SRM VALLIAMMAI ENGINEERING COLLEGE
(An Autonomous Institution)

S.R.M. Nagar, Kattankulathur - 603203

DEPARTMENT OF MATHEMATICS

QUESTION BANK

I SEMESTER
(Common to all Branches)

MA3122 - MATRICES AND CALCULUS

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DEPARTMENT OF MATHEMATICS

SUBJECT : MA3122 - MATRICES AND CALCULUS

SEM / YEAR: I / I year B.E./ B.Tech.(Common to all Branches)

UNIT I - MATRICES
Eigen values and Eigen vectors of a real matrix - Characteristic equation - Properties of Eigen values and Eigen vectors - Statement and Applications of Cayley-Hamilton Theorem - Reduction of a quadratic form into canonical form by orthogonal transformation.

Q.No.	Question	BT Level	Competence	Course Outcome
PART – A				
1.	Find the sum and product of the eigen values of $A = (2 \ -2 \ 2 \ -2 \ -1 \ -1 \ 2 \ -1 \ -1))$	BTL-1	Remembering	CO 1
2.	Find the sum of the eigen values of 2A, if $A = (8 \ -6 \ 2 \ -6 \ 7 \ -4 \ 2 \ -4 \ 3 \ )$	BTL-1	Remembering	CO 1
3.	Write any 2 applications of Cayley Hamilton theorem	BTL-2	Understanding	CO 1
4.	Find the constants a and b such that the matrix $(a \ 4 \ 1 \ b \ )$ has 3,-2 be the eigen values of A	BTL-1	Remembering	CO 1
5.	Find the quadratic form corresponding to the matrix A= $(2 \ 2 \ 0 \ 2 \ 5 \ 0 \ 0 \ 0 \ 3 \ )$	BTL-3	Applying	CO 1
6.	If $A = (1 \ 0 \ 4 \ 5 \ )$find A^3 using Cayley Hamilton theorem	BTL-3	Applying	CO 1
7.	Find the eigen values of A^{-1} if $A = (4 \ 1 \ 3 \ 2 \ )$	BTL-4	Analyzing	CO 1
8.	Define Index, Signature and Rank.	BTL-1	Remembering	CO 1
9.	Prove that the eigen values of A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3} \ldots\ldots\ldots\ldots \frac{1}{\lambda_n}$	BTL-2	Understanding	CO 1
10.	If the sum of 2 eigen values and the trace of a 3×3 matrix are equal, find the value of $\|A\|$	BTL-2	Understanding	CO 1
11.	A square matrix and its transpose have the same eigenvalues.	BTL-2	Understanding	CO 1

12.	Applications of Cayley-Hamilton theorem.	BTL-3	Applying	CO 1
13.	If the eigen values of the matrix A of order 3X3 are 2,3 and 1, then find the determinant of A	BTL-1	Remembering	CO 1
14.	Find the characteristic equation of $A = (1\ -2\ -5\ 4)$	BTL-1	Remembering	CO 1
15.	Find the matrix corresponding to the quadratic form $x^2+y^2+z^2$	BTL-1	Remembering	CO 1
16.	Prove that sum of eigen values of a matrix is equal to its trace.	BTL-3	Applying	CO 1
17.	For the given matrix A of order 3,|A|=32, and two eigen values are 8 & 2. Find the sum of the Eigenvalues.	BTL-2	Understanding	CO 1
18.	Two eigen values of $A = (2\ 2\ 1\ 1\ 3\ 1\ 1\ 2\ 2)$ are equal to unity each. Find the 3rd eigen value.	BTL-1	Remembering	CO 1
19.	If the characteristic equation of a matrix is $\lambda^2 - 3\lambda - 10 = 0$, then find the the eigen values of the matrix $10A^{-1} - 2I$.	BTL-4	Analyzing	CO 1
20.	Find the Rank,Index,Signature and Nature of given matrix $A = (3\ -1\ 4\ 0\ 5\ 2\ 0\ 0\ -1)$	BTL-2	Understanding	CO 1
21.	Find the sum and product of the eigen values of $A = (3\ 1\ 4\ 0\ 2\ 6\ 0\ 0\ 5)$	BTL-1	Remembering	CO 1
22.	The product of the 2 eigen values of $A = (6\ -2\ 2\ -3\ 3\ -1\ 2\ -1\ 3)$ is 14. Find the 3rd eigen value.	BTL-2	Understanding	CO 1
23.	Use Cayley Hamilton theorem to find A^{-1} if $A = (2\ 1\ 1\ -5)$	BTL-1	Remembering	CO 1
24.	State Cayley-Hamilton theorem.	BTL-2	Understanding	CO 1
25.	Find the matrix corresponding to the quadratic form 2xy-2yz+2xz.	BTL-4	Analyzing	CO 1
PART – B				
1.	Determine the nature of the quadratic form $2xy + 2yz - 2xz$ by reducing it into canonical form by orthogonal transformation	BTL-4	Analyzing	CO 1
2.(a)	Find the eigen values and eigen vectors of the matrix $A = (5\ 0\ 1\ 0\ -2\ 0\ 1\ 0\ 5)$	BTL-2	Understanding	CO 1
2.(b)	Find the Characteristic equation of the matrix $A = (2\ -1\ 2\ -1\ 2\ -1\ 1\ -1\ 2)$ and hence find A^4.	BTL-4	Analyzing	CO 1
3.	Use Cayley-Hamilton theorem to find the value of the matrix given by $A^6-5A^5+8A^4-2A^3-9A^2-35A+6I$ if the matrix $A = (1\ 2\ 1\ 0\ 1\ -1\ 3\ -1\ 1)$	BTL-4	Analyzing	CO 1

4(a)	Find the eigen values and eigen vectors of the matrix $A = (1\ 0\ -1\ 1\ 2\ 1\ 2\ 2\ 3)$	BTL-2	Understanding	CO 1
4(b)	Using Cayley-Hamilton theorem evaluate the matrix $A^4-4A^3-5A^2+A+2I$ given the matrix $A=(3\ 2\ 4\ 5)$	BTL-3	Applying	CO 1
5.	Diagonalize the matrix $A= (7\ -2\ 0\ -2\ 6\ -2\ 0\ -2\ 5)$ using an orthogonal transformation.	BTL-4	Analyzing	CO 1
6.(a)	Find the eigen values and eigen vectors of the matrix $A = (1\ 1\ -2\ -1\ 2\ 1\ 0\ 1\ -1)$	BTL-2	Understanding	CO 1
6.(b)	Verify Cayley-Hamilton theorem for the matrix $A = (1\ -2\ 3\ 2\ 4\ -2\ -1\ 1\ 2)$	BTL -4	Analyzing	CO 1
7.	Verify Cayley-Hamilton theorem and hence find A^{-1} of $A = (1\ 0\ 3\ 2\ 1\ -1\ 1\ -1\ 1)$	BTL-1	Remembering	CO 1
8(a)	Obtain the eigen values and eigen values and eigen vectors of the matrix $A = (3\ -1\ 0\ -1\ 2\ -1\ 0\ -1\ 3)$	BTL-4	Analyzing	CO 1
8(b)	Find the eigen values and eigen vectors of the matrix $A = (0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0)$	BTL-2	Understanding	CO 1
9.	Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal reduction.	BTL-3	Applying	CO 1
10.	Use Cayley-Hamilton theorem to find the value of the matrix given by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ if the matrix $A = (2\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 2)$	BTL -3	Applying	CO 1
11.	Reduce the quadratic form $6{x_1}^2 + 3{x_2}^2 + 3{x_3}^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form by an orthogonal reduction.	BTL-2	Understanding	CO 1
12	Diagonalize the matrix $A= (2\ 0\ 4\ 0\ 6\ 0\ 4\ 0\ 2)$ using an orthogonal transformation.	BTL -4	Analyzing	CO 1
13	Verify Cayley-Hamilton theorem for the matrix $A = (1\ 1\ 3\ 1\ 3\ -3\ -2\ -4\ -4)$ and also find A^{-1}.	BTL-4	Analyzing	CO 1
14.(a)	Find the eigen values and eigen vectors of the matrix $A = (1\ -1\ 4\ 3\ 2\ -1\ 2\ 1\ -1)$	BTL-2	Understanding	CO 1
14.(b)	Using Cayley-Hamilton theorem evaluate the matrix $A^4+A^3-18A^2-39A+2I$ given the matrix $A = (1\ 2\ 3\ 2\ -1\ 4\ 3\ 1\ -1)$	BTL -3	Applying	CO 1
15.	Reduce the quadratic form $2{x_1}^2 + {x_2}^2 + {x_3}^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ into canonical form by an orthogonal reduction.	BTL-1	Remembering	CO 1
16.	Reduce the quadratic form $2{x_1}^2 + 5{x_2}^2 + 3{x_3}^2 + 4x_1x_2$ into canonical form by an orthogonal reduction.	BTL-1	Remembering	CO 1

17.	Determine the nature of the quadratic form $2xy - 2yz + 2xz$ by reducing it into canonical form by orthogonal transformation	BTL -4	Analyzing	CO 1
18.	Diagonalize the matrix $A = (1\ 0\ 0\ 0\ 3\ -1\ 0\ -1\ 3\ )$ using an orthogonal transformation.	BTL -3	Applying	CO 1

UNIT-II DIFFERENTIAL CALCULUS OF ONE VARIABLE

Limit of a function - Continuity - Derivatives - Differentiation rules (sum, product, quotient, chain rules) - Implicit differentiation - Rolle's Theorem and Mean Value theorem -Taylor's series- Maxima and Minima of functions of one variable.

Q.No.	Question	BT Level	Competence	Course Outcome
	PART – A			
1.	Evaluate $\lim_{t\to 1} \frac{t^4-1}{t^3-1}$	BTL -1	Remembering	CO 2
2.	Check whether $\lim_{x\to -3} \frac{3x+9}{\|x+3\|}$ exist	BTL -2	Understanding	CO 2
3.	Find the domain of a function $y = \sqrt{x+4}$	BTL -1	Remembering	CO 2
4.	Find $\lim_{x\to 0} \frac{tanx-x}{x^3}$.	BTL -1	Remembering	CO 2
5.	If the function $f(x) = \{\frac{x^2-16}{x-4} if x \neq 4\ c if x = 4$ is continuous, what is the value of c?	BTL -2	Understanding	CO 2
6.	If $f(x) = xe^x$ find $f'(x)$	BTL -2	Understanding	CO 2
7.	Show that $\lim_{x\to 0} x^2 sin(\frac{1}{x}) = 0$ using Squeeze theorem	BTL -3	Applying	CO 2
8.	Predict the values of a and b so that the function f given by $f(x) = \{1 if x \leq 3\ ax + b if 3 < x < 5\ 7 if x \geq 5$ is continuous at x=3 and x=5.	BTL -3	Applying	CO 2
9.	Compute $\frac{d}{dx}((x)^{sinx})$	BTL -4	Analyzing	CO 2
10.	Evaluate $\frac{d}{dx}((sinx)^{lnx})$	BTL -3	Applying	CO 2
11.	Point out $\frac{dy}{dx}, if y = ln$.	BTL -3	Applying	CO 2
12.	Calculate $\frac{d}{dx}((x)^{\sqrt{x}})$	BTL -2	Understanding	CO 2
13.	Estimate $\frac{d}{dx}(cosh^{-1}x)$	BTL -3	Applying	CO 2
14.	Estimate y' if $x^3 + y^3 = 6xy$	BTL -2	Understanding	CO 2
15.	Estimate $\frac{d}{dx}((sinx)^{cosx})$	BTL -4	Analyzing	CO 2
16.	Evaluate $\frac{d}{dx}(sinhx)$	BTL -4	Analyzing	CO 2
17.	Using Rolle's theorem find the value of c for the function	BTL -4	Analyzing	CO 2

	$f(x) = (x-a)(b-x), a \le x \le b, a \ne b$			
18.	Verify Lagrange's law for the function $f(x) = x^3, [-2,2]$	BTL -2	Understanding	CO 2
19.	Using Rolle's theorem find the value of c for the function $f(x) = \sqrt{1-x^2}, -1 \le x \le 1$	BTL -4	Analyzing	CO 2
20.	Verify Lagrange's law for the function $f(x) = \frac{1}{x}, [1,2]$	BTL -1	Remembering	CO 2
21.	Find the critical numbers of the function$f(x) = 2x^3 - 3x^2 - 36x$	BTL -4	Analyzing	CO 2
22.	Find the critical points of $y = 5x^2 - 6x$	BTL -3	Applying	CO 2
23.	Find the Taylor's series expansion of the function $f(x) = sinx$about the point $x = \frac{\pi}{2}$	BTL -4	Analyzing	CO 2
24.	Find the critical points of $y = 5x^3 - 6x$	BTL -2	Understanding	CO 2
25.	Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so where?	BTL -4	Analyzing	CO 2
	PART – B			
1.(a)	For what value of the constant "c" is the function "f" continuous on$(-\infty, \infty), f(x) = \{cx^2 + 2x; x < 2\ x^3 - cx; x \ge 2$	BTL -4	Analyzing	CO 2
1.(b)	Obtain $y'' if x^4 + y^4 = 16$	BTL -4	Analyzing	CO 2
2(a)	Point out the domain where the function f is continuous Also find the number at which the function f is discontinuous when $f(x) = \{1 + x^2 if x \le 0\ 2 - x if 0 < x \le 2\ (x-2)^2 if x > 2$	BTL -4	Analyzing	CO 2
2(b)	Find$\frac{dy}{dx} if y = x^2 e^{2x}$.	BTL -3	Applying	CO 2
3.(a)	Find the values of a and b that make f continuous on $(-\infty, \infty)$ $f(x) = \{\frac{x^3-8}{x-2} if x < 2\ ax^2 - bx + 3 if 2 \le x < 3\ 2x - a + b if x \ge 3$	BTL -3	Applying	CO 2
3.(b)	Find y'if $x = a(cos\theta + logtan\frac{\theta}{2}), y = asin\theta$.	BTL -3	Applying	CO 2
4(a)	Find $y'$for $cos(xy) = 1 + siny$.	BTL -3	Applying	CO 2
4(b)	Find the derivative of $f(x) = cos^{-1}(\frac{b+acosx}{a+bcosx})$	BTL -3	Applying	CO 2
5.(a)	Find $\frac{dy}{dx}$ for the following functions $e^x + e^y = e^{x+y}$.	BTL -3	Applying	CO 2

5.(b)	Verify Rolle's theorem for the following $f(x) = 2x^3 - 5x^2 - 4x + 3, x \in [\frac{1}{2}, 3]$.	BTL -3	Applying	CO 2
6(a)	For what value of the constant b, is the function f continuous on $(-\infty, \infty)$ if f(x) = $\{bx^2 + 2x if x < 2\ x^3 - bx if x \geq 2$	BTL -3	Applying	CO 2
6(b)	Find $\frac{dy}{dx}$,when $y = tan^{-1}(\frac{acosx+bsinx}{bcosx-asinx})$	BTL -4	Analyzing	CO 2
7.	Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$is increasing and where it is decreasing. Also find the local maximum, local minimum, concavity and the inflection points of f(x).	BTL -4	Analyzing	CO 2
8.(a)	Verify mean value theorem for the following $f(x) = x^3 - 5x^2 - 3x, x \in [1,3]$.	BTL -3	Applying	CO 2
8.(b)	Find the Taylor's series expansion of $f(x) = \frac{1}{1+x}$ about x=0.	BTL -3	Applying	CO 2
9.	For the function f(x) =$x^4 - 4x^3$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity and the inflection points.	BTL -1	Remembering	CO 2
10.	If $f(x) = 2x^3 + 3x^2 - 36x$, find the intervals on which it is increasing or decreasing, local maximum , local minimum values , concavity and the inflection points of $f(x)$.	BTL -4	Analyzing	CO 2
11.	Find the tangent line to the equation$x^3 + y^3 = 6xy$ at the point (3,3) and at what point the tangent line is horizontal in the first quadrant.	BTL -3	Applying	CO 2
12(a)	Find the absolute maximum and minimum of f(x)= $x - 2tan^{-1}x \in [0,4]$.	BTL -4	Analyzing	CO 2
12(b)	Verify Lagrange's law for the following $f(x) = 2x^3 + x^2 - x - 1, x \in [0,2]$.	BTL -3	Applying	CO 2
13.	Use second derivative test to examine the relative maxima for $f(x) = x(12 - 2x)^2$	BTL -4	Analyzing	CO 2
14.(a)	Examine the local extreme of $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$. Also discuss the concavity and find the inflection points	BTL -4	Analyzing	CO 2
14.(b)	Verify Rolle's theorem for the following $f(x) = x(x-1)(x-2), x \in [0,2]$.	BTL -3	Applying	CO 2
15.	Discuss the curve $y = x^4 - 4x^3$find the intervals on which it is increasing or decreasing, local maximum , local minimum values , concavity and the inflection points of $f(x)$.	BTL -4	Analyzing	CO 2
16.(a)	Examine the local extreme of $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$. Also discuss the concavity and find the inflection points	BTL -4	Analyzing	CO 2

16.(b)	Find the Taylor's series expansion of $f(x) = tan^{-1}x$ about $x = 0$.	BTL -3	Applying	CO 2
17.	Find the local maximum and minimum values of $f(x) = \sqrt{x} - \sqrt[4]{x}$ using both first and second derivatives tests.	BTL -4	Analyzing	CO 2
18.(a)	Of all the right circular cones of given slant length l, find the dimensions and volume of the cone of maximum volume.	BTL -3	Applying	CO 2
18.(b)	Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$	BTL -3	Applying	CO 2

UNIT-III: DIFFERENTIAL CALCULUS OF SEVERAL VARIABLES

Partial derivatives - Total derivatives - Jacobians and properties - Taylor's series for functions of two variables - Maxima and Minima of functions of two variables -Lagrange's method of undetermined multipliers.

Q.No.	Question	BT Level	Competence	Course Outcome
	PART – A			
1.	Find $\frac{\partial r}{\partial x}, if\ x = r\cos\cos\theta$ & $y = r\sin\sin\theta$.	BTL -1	Remembering	CO 3
2.	Find the value of $\frac{du}{dt}$, given u= log (x + y + z) , x= e^t, $y = sint, z = cost$	BTL -2	Understanding	CO 3
3.	If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}, then\ find\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.	BTL -1	Remembering	CO 3
4.	Find $\frac{dy}{dx}$ if $x^3 + y^3 = 3ax^2y$	BTL -1	Remembering	CO 3
5.	Find $\frac{du}{dt} ifu = \sin\sin\left(\frac{x}{y}\right), wherex = e^t, y = t^2$	BTL -2	Understanding	CO 3
6.	Find $\frac{du}{dt} ifu = \frac{x}{y}, wherex = e^t, y = \log\log t$.	BTL -2	Understanding	CO 3
7.	Find the value of $\frac{du}{dt}$, given $u = x^2 + y^2, x = at^2, y = 2at$.	BTL -3	Applying	CO 3
8.	If $u = x^3y^2 + x^2y^3 where\ x = at^2 andy = 2at, then\ find\ \frac{du}{dt}$.	BTL -3	Applying	CO 3
9.	If $u = \frac{y^2}{2x} andv = \frac{x^2+y^2}{2x}, find\ \frac{\partial(u,v)}{\partial(x,y)}$.	BTL -4	Analyzing	CO 3
10.	If$x = uv, y = \frac{u}{v}. Find\ \frac{\partial(x,y)}{\partial(u,v)}$.	BTL -3	Applying	CO 3
11.	Write any two properties of Jacobians.	BTL -3	Applying	CO 3
12.	Find the Jacobian$\frac{\partial(u,v)}{\partial(r,\theta)}, ifx = r\cos\cos\theta$ &$y = r\sin\sin\theta$, $u = 2xy, v = x^2 - y^2$ without actual substitution	BTL -2	Understanding	CO 3
13.	Find the Taylor series expansion of x^ynear the point $(1, 1)$ up to first term	BTL -4	Analyzing	CO 3

14.	Expand $xy + 2x - 3y + 2 in\ powers\ of\ (x-1)\ \&\ (y+2)$, using Taylor's series up to first degree term.	BTL -1	Remembering	CO 3
15.	If $x = u^2 - v^2, y = 2uv$ find the Jacobian of x, y with respect to u and v	BTL -4	Analyzing	CO 3
16.	If u = $\frac{x+y}{1-xy}$ $and\ v = x + y, find \frac{\partial(u,v)}{\partial(x,y)}$	BTL -2	Understanding	CO 3
17.	State the necessary and sufficient condition for $f(x,y)$ to have a relative maximum at a point (a,b).	BTL -4	Analyzing	CO 3
18.	Find the minimum point of $f(x,y) = x^2 + y^2 + 6x + 12$.	BTL -3	Applying	CO 3
19.	Find the Stationary points of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.	BTL -4	Analyzing	CO 3
20.	Find the Stationary points of $x^2 - xy + y^2 - 2x + y$.	BTL -2	Understanding	CO 3
21.	If u = x^2+1, v = y^2 -2 then find $\frac{\partial(u,v)}{\partial(x,y)}$.	BTL -2	Understanding	CO 3
22.	Find the stationary points of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$	BTL -4	Analyzing	CO 3
23.	If u = x+y , y =uv then find $\frac{\partial(x,y)}{\partial(u,v)}$	BTL -2	Understanding	CO 3
24.	Find the nature of the stationary point (1,1) of the function f(x,y) if f_{yy}= $6x^3$y, f_{xx}= $6y^3$x, f_{xy}= $9x^2$y	BTL -4	Analyzing	CO 3
25.	Find the minimum value of f = $x^2 + y^2$ subject to the constraint x =1.	BTL -3	Applying	CO 3
	PART – B			
1.	Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225$	BTL -3	Applying	CO 3
2.(a)	A flat circular plate is heated so that the temperature at any point (x,y) is $u(x,y) = x^2 + 2y^2 - x$. Find the coldest point on the plate	BTL -3	Applying	CO 3
2.(b)	If $u = \frac{yz}{x}, v = \frac{zx}{y}\ and\ w = \frac{xy}{z}, find\ \frac{\partial(u,v,w)}{\partial(x,y,z)}$.	BTL -3	Applying	CO 3
3.	If u = log (x^2 + y^2) + ,then prove that u_{xx}+u_{yy} = 0	BTL -3	Applying	CO 3
4.(a)	Find $\frac{du}{dt}$ if $u = x^3 + y^3$,x = acost and y = bsint and verify the result.	BTL -4	Analyzing	CO 3
4.(b)	Find $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$ if $x = r sin\theta cos\varphi, y = r sin\theta sin\varphi, z = r cos\theta$	BTL -3	Applying	CO 3
5.	Expand $x^3y^2 + 2x^2y + 3xy^2$ in powers of (x+2) and (y-1) using Taylor's series up to third degree terms	BTL -1	Remembering	CO 3
6.(a)	If u = log $(x^3 + y^3 + z^3 - 3xyz)$ $then\ prove\ that\ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$	BTL -4	Analyzing	CO 3

6.(b)	If z = $e^{ax+by}.f(ax-by)$then prove that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}$ =2abz	BTL -3	Applying	CO 3
7.	If $u=f(x-y, y-z, z-x)$, $then\ show$ $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$	BTL -3	Applying	CO 3
8.(a)	$If x+y+z=u, y+z=uv, z=uvw$, prove $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$	BTL -3	Applying	CO 3
8.(b)	Find the extreme values of $f(x,y)=x^3+y^3-3x-12y+20$	BTL -3	Applying	CO 3
9.	Find the Taylors series expansion of $e^x\ siny$ at the point $\left(-1,\frac{\pi}{4}\right)$ up to third degree terms.	BTL -4	Analyzing	CO 3
10.(a)	Expand $\frac{y}{x}$ in the neighborhood of (1, 1) as Taylor's series up to second degree terms	BTL -3	Applying	CO 3
10.(b)	Find the Maximum value of $x^m y^n z^p when\ x+y+z=a$.	BTL -3	Applying	CO 3
11.	Discuss the extreme values of $f(x,y)=x^3y^2(1-x-y)$.	BTL -4	Analyzing	CO 3
12.(a)	Expand Taylor's series of $x^3+y^3+xy^2$ in powers of $(x-1)$ and $(y-2)$up to the second-degree term.	BTL -4	Analyzing	CO 3
12.(b)	If $u=xyz, v=x^2+y^2+z^2\ and\ w=x+y+z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	BTL -3	Applying	CO 3
13.	Find the shortest and longest distances from the point (1,2,-1) to the sphere $x^2+y^2+z^2=24$	BTL -4	Analyzing	CO 3
14.(a)	Examine $f(x,y)=x^3+3xy^2-15x^2+72x$ for extreme values	BTL -4	Analyzing	CO 3
14.(b)	Expand $e^x\ log\ log\ (1+y)$ in powers of x & y up to terms of second -degree using Taylor's series	BTL -3	Applying	CO 3
15.	Find the dimension of the rectangular box without a top of maximum capacity, whose surface area is 108 sq. cm.	BTL -4	Analyzing	CO 3
16.(a)	$If u=x+y+z, u^2v=y+z, u^3w=z$, prove$\frac{\partial(u,v,w)}{\partial(x,y,z)}=\frac{1}{u^5}$	BTL -3	Applying	CO 3
16.(b)	Find the minimum value of x^2yz^3 subject to 2x + y +3z = a.	BTL -3	Applying	CO 3
17.	Find the Taylors series expansion of $e^x\ siny$ at the point $\left(-1,\frac{\pi}{4}\right)$ up to third degree terms	BTL -4	Analyzing	CO 3
18.(a)	Find the extreme value of $x^2+y^2+z^2$subject to the condition $x+y+z=3a$.	BTL -3	Applying	CO 3
18.(b)	Expand e^{xy}in powers of $(x-1)$ and $(y-1)$upto second degree terms by Taylor's series	BTL -3	Applying	CO 3

UNIT-IV:MULTIPLE INTEGRALS

Double integrals in Cartesian and polar coordinates - Change of order of integration - Area enclosed by plane curves - Change of variables in Polar coordinates - Triple integrals - Volume of solids.

Q.No.	Question	BT Level	Competence	Course Outcome
	PART – A			
1.	Evaluate $\int_0^2 \int_0^x \frac{dxdy}{x^2+y^2}$	BTL -2	Understanding	CO 4
2.	Evaluate $\int \int dxdy$ over the region bounded by $x = 0, x = 2$, $y = 0\ and y = 2$	BTL -3	Applying	CO 4
3.	Change the order of integration $\int_0^1 \int_{y^2}^y f(x,y)\ dxdy$	BTL -3	Applying	CO 4
4.	Change the order of integration $\int_0^\infty \int_x^\infty f(x,y)\ dxdy$	BTL -2	Understanding	CO 4
5.	Evaluate $\int_0^2 \int_0^1 4xy\ dx\ dy$	BTL -1	Remembering	CO 4
6.	Evaluate $\int_2^3 \int_1^2 \frac{dxdy}{xy}$	BTL -1	Remembering	CO 4
7.	Evaluate $\int_0^1 \int_0^{\sqrt{1+y^2}} \frac{dxdy}{1+x^2+y^2}$	BTL -1	Remembering	CO 4
8	Find the limits of integration in the double integral $\iint_R f(x,y)dxdy$ where R is in the first quadrant and bounded x = 0, y = 0, $x + y = 1$	BTL -3	Applying	CO 4
9.	Find the limits of integration in the double integral $\iint_R f(x,y)dxdy$ where R is in the first quadrant and bounded x = 0, y = 0, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$	BTL -2	Understanding	CO 4
10.	Find the limits of integration in the double integral $\iint_R f(x,y)dxdy$ where R is in the first quadrant and bounded x=1, y=0, $y^2 = 4x$	BTL -2	Understanding	CO 4
11.	Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} sin\ sin\ (\theta + \varphi)\ d\theta\ d\varphi$	BTL -4	Analyzing	CO 4
12.	Find the area bounded by the lines $x = 0, y = 1\ and y = x$	BTL -2	Understanding	CO 4
13.	Evaluate $\int \int \int (x + y + z)dxdydz$ over the region bounded by $x = 0, x = 1 , y = 0\ and y = 1, z = 0, z = 1$	BTL -4	Analyzing	CO 4
14.	Evaluate $\int_0^\pi \int_0^{sin\theta} rdrd\theta$	BTL -3	Applying	CO 4
15.	Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dydx$	BTL -4	Analyzing	CO 4
16.	Evaluate $\int_1^{ln8} \int_0^{lny} e^{x+y}\ dxdy$	BTL -2	Understanding	CO 4

17.	Evaluate $\int_0^{\pi}\int_0^{a} rdrd\theta$	BTL -4	Analyzing	CO 4
18.	Evaluate $\int_0^{5}\int_0^{2}(x^2+y^2)dxdy$	BTL -1	Remembering	CO 4
19.	Evaluate $\int_a^{b}\int_c^{d}\int_f^{g} e^{x+y+z}dzdydx$	BTL -4	Analyzing	CO 4
20.	Write down the double integral to find the area of the circles $r = 2sin\theta, r = 4sin\theta$	BTL -3	Applying	CO 4
21.	Evaluate $\int_0^{1}\int_x^{\sqrt{x}} xy\,(x+y)dydx$	BTL -2	Understanding	CO 4
22.	Evaluate $\int_0^{1}\int_0^{x^2}(x^2+y^2)dydx$	BTL -4	Analyzing	CO 4
23.	Evaluate $\int_0^{\pi}\int_0^{5} r^4 sin\theta drd\theta$	BTL -4	Analyzing	CO 4
24.	Evaluate $\int_1^{3}\int_3^{4}\int_1^{4} xyz\,dzdydx$	BTL -3	Applying	CO 4
25.	Evaluate $\int_0^{1} dx\int_0^{2} dy\int_0^{3}(x+y+z)dz$	BTL -2	Understanding	CO 4
	PART – B			
1.	Change the order of integration $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xydxdy$ and hence evaluate it	BTL -2	Understanding	CO 4
2.(a)	Change the order of integration $\int_0^{\infty}\int_0^{\infty}\frac{e^{-y}}{y}dydx$ and hence evaluate it	BTL -4	Applying	CO 4
2.(b)	Find the area bounded by parabolas $y = 4 - x$ and $y^2 = x$ by double integration.	BTL -1	Remembering	CO 4
3.	Change the order of integration $\int_0^{a}\int_{a-y}^{\sqrt{a^2-y^2}} ydxdy$ and hence evaluate it	BTL -3	Analyzing	CO 4
4.	Evaluate $\iint xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$	BTL -4	Applying	CO 4
5.	Change the order of integration $\int_0^{1}\int_y^{2-y} xy\,dxdy$ and hence evaluate it	BTL -2	Understanding	CO 4
6.(a)	Change the order of integration $\int_0^{2}\int_0^{\sqrt{4-y^2}} xydxdy$ and hence evaluate it	BTL -2	Understanding	CO 4
6.(b)	Using double integral find the area of the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$	BTL -4	Applying	CO 4
7.	Find the area common to the cardioids $r = a(1+cos)$ and $r = a\,(1-cos)$	BTL -2	Understanding	CO 4
8.	Evaluate $\int_0^{a}\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-y^2-z^2}} x\,dx\,dy\,dz$	BTL -4	Analyzing	CO 4

9.	Evaluate $\iiint \frac{dx\,dy\,dz}{x^2+y^2+z^2}$, taken throughout the volume of the sphere $x^2+y^2+z^2=a^2$.	BTL -3	Applying	CO 4
10.(a)	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$	BTL -3	Applying	CO 4
10.(b)	Evaluate $\int_0^a \int_0^b \int_0^c (x^2+y^2+z^2)dxdydz$	BTL -3	Applying	CO 4
11.	Find the value of $\iiint xyz\,dxdydz$ through the positive spherical octant for which $x^2+y^2+z^2 \le a^2$.	BTL -4	Analyzing	CO 4
12.(a)	Find the area included between the curves $y^2 = 4x$ and $x^2 = 4y$	BTL -2	Understanding	CO 4
12.(b)	Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\,dzdydx$	BTL -2	Understanding	CO 4
13.	Change the order of integration $\int_0^a \int_y^a \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$ and hence evaluate it	BTL -2	Understanding	CO 4
14.	By change the order of integration and evaluate $\int_0^2 \int_{x^2}^{2-x} xydydx$	BTL -2	Understanding	CO 4
15.	By changing in to polar Co – ordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$. Hence find the value of $\int_0^\infty e^{-x^2}dx$.	BTL -2	Understanding	CO 4
16.	Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.	BTL -3	Applying	CO 4
17.	Change the integral into polar coordinates $\int_0^a \int_0^x \frac{x^3}{\sqrt{x^2+y^2}}dxdy$ and hence evaluate it	BTL -2	Understanding	CO 4
18.(a)	Find the volume bounded by the cylinder $x^2+y^2=1$ *and the planes* $x+y+z=3, z=0$	BTL -4	Analyzing	CO 4
18.(b)	Evaluate $\iint (x^2+y^2)dxdy$ over the area bounded by the parabola $y^2 = 4x$ and its latus rectum.	BTL -3	Applying	CO 4

UNIT-V: VECTOR CALCULUS

Gradient and directional derivative – Divergence and curl – Vector identities – Irrotational and Solenoidal vector fields – Vector Integration Green’s, Gauss divergence and Stoke’s theorems – Verification and application in evaluating line, surface and volume integrals.

Q.No.	Question	BT Level	Competence	Course Outcome
	PART – A			
1.	Find the maximum directional derivative of $\varphi = xyz^2$ at (1, 2, 3).	BTL -3	Applying	CO 5
2.	Is the position vector $\vec{r} = x\vec{i}+y\vec{j}+z\vec{k}$ irrotational? Justify.	BTL -1	Remembering	CO 5

3.	What is the value of m if the vector $\vec{F} = (x+3y)\vec{\imath} + (y-2z)\vec{\jmath} + (x+mz)\vec{k}$ is solenoidal	BTL -2	Understanding	CO 5
4.	If $\varphi = x^3 + y^3 + z^3$ then find $grad\ \varphi$ at (2, 0, 4).	BTL -1	Remembering	CO 5
5.	If $\varphi = x^3 + yz$ then find $grad\ \varphi$.	BTL -2	Understanding	CO 5
6.	Find the Directional derivative of $\varphi = 4xz^2 + x^2yz$ at (0, 2,4) in the direction $2\vec{\imath} + 3\vec{\jmath} + 4\vec{k}$.	BTL -1	Remembering	CO 5
7.	Give the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$ at (2,3,4).	BTL -2	Understanding	CO 5
8.	Find the unit normal to the surface $xy^3z^2 = 4$ at (1,1,3)	BTL -3	Applying	CO 5
9.	If $\varphi = 3xy - yz$, Find $grad\varphi$ at (1,0,1).	BTL -3	Applying	CO 5
10.	Show that the vector $\vec{F} = (x+3y)\vec{\imath} + (y-3z)\vec{\jmath} + (x-2z)\vec{k}$ is solenoidal.	BTL -3	Applying	CO 5
11.	What is the value of a, b, c if the vector $\vec{F} = (x+y+az)\vec{\imath} + (by+2y-z)\vec{\jmath} + (-x+cy+2z)\vec{k}$ may be irrotational	BTL -3	Applying	CO 5
12.	Give the unit normal vector to the surface $xyz = 4$ at (2, 1, 1).	BTL -2	Understanding	CO 5
13.	Show that the vector $\vec{F} = 3y^3z^2\vec{\imath} + 4x^2z^2\vec{\jmath} - 3x^2y^2\vec{k}$ is solenoidal.	BTL -4	Analyzing	CO 5
14.	Show that the vector $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{\imath} + (3xz + 2xy)\vec{\jmath} + (3xy - 2xz + 2z)\vec{k}$ is solenoidal.	BTL -3	Applying	CO 5
15.	Show that $curl(grad\varphi) = 0$.	BTL -3	Applying	CO 5
16.	If $\vec{r}$ is the position vector, Find $div\vec{r}$.	BTL -2	Understanding	CO 5
17.	Show that $\nabla(r^n) = nr^{n-2}\vec{r}$.	BTL -4	Analyzing	CO 5
18.	Find the unit normal to the surface $x^2 + y^2 - z = 10$ at (2,4,6)	BTL -3	Applying	CO 5
19.	Evaluate using Gauss Divergence theorem for $\vec{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + +yz\vec{k}$ taken over the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$.	BTL -2	Understanding	CO 5
20.	State stokes theorem**.**	BTL -4	Analyzing	CO 5
21.	State Greens theorem	BTL -3	Applying	CO 5
22.	If $\vec{F} = 3xy\vec{\imath} - y^2\vec{\jmath}$, Evaluate $\int \vec{F}.d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from the point (0,0) to the point (1,3).	BTL -2	Understanding	CO 5
23.	If $\vec{F} = (x^2)\vec{\imath} + (xy^2)\vec{\jmath}$, evaluate $\int \vec{F}.d\vec{r}$ from (0,0) to (2,2) along the path $y = x$.	BTL -4	Analyzing	CO 5

24.	Using Green's theorem evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the square enclosed by the lines $x = 0, y = 0, x = 3, y = 4$.	BTL -1	Remembering	CO 5
25.	State Gauss Divergence theorem.	BTL -4	Analyzing	CO 5
PART – B				
1.	Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the point (6, 4, 3).	BTL -2	Understanding	CO 5
2.(a)	Calculate the angle between the normals to the surface $xy = z^2$ at the points (4, 1, 2) and(3, 3, -3).	BTL -2	Understanding	CO 5
2.(b)	Find the work done when a force $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ moves a particle in the XY plane from (0,0,0) to (1,1,1) along the parabola $x = y^2$.	BTL -3	Analyzing	CO 5
3.	Find the Directional Derivative of $\varphi = 3x^2yz + 4xz^2 + xyz$ at (1, 2, 3) in the direction of $2\vec{i} + \vec{j} - \vec{k}$.	BTL -4	Applying	CO 5
4.(a)	Find the constants a and b, so that the surfaces $5x^2 - 2yz - 9x = 0$ and $ax^2y + bz^3 = 4$ may cut orthogonally at the point (1,-1,2).	BTL -2	Understanding	CO 5
4.(b)	Find the unit normal to the surface $x^2y + 2xz^2 = 8$ at the point (1, 0, 2).	BTL -4	Applying	CO 5
5.	Find the scalar potential, if the vector field $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$ is irrotational	BTL -2	Understanding	CO 5
6.(a)	Find the value of a, b, c so that the vector $\vec{F} = (x + y + az)\vec{i} + (bx - 2y - z)\vec{j} + (-x + cy + 2z)\vec{k}$ may be irrotational. Also find its scalar potential	BTL -3	Analyzing	CO 5
6.(b)	Find the work done by the force $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ where C is the arc of the parabola $y = 2x^2$ from (0,0) to (1,2).	BTL -2	Understanding	CO 5
7.	Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - 3 = z$ at the points (2,-1, 2).	BTL -3	Analyzing	CO 5
8.(a)	Find the work done by the force when it moves a particle from (1,-2, 1) to (3, 1,4) along any path?	BTL -4	Applying	CO 5
8.(b)	Find the values of a and b so that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ may cut orthogonally at (1,-1, 2).	BTL -1	Remembering	CO 5
9.	Find the Directional Derivative of $\varphi = xy^2\vec{i} + zy^2\vec{j} + xz^2\vec{k}$ at (2, 0, 3) in the direction of the outward normal to the sphere $x^2 + y^2 + z^2$ at the point(3,2,1)	BTL -2	Understanding	CO 5

10.(a)	Verify Green's theorem in the plane for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by $x = 0, y = 0, x + y = 1$.	BTL -3	Analyzing	CO 5
10.(b)	Find $div\vec{F}$ and $curl\vec{F}$ where $\vec{F} = grad\ (x^3 + y^3 + z^3$ - 3xyz)	BTL -4	Applying	CO 5
11.	Prove that area bounded by a simple closed curve is given by $\frac{1}{2}\int_c (xdy - ydx)$.Hence find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	BTL -2	Understanding	CO 5
12.(a)	Find the Directional Derivative of $\varphi = xyz$ at P(1, 1, 3) in the direction of the outward drawn normal to the sphere $x^2 + y^2 + z^2 = 11$ through the point P.	BTL -2	Understanding	CO 5
12.(b)	Verify Green's theorem in the plane for $\int_c [(xy + y^2)dx + (x^2)dy]$ where c is a closed of the region bounded by $x = y$ and $y = x^2$.	BTL -2	Understanding	CO 5
13.	Verify Gauss divergence theorem for $\vec{F} = (x^3)\vec{i} + (y^3)\vec{j} + z^3\vec{k}$ where s is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0,\ y = a, z = 0, z = a$.	BTL -3	Applying	CO 5
14.(a)	Show that Stokes theorem is verified for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ where S is the surface of the cube formed by $x = 0, x = 2, y = 0, y = 2, and\ z = 2$ above the XY- plane.	BTL -4	Analyzing	CO 5
14.(b)	Verify Gauss divergence theorem for $\vec{F} = (x^2)\vec{i} + (y^2)\vec{j} + (z^2)\vec{k}$ where s is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0,\ y = b, z = 0, z = c$.	BTL -4	Analyzing	CO 5
15.	Verify Gauss divergence theorem for $\vec{F} = (x^3 - yz)\vec{i} - (2x^2y)\vec{j} + 2\vec{k}$ where s is the surface of the cuboid formed by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.	BTL -3	Applying	CO 5
16.(a)	Verify the Stokes theorem is verified for $\vec{F} = (x^2)\vec{i} + xy\vec{j}$ integrated round the square those sides formed $x = 0, x = a, y = 0, y = a\ in\ the\ plane\ z = 0$	BTL -4	Analyzing	CO 5
16.(b)	Verify Gauss divergence theorem for $\vec{F} = (x^2)\vec{i} + z\vec{j} + yz\vec{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1,\ z = 0, z = 1$.	BTL -3	Applying	CO 5
17.	Verify Stoke's theorem for $\vec{F} = (xy + y^2)\vec{i} + x^2\vec{j}$ in the XOY plane bounded by $x = y$ and $y = x^2$	BTL -2	Understanding	CO 5
18.(a)	Show that the field $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ is irrotational and find its scalar potential.	BTL -4	Analyzing	CO 5
18.(b)	Find the angle between the surface $x\ logz = y^2 - 1$ and $x^2y = 2 - z$ at the point (1,1, 1).	BTL -3	Applying	CO 5