SRM VALLIAMMAI ENGINEERING COLLEGE

(An Autonomous Institution)

(S.R.M.NAGAR, KATTANKULATHUR-603 203)

DEPARTMENT OF MATHEMATICS

QUESTION BANK

IV SEMESTER

B.TECH- ARTIFICIAL INTELLIGENCE &DATA SCIENCES

1918406 – NUMERICAL LINEAR ALGEBRA

Regulation – 2019

Academic Year 2022- 2023

Prepared by

Dr. G. Sasikala, Assistant Professor/ Mathematics

SRM VALLIAMMAI ENGINEERING COLLEGE

(An Autonomous Institution)

SRM Nagar, Kattankulathur – 603 203.

DEPARTMENT OF MATHEMATICS

QUESTION BANK

SUBJECT : 1918406 – NUMERICAL LINEAR ALGEBRA

SEMESTER / YEAR: IV/ II (AI&DS)

UNIT I - VECTOR SPACES

Vector Spaces- Subspaces-Linear combinations and linear system of equations-Linear independence and linear dependence-Bases and Dimensions.

PART- A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Define Vector Space	**BTL-1**	Remembering
2.	Define Subspace of a vector space	**BTL-1**	Remembering
3.	What are the possible subspace of R^2	**BTL-1**	Remembering
4.	In a Vector Space V (F) if αv=0 then either α=0 or v=0 prove.	**BTL-2**	Understanding
5.	Is $\{(1,4,-6),(1,5,8),(2,1,1),(0,1,0)\}$ is a linearly independent subset of R^3 ? Justify your answer	**BTL-2**	Understanding
6.	State Replacement Theorem	**BTL-3**	Applying
7.	In a vector Space V(F),prove that 0v=0 , for all $v \in V$	**BTL-3**	Applying
8.	Write the vectors $v=(1,-2,5)$ as a linear combination of the vectors $x=(1,1,1), y=(1,2,3)$ and $z=(2,-1,1)$	**BTL-2**	Understanding
9.	What is the Dimension of $M_{2x2}(R)$?	**BTL-3**	Applying
10.	Determine whether the set W=$\{(a_1,a_2,a_3)\epsilon R^3: a_1+2a_2-3a_3=1\}$ is a subspace of R^3 under the operations of addition and scalar multiplication.	**BTL-2**	Understanding
11.	Determine whether $w=(4,-7,3)$ can be written as a linear combination of $v_1=(1,2,0)$ and $v_2=(3,1,1)$ in R^3	**BTL-2**	Understanding
12.	For which value of k will the vector $u=(1,-2,k)$ in R^3 be a linear combination of the vectors $v=(3,0,-2)$ and $w=(2,-1,5)$?	**BTL-3**	Applying
13.	Define Infinite dimensional vector Space	**BTL-2**	Understanding
14.	Point out whether the set $W_1=\{(a_1,a_2,a_3)\in R^3: a_1-4a_2-a_3=0\}$ is a subspace of R^3 under the operations of addition and scalar multiplication defined on R^3	**BTL-4**	Analyzing
15.	If W is a Subspace of the Vector Space V(F) prove that W must contain 0 vector in V	**BTL-4**	Analyzing
16.	Point out whether $w=(3,4,1)$ can be written as a linear combination of $v_1=(1,-2,1)$ and $v_2=(-2,-1,1)$ in R^3	**BTL-4**	Analyzing

17.	What are the possible subspaces of R^3	**BTL-4**	Analyzing
18.	Show that the vectors $\{(1,1,0),(1,0,1)$ and $(0,1,1)\}$ genarate R^3	**BTL-3**	Applying
19.	If $v_1,v_2 \in V(F)$ and $\alpha_1, \alpha_2 \in F$.Show that the set $\{v_1, v_2, \alpha_1 v_1 + \alpha_2 v_2\}$ is linearly dependent	**BTL-4**	Analyzing
20.	Test whether S={(2,1,0) , (1,1,0),(4,2,0)} in R^3 is a basis of R^3 over R	**BTL-5**	Evaluating
21.	Define finite dimensional Vector Space	**BTL-1**	Remembering
22.	Is $v = (2,-5,4)$ a linear combination of (1,-3,2) and (2,-1,1) in R^3(R) ?	**BTL-2**	Understanding
23.	Define linear span	**BTL-1**	Remembering
24.	Show that the set S= $\{(0,1,0),(1,0,1)$ and $(1,1,0)\}$ in R^3 is a basis over R	**BTL-3**	Applying
25.	Define linear combination of vectors	**BTL-1**	Remembering
	PART-B		
1.	Determine whether the following set is linearly dependent or linearly independent $\begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix},\begin{pmatrix}0 & -1\\ 1 & 1\end{pmatrix},\begin{pmatrix}-1 & 2\\ 1 & 0\end{pmatrix}$ and $\begin{pmatrix}2 & 1\\ 2 & -2\end{pmatrix}$ generate $M_{2\times2}(R)$	**BTL-3**	Applying
2.	If x, y and z are vectors in a vector space V such that $x+z=y+z$, then prove that $x=y$ i) The vector 0 (identity) is unique ii) The additive identity for any $x \in V$ is unique	**BTL-5**	Evaluating
3.	Show that the set ,S={(1,3,-4,2),(2,2,-4,0),(1,-3,2,-4),(-1,0,1,0)} is linearly dependent of the other vectors	**BTL-4**	Analyzing
4.	Determine whether the following subset of vector space R^3(R) is a subspace $W_1=\{(a_1,a_2,a_3): 2a_1-7a_2+a_3=0\}$	**BTL-5**	Evaluating
5.	Illustrate that set of all diagonal matrices of order $n \times n$ is a subspace of the vector space $M_{n\times n}(F)$, where $M_{n\times n}$ is the set of all square matrices over the field F	**BTL-4**	Analyzing
6.	Evaluate that $W_1 = \{(a_1,a_2,\ldots a_n) \in F^n; a_1+a_2+\cdots+a_n=0\}$ is a subspace of F^n and $W_2 = \{(a_1,a_2,\ldots a_n) \in F^n; a_1+a_2+\cdots+a_n=1\}$ is not a subspace	**BTL-2**	Understanding
7.	Illustrate that the vectors $\{(1,1,0),(1,0,1),(0,1,1)\}$ generate R^3	**BTL-5**	Evaluating
8.	Determine the following sets { $1-2x-2x^2$, $-2+3x-x^2$, $1-x+6x^2$} are bases for P_2(R)	**BTL-3**	Applying
9.	Analyze that the matrices $\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix},\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix},\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$ and $\begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}$ generate $M_{2\times2}(R)$	**BTL-2**	Understanding
10.	Identify whether the set $\{x^3+2x^2, -x^2+3x+1, x^3-x^2+2x-1\}$in $P_3(R)$ is linearly independent or not	**BTL-3**	Applying
11.	Determine the following sets { $1+2x-x^2$, $4-2x+x^2$, $-1+18x-9x^2$} are bases for P_2(R)	**BTL-3**	Applying
12.	Illustrate that the set $\{1, x, x^2, \ldots. x^n\}$ is a basis for $P_n(F)$	**BTL-3**	Applying
13.	Determine whether the set of vectors { (1,0,0,-1),(0,1,0,-1),(0,0,1,-1),(0,0,0,1)} is a basis for R^4	**BTL-3**	Applying
14.	Determine the basis and dimension of the solution space of the linear homogeneous system x+y-z=0,-2x-y+2z=0,-x+z =0.	**BTL-3**	Applying
15.	Determine x so that the vectors (1,-1,x-1),(2,x,-4),(0,x+2,-8) are linearly dependent over R	**BTL-3**	Applying
16.	Whether the sets { $1-2x-2x^2$, $-2+3x-x^2$, $1-x+6x^2$}are bases for P_2(R)	**BTL-5**	Evaluating
17.	Determine whether the set of vectors X_1=(1,0,-1), X_2=(2,5,1),and X_3=(0,-4,3) is a basis for R^3	**BTL-3**	Applying

18.	The set of solutions to the system of linear equations $x_1-2x_2+x_3=0$, $2x_1-3x_2+x_3=0$ is a subspace of R^3. Find a basis for this subspace.	**BTL-4**	Analyzing
	PART-C		
1	Determine whether the vectors $v_1=(1,-2,3), v_2=(5,6,-1), v_3=(3,2,1)$ form a linearly dependent or linearly independent set in R^3.	**BTL-4**	Analyzing
2	Prove that the upper triangular matrices form a subspace of $M_{mxn}(F)$.	**BTL-4**	Analyzing
3	Decide whether or not the set $S = \{x^3 + 3x - 2, 2x^2 + 5x - 3, -x^2 - 4x + 4\}$ is a basis for $P_2(R)$	**BTL-2**	Understanding
4	Determine whether the set of vectors $X_1=(1,1,2)$, $X_2=(1,0,1)$, and $X_3=(2,1,3)$ span R^3	**BTL-3**	Applying
5	Determine whether the vector $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is in the span of S= $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$	**BTL-4**	Analyzing

UNIT II LINEAR TRANSFORMATION AND DIAGONALIZATION

Linear transformations –Null space-Range- Matrix representation of a linear transformation- Eigen values and Eigen vectors –Diagonalization

PART –A

Q.No	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	Define linear transformation of a function	**BTL-3**	Applying
2.	If $T: V \to W$ be a linear transformation then prove that $T(-v) = -v$ for $v \in V$	**BTL-3**	Applying
3.	If $T: V \to W$ be a linear transformation then prove that $T(x - y) = x - y$ for all $x, y \in V$	**BTL-3**	Applying
4.	Prove that the transformation T is linear if and only if $T(cx + y) = cT(x) + T(y)$	**BTL-3**	Applying
5.	Illustrate that the transformation $T: R^2 \to R^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_2)$ is linear	**BTL-2**	Understanding
6.	Evaluate that the transformation $T: R^3 \to R^2$ defined by $T(x, y, z) = (x, 0,0)$ a linear transformation.	**BTL-5**	Evaluating
7.	Describe explicitly the linear transformation $T: R^2 \to R^2$ such that $T(2,3) = (4,5)$ *and* $T(1,0) = (0,0)$	**BTL-1**	Remembering
8.	Illustrate that the transformation $T: R^2 \to R^3$ defined by $T(x, y) = (x + 1, 2y, x + y)$ is not linear	**BTL-2**	Understanding
9.	Is there a linear transformation $T: R^3 \to R^3$ such that $T(1,0,3) = (1,1)$ and $T(-2,0,-6) = (2,1)$?	**BTL-5**	Evaluating
10.	Define null space .	**BTL-1**	Remembering
11.	Define matrix representation of T relative to the usual basis $\{e_i\}$	**BTL-1**	Remembering
12.	Find the matrix $[T]_e$ whose linear operator *is* $T(x, y) = (5x + y,\ 3x - 2y)$	**BTL-2**	Understanding
13.	Find a basis for the null space of the matrix $A = \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix}$	**BTL-2**	Understanding
14.	Define diagonalizable of a matrix with linear operator T.	**BTL-1**	Remembering
15.	Find the matrix representation of usual basis $\{e_i\}$ to the linear operator $T(x, y, z) = (2y + z, x - 4y, 3x)$	**BTL-2**	Understanding
16.	Define Eigen value and Eigen vector of linear operator T.	**BTL-1**	Remembering
17.	State Cayley-Hamilton Theorem	**BTL-1**	Remembering
18.	Find the Eigenvalue of the matrix $A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$	**BTL-2**	Understanding

19.	Find the matrix A whose minimum polynomial is t^3-5t^2+6t+8.	**BTL-2**	Understanding
20.	State the dimension theorem for matrices.	**BTL-1**	Remembering
21.	Define Range	**BTL-1**	Remembering
22.	Write the properties of an Eigen vector	**BTL-1**	Remembering
23.	Find the Eigenvalue of the matrix $A=\begin{pmatrix}4 & 1\\ 3 & 2\end{pmatrix}$	**BTL-2**	Understanding
24.	Find the sum and product of the eigenvalues of the matrix $\begin{pmatrix}1 & 2 & -2\\ 1 & 0 & 3\\ -2 & -1 & -3\end{pmatrix}$	**BTL-2**	Understanding
25.	If 2 and 3 are eigenvalues of A=$\begin{pmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{pmatrix}$,find the third eigenvalue	**BTL-3**	Applying

PART –B

1. a)	For each of the following linear operators T on a vector space V and ordered basis β,compute $[T]_\beta$,and determine whether β is a basis consisting of eigenvectors of T.V=R^2,,T$\begin{pmatrix}a\\ b\end{pmatrix}$=$\begin{pmatrix}10a-6b\\ 17a-10b\end{pmatrix}$,β={$\begin{pmatrix}1\\ 2\end{pmatrix}$,$\begin{pmatrix}2\\ 3\end{pmatrix}$}	**BTL-3**	Applying
1.b)	Let $T: P_2(R)\to P_3(R)$be defied by $T[f(x)]=2f'(x)+\int_0^x 3f(t)dt$. Prove that T is linear, find the bases for$N(T)$and $R(T)$. Compute the nullity and rank of T. Determine whether T is one-to-one or onto.	**BTL-2**	Understanding
2.b)	Let $T: P_2(R)\to P_3(R)$be defined by $T[f(x)]=xf(x)+f'(x)$is linear. Find the bases for both $N(T)$, $R(T)$, nullity of T, rank of T and determine whether T is one –to-one or onto	**BTL-2**	Understanding
3.	Let $T: R^3\to R^3$be a linear transformation defined by $T(x,y,z)=(x+2y-z, y+z, x+y-2z)$. Evaluate a basis and dimension of null space N(T) and range space R(T) and range space R(T). Also verify dimension theorem	**BTL-5**	Evaluating
4.	Find a linear map $T: R^3\to R^4$whose image is generated by (1,2,0,-4) and (2,0,-1,-3)	**BTL-2**	Understanding
5.	Point out that T is a linear transformation and find bases for both N(T) and R(T). Compute nullity rank T. Verify dimension theorem also verify whether T is one –to-one or onto where $T: P_2(R)\to P_3(R)$ defined by $T[f(x)]=xf(x)+f'(x)$	**BTL-4**	Analyzing
6.	For each of the following linear operators T on a vector space V and ordered basis β,compute $[T]_\beta$,and determine whether β is a basis consisting of eigen vectors of T.V=$P_1(R)$,T(a+bx)=(6a-6b)+(12a-11b)x and β={3+4x,2+3x}	**BTL-3**	Applying
7.	Let $T: R^2\to R^3$ be defined by $T(x,y)=(x+3y,0,2x-4y)$. Compute the matrix of the transformation with respect to the standard bases of R^2 and R^3. Find N(T) and R(T) . Is T one –to-one ? IsT onto. Justify your answer.	**BTL-4**	Analyzing
8.	Let T be the linear operator on R^3 defined by$T(x,y,z)=(2x-7y-4z, 3x+y+4z, 6x-8y+z)$ (i) Find the matrix of T in the basis { $f_1=(1,1,1)$, $f_2=(1,1,0)$ $f_3=(1,0,0)$ and (ii) Verify $[T]_f\,[T]_v=[T(v)]_f$ for any vector $v\epsilon R^3$	**BTL-2**	Understanding
9.	For the given matrix Evaluate all Eigen values and a basis of each Eigenspace . $A=\begin{pmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{pmatrix}$	**BTL-5**	Evaluating
10.	Let α={$\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$,$\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$,$\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$,$\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$} β={1,x,$x^2$} and γ={1} ,Define T:$M_{2x2}(F)\to M_{2x2}(F)$ by T(A)=A^T. Compute $[T]_\alpha$.	**BTL-3**	Applying

11.	Let $T: p_2(R) \to p_2(R)$ be defined as $T(f(x)) = f(x) + (x+1)f'(x)$ FInd eigen values and corresponding eigen vectors of T with respect to standard basis of $p_2(R)$.	**BTL-5**	Evaluating
12.	Let V be the space of 2X 2 matrices over R and let M=$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Let T be linear operator defined by T(A)=MA .Find the trace of T.	**BTL-2**	Understanding
13.	Let V and W be vector spaces over F,and suppose that$\{v_1, v_2 \dots \dots \dots . v_n\}$ is a basis for V, For $w_1, w_2 \dots \dots \dots . w_n$ in W Prove that there exists exactly one linear transformation $T: V \to W$ such that T$(v_i) = w_i$ for i=1,2,…n	**BTL-3**	Applying
14.	Consider the basis S={v_1,v_2,v_3} for R^3 where v_1=(1,1,1) , v_2=(1,1,0) and v_3=(1,0,0). Let T:$R^3 \to R^2$ be the linear transformation such that T(v_1)=(1,0), T(v_2)=(2,-1) and T(v_3)=(4,3) . Find the formula for T(x_1,x_2,x_3), then use this formula to compute T(2,-3,5)	**BTL-4**	Analyzing
15.	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{pmatrix}$	**BTL-3**	Applying
16.	Diagonalise the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ using similarity transformation.	**BTL-4**	Analyzing
17	For each of the following matrices A∈ M_{nxn}(R) test A for diagonalizability and if A is diagonalizable find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ$ =D,A= $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$.	**BTL-5**	Evaluating
18	Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$	**BTL-3**	Applying
	PART-C		
1	Let T be a linear operator $T(a,b,c) = (-4a + 3b - 6c, 6a - 7b + 12c, 6a - 6b + 11c)$, β be the ordered basis then find $[T]_\beta$ which is a diagonal matrix	**BTL-2**	Understanding
2	Let $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ Point out all eigen values of A and corresponding Eigen vectors find an invertible matrix P such that $P^{-1}AP$ is diagonal.	**BTL-3**	Applying
3	For each of the following matrices A∈ M_{nxn}(R) test A for diagonalizability and if A is diagonalizable find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ$ =D,A= $\begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$	**BTL-5**	Evaluating
4	Let $T: R^3 \to R^2$ be defined by $T(x,y,z) = (2x - y, 3z)$ verify whether T is linear or not. Find N(T) and R(T) and hence verify the dimension theorem	**BTL-3**	Applying
5	Diagonalise the matrix $A = \begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$ using similarity transformation	**BTL-4**	Analyzing

UNIT III –INNER PRODUCT SPACE

Inner product and norms - Gram Schmidt Orthonormalization process - Orthogonal Complement - Least square approximation.

PART -A

Q. No.	Questions	Bloom's Taxonomy Level	Domain
1.	Define inner Product Space and give its axioms.	BTL-1	Remembering
2.	Define orthogonal	BTL-1	Remembering
3.	Find the norm of $v = (3,4) \in R^2$ with respect to the usual product.	BTL-2	Understanding
4.	In $c([0,1])$let $f(t) = t, g(t) = e^t$ Evaluate $< f, g >$.	BTL-5	Evaluating
5.	If x, y and z are vector of inner product space such that $< x, y > = < x, z >$then prove that $y = z$.	BTL-3	Applying
6.	Normalize $u = (2,1,-1)$ in Euclidean space R^2.	BTL-2	Understanding
7.	Prove that the norm in a inner product space satisfies $\|v\| \geq 0$ and $\|v\| = 0$ if and only if v = 0.	BTL-3	Applying
8.	Find the norm of $v = (1,2) \in R^2$with respect to the inner product $< u, v > = x_1y_1 - 2x_1y_2 - 2x_2y_1$.	BTL-2	Understanding
9.	Define unit vector	BTL-1	Remembering
10.	Let S={(1,0, i)(1,2,1)} in c^3 Pointout $S^{\perp}$	BTL-4	Analyzing
11.	Let W= span ({i,0,1}) in c^3 find the orthonormal bases of w and $w^{\perp}$	BTL-2	Understanding
12.	What is an adjoint of linear operator.	BTL-3	Applying
13.	Let T be a linear operator on v,β is an orthonormal basis then prove that $[T^*]_\beta$=$[T]_\beta$	BTL-3	Applying
14.	Let S and T be linear operators on V then prove that $(S + T)^*$=$S^* + T^*$	BTL-3	Applying
15.	Let V=R^2,T(a,b)=(2a+b,a-3b) x=(3,5) find T^*at the given vector in V, when T is a Linear operator.	BTL-2	Understanding
16.	Let V be avector space of polynomials with inner product defined by $< f(x), g(x) > = \int_0^1 f(x)g(x)dx$ .If $f(x) = x^2 + x - 4$, g(x)=x-1, then find $< f, g >$	BTL-3	Applying
17.	Let $g: v \to f$ be the linear transformation ,find a vector y such that g(x)=$< x, y >$ for all x∈v such that V=R^3 g(a_1, a_2, a_3)= $a_1 - 2a_2 + 4a_3$	BTL-2	Understanding
18.	Show that I^*=I for every u,v∈ v	BTL-3	Applying
19.	Define orthonormal.	BTL-1	Remembering
20.	What is an adjoint of linear operator.	BTL-3	Applying
21.	Define norm or Length of a function	BTL-2	Remembering
22.	Find the norm of $v = (3,-4,0) \in R^3(R)$ with the standard inner product	BTL-2	Understanding
23.	Find the value of x so that (x,-3,-4) and (x,-x,1) are orthogonal in R^3 with standard inner product	BTL-2	Understanding
24.	Find the distance between the vectors (7,1),(3,2) in R^2 with standard inner product	BTL-2	Understanding
25.	Find the norm of $v = (-2,5) \in R^2$ with respect to the usual product.	BTL-2	Understanding

PART-B

1.	Let V be an inner product space. Prove that (a) $\|x \pm y\|^2 = \|x\|^2 \pm 2R < x,y > + \|y\|^2$ for all x, y ∈ V, where $R < x,y >$ denotes the real part of the complex number $< x,y >$. (b) $\|\|x\| - \|y\|\|^2 \le \|x - y\|$ for all x, y ∈ V.	**BTL-3**	Applying
2.	Let V be an inner product space, for x, y, z ∈ V and C∈ F, checkwhether the following are true. (i) $< x,y+z > = < x,y > + < x,z >$ (ii) $< x,cy > = \bar{c} < x,y >$ (iii) $< x,0 > = < 0,x > = 0$ (iv) $< x,x > = 0$ if and only if x=0. (v) $< x,y > = < x,z >$ for all x∈V then y=z	**BTL-4**	Analyzing
3.	In $C([0,1])$, let f(t) = t and g(t) = e^t. Compute $< f,g >$, $\|f\|$, $\|g\|$ and $\|f + g\|$. Then verify both the Cauchy-Schwarz inequality and the triangle inequality.	**BTL-4**	Analyzing
4.	Let P_2 be a family of polynomials of degree 2 atmost. Define an inner product on P_2 As $< f(x), g(x) > = \int_0^1 f(x)g(x)dx$. Let {1,x, x^2} be a basis of the inner product space P_2 . Find out an orthonormal basis from this basis.	**BTL-5**	Evaluating
5.	Evaluate by the Gram Schmidt Process to the given subset $S = \{(1,-2,-1,3),(3,6,3,-1),(1,4,2,8)\}$ and $x = (-1,2,1,1)$ of the inner product space $V = R^4$ to obtain an orthogonal basis for span(S). Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S), and compute the Fourier coefficients of the given vector relative to β.	**BTL-5**	Evaluating
6.	Apply the Gram-Schmidt process to the vectors u_1=(1,0,1) , u_2=(1,0,-1), u_3=(0,3,4) to obtain an orthonormal basis for R^3(R) with standard inner product.	**BTL-4**	Analyzing
7.	Evaluate byapplying the Gram Schmidt Process to the given subset with the inner product $< f(x), g(x) > = \int_0^1 f(t)g(t)dt$, $S = \{1, x, x^2\}$, and $h(x) = 1 + x$ .of the inner product space $V = P_2(R)$ to obtain an orthogonal basis for span(S). Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S), and compute the Fourier coefficients of the given vector relative to β.	**BTL-2**	Understanding
8.	Let V=C({-1,1}) with the inner product $< f,g > = \int_{-1}^1 f(t)g(t)dt$, and let W be the subspace $P_2(R)$, viewed as a space of functions. Use the orthonormal basis obtained to compute the “best”(closet) second degree polynomial approximation of the function h(t)=e^t on the interval [-1,1]	**BTL-2**	Understanding
9.	Compute < x,y > for x=(1-i,2+3i) and y=(2+i,3-2i)	**BTL-3**	Applying
10.	For each of the sets of data that follows, use the least squares approximation to find the best fits with both (i) a linear function and Compute the error E in both cases. {(-3, 9), (-2, 6), (0, 2),(1, 1)}	**BTL-2**	Understanding
11.	Consider the system $x + 2y - z = 1$; $2x + 3y + z = 2$; $4x + 7y - z = 4$; find the minimal solution	**BTL-2**	Understanding

12.	For each of the following inner product spaces V and linear operators Ton V, evaluate T* at the given vector in V. $V = R^2, T(a,b) + (2a + b, a - 3b), x = (3,5)$	**BTL-5**	Evaluating
13.	Let V be an inner product space, and let T and U be linear operators on V. then verify(a) (T+U)*=T*+U*; (b) (cT)*=$\bar{c}\,T$* for any $c \in F$; (c) (TU)* = U*T*;(d) T** = T; I* = I	**BTL-4**	Analyzing
14.	For each of the sets of data that follows, use the least squares approximation to find the best fits with linear function and Compute the error E in both cases.{(-2, 4), (-1, 3), (0, 1), (1, -1), (2, -3)}	**BTL-4**	Analyzing
15.	Find the minimal solution to the following system of linear equation x+2y-z=12	**BTL-3**	Applying
16.	Let P_2 have the inner product $< p,q > = \int_{-1}^{1} p(x)q(x)dx$. Find the angle between p and q, where p=x and q=x^2 with respect to the inner product on P_2.	**BTL-4**	Analyzing
17.	Let x=(1,1,0),y=(1,-1,1),z=(-1,1,2) be the vectors in F^3 obtain the orthonormal set	**BTL-4**	Analyzing
18.	Using least square approximation determine the best linear fit for the data {(1,2),(2,3),(3,5),(4,7)}	**BTL-2**	Understanding
	Part-C		
1.	Let x = (2, 1+i, i) and y = (2-i, 2, 1+2i) be vectors in C^3. Compute $< x,y >$.$\|x\|$, $\|y\|$ and $\|x + y\|$. Then verify both the Cauchy Schwarz inequality and the triangle inequality.	**BTL-1**	Remembering
2.	Find the minimal solution of to the following system of linear equations x+2y+z = 4 , x-y+2z= -11 ,x+5y=19	**BTL-3**	Applying
3.	Let A and B be $n \, X \, n$ matrices. Then prove that (a) $(A + B)^* = A^* + B^*$ (b) $(cA)^* = \bar{c}A^*$ for all $c \in F$ (c) (AB)* = B*A*(d) A** = A (e) I* = I	**BTL-3**	Applying
4.	For each of the following inner product spaces V and linear operators T on V, evaluate T* at the given vector in V. $V = P_1$(R) with $< f,g > = \int_{-1}^{1} f(t)g(t)dt . T(f) = f' + 3f, f(t) = 4 - 2t$	**BTL-5**	Evaluating
5.	Find Least square solution to the inconsistent system x+5y=3,2x-2y=2,x+y=5.Also find the least square error.	**BTL-3**	Applying

UNIT IV-NUMERICAL SOLUTION OF LINEAR SYSTEM OF EQUATIONS

Solution of linear system of equations – Direct method: Gauss elimination method – Pivoting – Gauss-Jordan method - LU decomposition method – Cholesky decomposition method - Iterative methods: Gauss-Jacobi and Gauss-Seidel.

PART- A

Q. No	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	State the order and condition for Convergence of Iteration method.	BTL -2	Understanding
2.	State the principle used in Gauss Jordon method.	BTL -2	Understanding
3.	Solve the following equations by Gauss Jordan method x +y=2,2x+3y=5	BTL -3	Applying

4.	Solve by Gauss Elimination method 2x + 3y = 5 and 3x-y=2	BTL -2	Understanding
5.	Distinguish the advantages of iterative methods over direct method of solving a system of linear algebraic equations.	BTL -4	Analyzing
6.	Solve by Gauss Elimination method 2x + y = 3 and 7x-3y=4.	BTL -3	Applying
7.	Compare Gauss Elimination, Gauss Jordan method.	BTL -4	Analyzing
8.	State the condition for the convergence of Gauss Seidel iteration method for solving a system of linear equation.	BTL -2	Understanding
9.	Compare Gauss Seidel method, Gauss Jacobi method.	BTL -4	Analyzing
10.	Which of the iterative methods is used for solving linear system of equations it converges fast? Why?	BTL -1	Remembering
11.	Compare Gauss Seidel method, Gauss Elimination method.	BTL -4	Analyzing
12.	Write down the condition for the convergence of Gauss Jacobi iteration method for solving a system of linear equation.	BTL -2	Understanding
13.	Solve by Gauss Elimination method 5x -2y = 1 and 4x+28y=23.	BTL -4	Analyzing
14.	Write a sufficient condition for Gauss Seidel method will converge	BTL -3	Applying
15.	Write the necessary conditions for Cholesky decomposition of a matrix.	**BTL -1**	Remembering
16.	Find the Cholesky decomposition **of** $\begin{pmatrix} 4 & 2 \\ 2 & 10 \end{pmatrix}$	**BTL -2**	Understanding
17.	Give two indirect methods to solve a system of linear equations.	**BTL -2**	Understanding
18.	Define LU decomposition method	**BTL -1**	Remembering
19.	Solve by Gauss Elimination method 4x -3y = 11 and 3x+2y=4.	BTL -2	Understanding
20.	Solve by Gauss jordan method 2x + y = 3 and 7x-3y=4.	BTL -2	Understanding
21.	State the principle used in Gauss Jordan method.	BTL -1	Remembering
22.	Solve by Gauss Elimination method x + y = 2 and 2x+3y=5.	BTL -4	Analyzing
23.	State a sufficient condition for Gauss Seidel method will converge.	BTL -1	Remembering
24.	Solve by Gauss Elimination method x -2y = 0 and 2x+y=5.	BTL -4	Analyzing
25.	What type of eigen value can be obtained using power method?	**BTL-3**	Applying
	Part-B		
1	Solve by Gauss Elimination method 3x + y - z = 3 ; 2x – 8y + z = -5 ; x -2y + 9z = 8	BTL -3	Applying

2	Solve by Gauss Jordan method 10x + y+ z = 12 ; x+10y + z = 12 ; x +y +10z = 12	BTL -3	Applying
3	Solve by Gauss Jacobi method 14x -5y =5.5 ; 2x +7y =19.3.	BTL -3	Applying
4.	Apply Gauss Seidel method to solve system of equations 10x – 5y -2z = 3 ; 4x -10y +3z = -3 ;x + 6 y +10z = -3	BTL -3	Applying
5.	Solve by Gauss Elimination method 2x + 4y+z = 3 ; 3x +2y -2z = -2 ;x -y + z =6	BTL-2	Understanding
6.	Solve by Gauss Jacobi method 8x -3y +2z =20 ; 4x +11y -z = 33 ; 6x +3y + 12z = 35.	BTL-2	Remembering
7.	Find the Cholesky decomposition of the matrix $\begin{bmatrix} 4 & 2i & 2 \\ -2i & 10 & 1-i \\ 2 & 1+i & 9 \end{bmatrix}$	BTL -2	Understanding
8.	Solve by Gauss Jordan method 10 x + y + z =12 ; 2x + 10y + z = 13; x + y + 5z =7.	BTL -3	Applying
9.	Solve by Gauss Elimination method x + 5y +z =14 ; 2x + y + 3z = 13 ; 3x+y+4z =17	BTL -3	Applying
10.	Apply Gauss seidel method to solve system of equations x – 2y +5z = 12 ; 5x + 2y - z = 6 ; 2x + 6 y – 3z = 5 (Do up to 6 iterations)	BTL -3	Applying
11.	Solve by Gauss Elimination method 6x -y+z = 13 ; x + y +z = 9 ;10x +y -z =19	BTL -3	Applying
12.	By Gauss seidel method to solve system of equations x + y + 54z = 110; 27x + 6y –z = 85; 6x + 15y – 2z =72.	BTL -4	Analyzing
13.	Solve by Gauss Elimination method 3x + 4y+5z = 18 ; 2x – y + 8z = 13 ;5x -2y + 7z =20	BTL -3	Applying
14.	Solve by using Gauss-Seidal method $8x - 3y + 2z = 20,\ 4x + 11y - z = 33,\ 6x + 3y + 12z = 35$.	BTL -5	Evaluating
15.	Find the Cholesky decomposition of the matrix $\begin{pmatrix} 4 & 2i & -i \\ -2i & 10 & 1 \\ i & 1 & 9 \end{pmatrix}$	BTL -2	Understanding
16.	Apply Gauss Seidel method to solve system of equations 8x +y +z = 8 ; 2x –0y +3z = -3 ;x + 6 y +10z = -3	BTL -3	Applying
17.	Solve by Gauss Jacobi method 10x -2y -2z =6 ; -x -10y -2z = 7 ; x -y + 10z = 8.	BTL -5	Evaluating

18.	Solve by Gauss Jordan method 4x - y - z =-7 ; x - 5y + z =-10; x + 2y + 6z =9.	BTL -5	Evaluating
	Part-C		
1	Solve by Gauss Jordan method 10 x + y + z =12 ; 2x + 10y + z = 13; x + y + 5z =7.	BTL -2	Understanding
2	Solve by Gauss Elimination method 3x – y+ 2z = 12 ; x+2y +3z =11 ; 2x -2y -z =2.	BTL -2	Understanding
3	Solve the system of equations using Cholesky decomposition $4x_1 - x_2 - x_3 = 3$; $x_1 + 4x_2 - x_3 = -0.5$; $-x_1 - 3x_2 + 5x_3 = 0$.	BTL -5	Evaluating
4	Solve the linear system 6x +18y +3z = 3,2x +12y + = 19 ,4x +15y +3z = 0 by LU decomposition method.	BTL -2	Understanding
5.	Solve by using Gauss-Seidal method $28x + 4y - z = 32,\ x + 3y + 10z = 24,\ 2x + 17y + z = 35$.	BTL -5	Evaluating

UNIT V -NUMERICAL SOLUTION OF EIGEN VALUE PROBLEMS AND GENERALISED INVERSES

Eigen value Problems: Power method – Jacobi 's rotation method – Conjugate gradient method – QR decomposition - Singular value decomposition method- Singular value decomposition

Q. No	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	Define Eigen value	BTL -2	Understanding
2.	Find the sum and product of all Eigen values of $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$	BTL -5	Evaluating
3.	Find the sum and product of all Eigen values of $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{bmatrix}$	BTL -5	Evaluating
4.	Define singular matrix with an example.	BTL -2	Understanding
5.	Find the all Eigen values of $\begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$	BTL -5	Evaluating
6.	If the sum of two eigenvalues and trace of a 3x3 matrix A are equal find the value of |A|	BTL -2	Understanding
7.	Define Jacobi rotation method	BTL -2	Understanding
8.	Let $\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} = \boldsymbol{A_1}$ .Compute A_2 using QR algorithm.	BTL -5	Evaluating
9.	Find eigen values of the matrix A = $\begin{bmatrix} 3 & 5 \\ -2 & -4 \end{bmatrix}$.	BTL -3	Applying

10.	Define the generalized Eigen vector, chain of rank m, for a square matrix.	BTL -2	Understanding
11.	Find the eigen values of $\begin{bmatrix} 15 & 1 \\ 0 & 1 \end{bmatrix}$	BTL -5	Evaluating
12.	Find the determinant value of A if A= $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$	BTL4	Analyzing
13.	Give the nature of quadratic form without reducing into canonical form $x_1^2 - 2x_1x_2 + x_2^2 + x_3^2$	BTL -4	Analyzing
14.	Find the eigen values of $\begin{bmatrix} -1 & 1 \\ 9 & 1 \end{bmatrix}$	BTL-4	Analyzing
15.	Write short note on Singular value decomposition of complex matrix A.	BTL-1	Remembering
16.	State Singular value decomposition theorem	BTL-1	Remembering
17.	If A is a nonsingular matrix, then what is A^+?	BTL -2	Understanding
18.	Define conjucate gradient method	BTL-1	Remembering
19.	Write the numerical example of a conjucate gradient method	BTL-1	Remembering
20.	Find the generalized inverse of $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	BTL-3	Applying
21.	Explain Power method to find the dominant Eigen value of a square matrix A	BTL -2	Understanding
22.	How will you find the smallest Eigen value of a matrix A.	BTL-1	Remembering
23.	Find the dominant Eigen value of $A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ by power method upto 1 decimal place accuracy. Start with $X^{(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	BTL-4	Analyzing
24.	Define orthogonal Vectors.	BTL-1	Remembering
25.	Find the dominant eigen value of A= $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method.	BTL-3	Applying
	Part-B		
1	Find the dominant eigen value and vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ using Power method.	BTL-4	Analyzing
2	Find the largest Eigen value and Eigen vector of $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ using power method.	BTL-4	Analyzing
3	Find the QR factorization of $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 0 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}$	BTL-4	Analyzing
4	Evaluate the singular value decomposition of $\begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix}$	BTL-5	Evaluating

5	Evaluate the dominant Eigen value and vector of $A = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 4 & 10 \\ 1 & 3 & -1 \end{pmatrix}$ using Power method.	BTL-5	Evaluating
6	Find the QR decomposition of $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$	BTL-4	Analyzing
7	Consider the decomposition of $A = \begin{bmatrix} 12 & -51 & 4 \\ 6 & 167 & -68 \\ -4 & 24 & -41 \end{bmatrix}$	BTL-4	Analyzing
8	Find the singular value decomposition of $A=\begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 4 & -2 \end{bmatrix}$	BTL-4	Analyzing
9	Using Power method , Identify all the eigen values of $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$	BTL-3	Applying
10	Get the singular value decomposition of $\begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}$	BTL-3	Applying
11	Find the dominant Eigen value and corresponding eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ using Power method.	BTL-3	Applying
12	Find the conjucate gradient method of 5x+y=2 and x+2y = 2	BTL-3	Applying
13	Determine the largest eigenvalue and the corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$	BTL-3	Applying
14	Find the dominant Eigen value of $A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$ by power method up to 1 decimal place accuracy. Start with $X^{(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	BTL -3	Applying
15	Find the QR decomposition of $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	BTL -3	Applying
16	Find the singular value decomposition of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \end{pmatrix}$	BTL -3	Applying
17	Find the dominant Eigen value and the corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$	BTL -3	Applying

18	Find the singular value decomposition of A $=\begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{bmatrix}$	BTL -3	Applying
	Part-C		
1	Evaluate the dominant Eigen value and vector of $A=\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ using Power method.	BTL-5	Evaluating
2	Obtain A^{+} of $A=\begin{bmatrix} 2 & -1 \\ -1 & 1 \\ 4 & -3 \end{bmatrix}$ the generalized inverse.	BTL-3	Applying
3	Determine the largest eigenvalue and the corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$ by using Power method.	BTL-3	Applying
4	Find the singular value decomposition of $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$.	BTL-3	Applying
5	Using Jacobi's method ,Find all the eigen values and eigenvectors for the given matrix $A=\begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$	BTL-3	Applying