barber shop according to a Poisson with a	BTL2	Understanding



	<p>mean inter arrival time of 12 min Customers spend an average of 10min in the barber's chair</p> <ol style="list-style-type: none"> <li>1) What is the expected number of customers in the barber shop?</li> <li>2) What is the expected number of customers In the Queue?</li> <li>3) What is the probability that a customer will not have to wait for a hair cut?</li> <li>4) How much can a customer expect to spend in the barbershop?</li> <li>5) What are the average time customers spend in the queue?</li> <li>6) What is the probability that the waiting time in the system is greater than 30 min?</li> </ol>		
5.	<p>In a given M / M / 1 queueing system, the average arrivals is 4 customers per minute, <math>\rho = 0.7</math>. Find the</p> <ol style="list-style-type: none"> <li>1 ) mean number of customers <math>L_s</math> in the system</li> <li>2) mean number of customers <math>L_q</math> in the queue</li> <li>3 ) probability that the server is idle</li> <li>4) mean waiting time <math>W_s</math> in the system</li> <li>5) mean waiting time <math>W_q</math> in the queue</li> </ol>	BTL2	Understanding
6.	<p>Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes, and the service time is exponential random variable with 8 minutes. Apply M/M/1 queueing model</p> <ol style="list-style-type: none"> <li>a) Find the average number of customers <math>L_s</math> in the shop.</li> <li>b) Find the average number of customers <math>L_q</math> in the queue.</li> <li>c) Find the average time a customer spends in the system in the shop <math>W_s</math>.</li> <li>d) What is the probability that the server is idle?</li> </ol>	BTL3	Applying
7.	<p>A TV repairman finds that the time spend on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 – hour day. Point out the repairman's expected idle time in each day? How many jobs are ahead of the average set just brought in?</p>	BTL3	Applying
8.	<p>On average 96 patients per (24 hour) day require the service of an emergency clinic. Also an average a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs.100 per patient treated to obtain an average service time of 10 minutes, and that each minute of decrease in this average time would cost Rs .10 per patient treated .Analyze how much would have to be budgeted by the clinic to decrease the average size of the queue from <math>1 \frac{1}{3}</math> patients to <math>\frac{1}{2}</math> patient?</p>	BTL4	Analyzing
9.	<p>A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only .It has been found that the service time distributions for both deposits and withdrawals are exponential with mean time of 3 min per customer. Depositors are found to arrive in Poisson fashion throughout the day with mean arrival rate of 16 per hour Withdrawers also arrive in a Poisson fashion with mean arrival rate 14 per hour. What would be the effect on the average waiting time for the customers if each teller handles both withdrawals and deposits? What would be the effect, if this could only be accomplished by increasing the service time to 3.5 min?</p>	BTL4	Analyzing
10.	<p>There are three typists in an office. Each typist can type an average of 6</p>		

	<p>Letters per hour .If letters arrive for being typed at the rate of 15 letters per hour, Analyze the following</p> <p>a) What fraction of the time all the typists will be busy?</p> <p>b) What is the average number of letters waiting to be typed?</p> <p>c) What is the average time a letter has to spend for waiting and for being typed?</p> <p>d) What is the probability that a letter will take longer than 20 min waiting to be typed?</p>	BTL4	Analyzing
11.	<p>A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min and cars arrive for service in a Poisson process at the rate of 30 cars per hour. Evaluate the following</p> <p>(a)What is the probability that an arrival would have to wait in line?</p> <p>(b)Find the average waiting time, average time spent in the system and the average number of cars in the system</p> <p>(c) For what percentage of time would a pump be idle on an average?</p>	BTL5	Evaluating
12.	<p>A supermarket has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 min and if people arrive in Poisson fashion at the rate of 10 per hour</p> <p>a) What is the probability that a customer has to wait for service?</p> <p>b) What is the expected percentage of idle time for each girl?</p> <p>c) If the customer has to wait in the queue, what is the expected length of the waiting time?</p>	BTL5	Evaluating
13.	<p>A tele phone exchange has two long distance operators. The telephone company finds that during peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service of these calls approximately exponentially distributed with mean length of 5 minutes.</p> <p>(1) What is the probability that a subscriber will have to wait for his long distance calls during the peack hours of the day?</p> <p>(2) If the subscribers will wait and are serviced in turn, what is the expected waiting time?</p>	BTL5	Evaluating
14.	<p>Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains.</p> <p>1.Find the probability that the yard is empty</p> <p>2. Find the average number of trains in the system</p>	BTL4	Analyzing
15.	<p>Consider a single server queueing system with Poisson input exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hours and the maximum permissible number calling units in the system is 2. Apply single server finite capacity to</p> <p>1. Find the steady state probability distribution of the number of calling units in the system.</p> <p>2. Find the expected number of calling units in the system and in queue.</p> <p>3. Find the average waiting time in the system and in the queue.</p>	BTL3	Applying
16.	<p>A 2 – person barber shop has 5 chair to accommodate waiting customers.Potential customers ,who arrive when all 5 chairs are full,leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber’s chair .Compute <math>P_0</math>, <math>P_1, P_7</math>, <math>E(N_q)</math> and <math>E(W)</math></p>	BTL4	Analyzing
17.	<p>A car servicing station has 2 bays where service can be offered</p>		

	simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both bays is exponentially distributed with $\mu = 8$ cars per day per bay. Write the average number of cars in the service station, the average number of cars waiting for service time a car spends in the system	BTL6	Creating
18.	The railway marshalling yard is sufficient only for trains (there being 11 lines, one of which is earmarked for the shunting engine to reverse itself from the crest of the hump to the rear of the train). Trains arrive at the rate of 25 trains per day, inter-arrival time and service time follow exponential with an average of 30 minutes. Estimate the probability that the yard is empty and average queue length.	BTL6	Creating
<b>PART –C (15 Marks Questions)</b>			
1.	A departmental store has single cashier. During the rush hours customers arrive at the rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Point out the following 1. What is the probability that the cashier is idle? 2. What is the average number of customers in the queueing system? 3. What is the average time a customer spends in the system? 4. What is the average number of customers in the queue? 5. What is the average time a customer spends in the queue?	BTL2	Understanding
2.	A repairman is to be hired to repair machines which breakdown at the average rate of 3 per hour the breakdown follow Poisson distribution. Non-productive time of machine is considered to cost Rs 16/hour. Two repair men have been interviewed. One is slow but cheap while the other is fast and expensive. The slow repairman charges Rs.8 per hour and he services at the rate of 4 per hour The fast repairman demands Rs .10 per hour and services at the average rate 6 per hour. Which repairman should be hired?	BTL3	Applying
3.	Patients arrive at a clinic according to a Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour. 1. Derive the effective arrival rate at the clinic 2. What is the probability that an arriving patient does not have to wait? 3. What is the expected waiting time until a patient is discharged from the clinic?	BTL4	Analyzing
4.	A petrol pump station has 2 pumps. The service times follow the exponential distribution with a mean of 4 minutes and cars arrive for service in a Poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What proportion of time the pumps remain idle?	BTL5	Evaluating
5.	At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms. down the river. Tankers arrive according to a Poisson process with a mean of 1 for every 2 hours. It takes for an unloading crew, on the average, 10 hours to unload a tanker, the unloading time follows an exponential distribution Develop and Determine (i) How many tankers are at the port on the average? (ii) How long does a tanker spend at the port on the average? (iii) What is the average arrival rate at the overflow facility?	BTL6	Creating

**UNIT V: ADVANCED QUEUEING MODELS**

Finite source models – M/G/1 queue – Pollaczek Khinchine formula – M/D/1 and M/E<sub>k</sub>/1 as special cases – Series queues – Open Jackson networks.

**PART-A (2 Marks Questions)**

1.	Express Pollaczek- Khinchine formula.	BTL1	Remembering
2.	Write P-K formula for the case when the service time is constant.	BTL1	Remembering
3.	Write down the formula for (M/D/1) : ( $\infty/GD$ ) model		
4.	Define effective arrival rate with respect to an (M   M   1): (GD / N/ $\infty$ ) queueing model.	BTL1	Remembering
5.	What do you mean by $E_k$ in the M/E <sub>k</sub> / 1 queueing model?	BTL2	Understanding
6.	For an M/G/1 model if $\lambda=5$ and $\mu=6$ min and $\sigma =1/20$ , find the length of the queue.	BTL1	Remembering
7.	A one man barber shop taken 25 minutes to complete a haircut. If customers arrive in a Poisson fashion at an average rate of 1 per 40 minutes find the average length of the queue.	BTL1	Remembering
8.	In an M/D/1 queueing system, an arrival rate of customer is 1/6 per minute and the server takes exactly 4 minutes to serve a customer. Calculate the mean number of customers in the system.	BTL2	Understanding
9.	The arrival of trucks to a factory for unloading is Poisson with arrival rate of 3 trucks per hour. The unloading time is constant with exactly 4 customers per hour. What is the expected number of trucks in queue?	BTL2	Understanding
10.	Describe series queue.	BTL1	Remembering
11.	Define a two-stage series queue.	BTL3	Applying
12.	Define Series Queue with blocking.	BTL1	Remembering
13.	Define a tandem queue.	BTL3	Applying
14.	Draw the state transition diagram of a two stage sequential queue model with blocking for the stage2	BTL4	Analyzing
15.	State any two examples for series queues.	BTL4	Analyzing
16.	A transfer line has two machines M1 and M2 with unlimited buffer space in between. Parts arrives the transfer line at the rate of 1 part every 2 minutes. The processing rates of M1 and M2 are 1 per minutes and 2 per minutes respectively. Find the average number of parts in M1.	BTL1	Remembering
17.	Consider a series facility with two sequential stations with respective service rates 3/min and 4/min. The arrival rate is 2/min. What is the average service time of the system, if the system could be approximated by a two stage tandem queue?	BTL1	Remembering
18.	Define an open Jackson network	BTL5	Evaluating
19.	State Jackson's theorem for an open network.	BTL3	Applying
20.	Compose classification of queueing networks.	BTL6	Creating
21.	Distinguish between open and closed networks.	BTL1	Remembering
22.	What do you mean by bottleneck of a network?	BTL2	Understanding
23.	Write down the characteristics of an open Jackson network.	BTL1	Remembering
24.	Define a closed Jackson network.	BTL1	Remembering
25.	State Burke's theorem used in queueing theory.	BTL1	Remembering

**PART – B (13-MARK QUESTIONS)**

1.	State and Derive Pollaczek - Khinchine formula.	BTL6	Creating
2.	In a college canteen, it was observed that there is only one waiter who takes exactly 4 minutes to serve a cup of coffee once the order has been placed with him. If students arrive in the canteen at an average rate of 10 per hour,	BTL1	Remembering

	how much time one is expected to spend waiting for his turn to place the order.		
3.	A one-man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, Find the average time a customer spends in the shop. Also, Find the average time a customer must wait for service?	BTL2	Understanding
4.	For $(M/E_2/1)$ : (FIFO/ $\infty/\infty$ ) queueing model with $\lambda = \frac{6}{5}$ per hour and $\mu = \frac{3}{2}$ per hour, find the average waiting time of a customer. Also find the average time he spends in the system	BTL4	Analyzing
5.	Consider a queueing system where arrivals according to a Poisson distribution with mean 5/hr. Find expected waiting time in the system if the service time distribution is Uniform from $t = 5$ min to $t = 15$ minutes	BTL5	Evaluating
6.	Find $L_s, L_q, W_s$ and $W_q$ . Automatic car wash facility operates with only one Bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. (i) If the service time for all cars is constant and equal to 10 min (ii) Uniform distribution between 8 and 12 minutes (iii) Normal distribution with mean 12 minutes and SD 3 minutes (iv) Follows discrete distribution 4, 8 & 15 minutes with corresponding probability 0.2, 0.6 & 0.2	BTL5	Evaluating
7.	If a patient who goes to a single doctor clinic for a general check up has to go through 4 phases. The doctor takes on the average 4 minutes for each phase of the check up and the time taken for each phase is exponentially distributed. If the arrivals of the patients at the clinic are approximately Poisson at the average rate of 3 per hour, what is the average time spent by a patient (i) in the examination (ii) waiting in the clinic?	BTL6	Creating
8.	In a computer programs for execution arrive according to Poisson law with a mean of 5 per minute. Assuming the system is busy, Find $L_q, L_s, W_q, W_s$ if the service time is uniform between 8 and 12 sec.	BTL1	Remembering
9.	Find the average calling rate for the services of the crane and what is the average delay in getting service? In a heavy machine shop, the overhead crane is 75% utilized. Time study observations gave the average slinging time as 10.5 minutes with a standard deviation of 8.8 minutes. If the average service time is cut to 8.0 minutes with a standard deviation of 6.0 minutes, how much reduction will occur on average in the delay of getting served?	BTL3	Applying
10.	In a two-station service facility, queues are not allowed. Customers arrive at the facility at an average rate of 4 per hour; the server at each station serves at the rate of 5 customers per hour. If arrivals are Poisson and service times are exponential, find the probability that an arriving customer enter the system (a) The probability that an arriving customer enters the system. (b) effective arrival rate. (c) Average (expected) number of customers in the system. (d) Expected time of a customer spends in the system.	BTL2	Understanding
11.	In a charity clinic there are two doctors, one assistant doctor D1 and his senior doctor D2. The Junior doctor tests and writes the case sheet and then sends to the senior for diagnosis and Prescription of medicine. Only one patient is allowed to enter the clinic at a time due to capacity of space. A patient who has finished with D1 has to wait till the patient with D2 has	BTL2	Understanding

	finished. If Patients arrive according to Poisson with rate 1 per hour and service times are independent and Follow exponential with parameters 3 and 2, Find (i) the probability of a customer entering the Clinic, (ii) the average number of customers in the clinic, (iii) the average time spent by a patient Who entered the clinic.		
12.	A repair facility is shared by a large number of machines for repair. The facility has two sequential stations with respective rates of service 2 per hour and 3 per hour. The cumulative failure rate of all the machines is 1 per hour. Assuming that the system behavior may be approximated by a two-station tandem queue Find (i) the average number of customers in both stations, (ii) The average repair time. (iii) The probability that both service stations are idle.	BTL3	Applying
13.	There are two service stations S1 and S2 in a line with unlimited buffer space in between. Customers arrives S1 at a rate of 1 per every 2 min. The service time rates of S1 and S2 are 1 and 2 per min. respectively. Find (i) the average number of customers at S1 and S2 (ii) The average waiting times at S1and S2 (iii) the total waiting time in the system.	BTL2	Understanding
14.	In a book shop there are two sections, one for text books and the other for note books Customers from side arrived at the text book section at a Poisson rate of 4 per hour and at the notebook section at a Poison rate of 3per hour. The service rates of T.B and N.B sections respectively 8 and 10 per hour. customer upon completion of service at T.B section is equally likely to go to the N.B section or to leave the book shop, where as a customer upon completion of service at N.B section will go to the T.B section with probability 1/3 and will leave the book shop otherwise. Find the joint steady state probability that there are 4 customers in the T.B section and 2 customers In the N.B section. Find also the average number of customers in the book shop and the average waiting time of the customers in the shop. Assume that there is only one sales man in each section.	BTL2	Understanding
15.	In a departmental store, there are two sections namely grocery section and perishable section. Customers from outside arrive the G-section according to a Poisson process at a mean rate of 10 per hour and they reach the p-section at a mean rate of 2 per hour. The service times at both the sections are exponentially distributed with parameters 15 and 12 respectively. On finishing the job in G-section, a customer is equally likely to go to the P-section or leave the store, where as a customer on finishing his job in the P-section will go to the G- section with probability 0.25 and leave the store otherwise. Assuming that there is only one salesman in each section, find (i) the probability that there are 3 customers in the G-section and 2 customers in the P-section, (ii) the average waiting time of a customer in the store.	BTL4	Analyzing
16.	Consider a system with two servers where customers arrive from outside the system in a Poisson fashion at server 1 at the rate of 4 per hour and at server 2 at a rate of 5 per hour. The customers are served at station 1 and station2 at the rate of 8 hour and 10 hour respectively. A customer after completion of service at server 1 is equally likely will go to server 2 or to leave the system. A departing customer from server 2 will go to server 1, 25% of the time and will depart from the system otherwise. Find the	BTL2	Understanding

	<ul style="list-style-type: none"> <li>(i) The total arrival rates at server1 and server 2.</li> <li>(ii) The limiting probability of n customers at server 1 and m customers at server2</li> <li>(iii) Expected number of customers in the system</li> <li>(iv) Expected time a customer spends in the system</li> </ul>																																					
17.	<p>In a network of 3 service station 1,2, 3 customer arrive at 1,2,3 from outside in accordance with Poisson process having rate 5, 10, 15 respectively. The service time at the stations are exponential with respect rate 10, 50, 100, A customer completing service at station -1 is equally likely to (i) go to station 2 (ii) go to station 3 or (iii) leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go station 2 or leave the system. (a) Find the average number customer in the system consisting of all the three stations? (b) Examine the average time a customer spend in the system?</p>	BTL5	Evaluating																																			
18.	<p>The open Jackson network the following information are given:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="4"></th> <th colspan="3"><math>\Gamma_{ij}</math></th> </tr> <tr> <th>Station</th> <th><math>C_j</math></th> <th><math>\mu_j</math></th> <th><math>r_j</math></th> <th>i = 1</th> <th>i =2</th> <th>i =3</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>10</td> <td>1</td> <td>0</td> <td>0.1</td> <td>0.4</td> </tr> <tr> <td>2</td> <td>2</td> <td>10</td> <td>4</td> <td>0.6</td> <td>0</td> <td>0.4</td> </tr> <tr> <td>3</td> <td>1</td> <td>10</td> <td>3</td> <td>0.3</td> <td>0.3</td> <td>0</td> </tr> </tbody> </table> <p>Find (i) the joint probability for the number of customers in 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> stations are 2,3,4 respectively.  (ii) the expected number of customer in each station.  (iii) the expected total number of customers in the system  (iv) the expected total waiting time in the system.</p>					$\Gamma_{ij}$			Station	$C_j$	$\mu_j$	$r_j$	i = 1	i =2	i =3	1	1	10	1	0	0.1	0.4	2	2	10	4	0.6	0	0.4	3	1	10	3	0.3	0.3	0	BTL2	Understanding
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3	1	10	3	0.3	0.3	0																																
<b>PART-C(15 Marks questions)</b>																																						
1	<p>Consider a single server. Poisson input queue with a mean arrival rate of 10/hr. Currently the server works according to an exponential distribution with a mean service time of 5 minutes. Management has a training course after which service time will follow non-exponential distribution and the mean service time will increase to 5.5 minutes, but the standard deviation will decrease from five minutes (exponential case) to 4 minutes. Should the server undergo training?</p>	BTL2	Understanding																																			
2	<p>In a car manufacturing plant the loading crane takes exactly 10 minutes to load a car into a wagon and again come back position to load another car.If the arrivals of the car is a Poisson stream at an average of one every 20 minutes. Calculate the following</p> <ul style="list-style-type: none"> <li>(1) Average number of cars in the system</li> <li>(2) Average number of cars in the queue</li> <li>(3) The Average waiting time of cars in the system</li> <li>(4) The Average waiting time of cars in the queue</li> </ul>	BTL-4	Analyzing																																			
3	<p>An average of 120 students arrives each hour (inter arrival times are exponential) at the controller's office to get their hall tickets. To complete the process a candidate must pass through counters. Each counter consists of a single server, service times at each counter are exponential with the following mean times: counter1, 20 seconds; conuter2, 15 seconds and counter3, 12 seconds. On the average evaluate how many students will be</p>	BTL-5	Evaluating																																			

	present in the controller's office?		
4	<p>Consider two servers. An average of 8 customers per hour from outside at server1 and an average of 17 customers arrive at server2. Inter arrival times are exponential server1 can serve at an exponential rate of 20 customers per hour and server2 can serve at an exponential rate of 30 customers per hour. After completing service at station1, half the customers leave the system and half go to server2. After completing service at station 2 <math>\frac{3}{4}</math> of the customers complete the server and <math>\frac{1}{4}</math> return to server1. Find the expected number of customers at each server. Find the average time a customer spends in the system.</p>	BTL-6	Creating
5.	<p>There are two salesman in a ration shop one in charge of billing and receiving payment and the other in charge of weighing and delivering the items. Due to limited availability of space, only one customer is allowed to enter the shop that too when the billing clerk is free. The customer who has finished his billing job has to wait until the delivery section becomes free. If customers arrive in accordance with a Poisson process at rate 1 and the service times of two clerks are independent and have exponential rates of 3 and 2. Find</p> <ol style="list-style-type: none"> <li>The proportion of customers who enter the ration shop</li> <li>The average number of customers in the shop</li> <li>The average amount of time that an entering customer spends in the shop.</li> </ol>	BTL-3	Applying