SRM VALLIAMMAI ENGINEERING COLLEGE

(An Autonomous Institution)

S.R.M. Nagar, Kattankulathur - 603203

DEPARTMENT OF MATHEMATICS

QUESTION BANK

IV SEMESTER

B. E / B.Tech - CSE, IT & Cyber

MA3424- APPLIED MATHEMATICS FOR INFORMATION SCIENCE

Regulation – 2023

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DEPARTMENT OF MATHEMATICS

SUBJECT: MA3424- APPLIED MATHEMATICS FOR INFORMATION SCIENCE

SEM / YEAR: IV / II Year B.E. / B.Tech CSE, IT & Cyber

	UNIT I - GROUPS AND RINGS Algebra: groups, rings, fields, finite fields – Definitions - Examples - Properties			
Q.No.	**Question**	**BT Level**	**Competence**	**CO s**
	PART – A			
1.	Define group with examples.	BTL -1	Remembering	CO 1
2.	State any two properties of a group.	BTL -1	Remembering	CO 1
3.	Show that the cancellation laws are true in a group $(G,*)$	BTL -1	Remembering	CO 1
4.	Prove that identity element in a group is unique.	BTL -2	Understanding	CO 1
5.	Prove that the inverse of each element of the group $(G,*)$ is unique.	BTL -1	Applying	CO 1
6	In a group $(G,*), prove\ that\ (a*b)^{-1} = b^{-1}*a^{-1}$for all $a,b \in G$	BTL -2	Applying	CO 1
7.	If $(G,*)$ is a group infer that the only idempotent element of a is the identity element.	BTL -2	Understanding	CO 1
8	Let Z be a group of integers with binary operation * defined by $a*b = a+b-2$ for all $a,b \in Z$. Find the identity element of the group$\langle Z,*\rangle$.	BTL -2	Understanding	CO 1
9.	Let R be the set of non-zero real numbers and * is the binary operation defined as $a*b = \frac{ab}{2}, for\ a,b \in R$. Find the inverse of any element.	BTL -2	Applying	CO 1
10.	Define Semi group.			CO 1
11.	Define Monoid.	BTL -2	Understanding	CO 1
12.	Prove that if G is abelian group, then for all $a,b \in G, (a*b)^2 = a^2*b^2$	BTL -1	Analyzing	CO 1
13.	If $(G,*)$ is a group for any $a \in G$ prove that $(a^{-1})^{-1} = a$.	BTL -1	Applying	CO 1

14.	Prove that the order of an element a of a group G is the same as that of its inverse (a^{-1}).	BTL -1	Analyzing	CO 1
15.	Let R be the set of non-zero real numbers and * is the binary operation defined as $a * b = \frac{ab}{2}, for\ a, b \in R$. Find the identity element.	BTL -2	Understanding	CO 1
16.	Let Z be a group of integers with binary operation * defined by $a * b = a + b - 1$ for all $a, b \in Z$. Find the identity of the element of the group$\langle Z,*\rangle$.	BTL -1	Analyzing	CO 1
17.	Let Z be a group of integers with binary operation * defined by $a * b = a + b - 2$ for all $a, b \in Z$. Find the inverse of the element of the group$\langle Z,*\rangle$.	BTL -1	Remembering	CO 1
18.	Let R be the set of non-zero real numbers and * is the binary operation defined as $a * b = \frac{ab}{3}, for\ a, b \in R$. Find the identity element.	BTL -2	Analyzing	CO 1
19.	If G is a group of order n and $a \in G$, prove that $a^n = e$	BTL -2	Understanding	CO 1
20.	Give an example of a ring which is not a field.	BTL -1	Analyzing	CO 1
21.	Discuss a ring and give an example	BTL -1	Applying	CO 1
22.	Define commutative ring and Ring with unity	BTL -2	Understanding	CO 1
23.	Discuss a sub ring with example	BTL -1	Analyzing	CO 1
24.	Define integral domain and give an example.	BTL -1	Remembering	CO 1
25.	Define a field with example	BTL -1	Remembering	CO 1
	PART – B			
1.	Prove that $G = \left\{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}\right\}$ forms an abelian group under matrix multiplication.	BTL -3	Applying	CO 1
2.(a)	Prove that in a group G the equations $a * x = b\ and\ y * a = b$ have unique solutions for the unknowns x and y as $x = a^{-1} * b, y = b * a^{-1}$ when $a, b \in G$.	BTL -4	Understanding	CO 1
2.(b)	Show that the set of all matrices $\begin{bmatrix}a & b\\-b & a\end{bmatrix}$forms an abelian group with respect to matrix multiplication.	BTL -4	Evaluating	CO 1
3.	Apply the definition of a group to Prove that $(G,*)$ is a non-abelian group where $G = R \times R$ and the binary operation * is defined as $(a, b) * (c, d) = (ac, bc + d)$	BTL -3	Applying	CO 1

4.(a)	If $(G,*)$ is an abelian group and if $\forall\ a, b\ \in G$. Show that $(a * b)^n = a^n * b^{\,n}$, for every integer n.	BTL -3	Applying	CO 1
4.(b)	Show that $(Q^+,*)$ is an abelian group where $*$ is defined as $a * b = ab/2, \forall a, b \in Q^+$	BTL -3	Applying	CO 1
5.	Determine whether $(Q, \oplus, \odot)$ is a ring with the binary operations $x \oplus y = x + y + 7, x \odot y = x + y + \frac{xy}{7}$ for all $x, y \in Q$	BTL -3	Applying	CO 1
6.(a)	Prove that the intersection of two subgroups of a group G is again a subgroup of G	BTL -3	Applying	CO 1
6.(b)	Prove that the set $\{1, -1, i, -i\}$ is a finite abelian group with respect to the multiplication of complex numbers.	BTL -3	Applying	CO 1
7.	If (G, *) is a finite cyclic group generated by an element $a \epsilon G$ and is of order n then $a^n = e$ so that $G = \{a, a^2, \ldots a^n (= e)\}$. Also, n is the least positive integer for which $a^n = e$.	BTL -4	Remembering	CO 1
8.(a)	Show that $\left(Z_{7,}\ \cdot_7\right)$ is a abelian group.	BTL -4	Analyzing	CO 1
8.(b)	Prove that $G = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix},\ ad \neq 0 \quad a, b, d\ \in R \right\}$ is a group under matrix multiplication.	BTL -3	Applying	CO 1
9.	Prove that $(S_3,*)$,where $S = (1,2,3)$ is a group under the operation of right composition. Is it abelian?	BTL -4	Evaluating	CO 1
10.(a)	Let G be a group and $a \in G$. Let $f: G \to G$ be given by $f(x) = axa^{-1}, \forall\ x \in G$. Prove that f is an isomorphism of G onto G	BTL -4	Creating	CO 1
10.(b)	Prove that $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix},\ a \neq 0 \quad a \in R \right\}$ is a group under matrix multiplication.	BTL -3	Applying	CO 1
11.	Analyze that Z_n is a field if and only if n is prime	BTL -4	Analyzing	CO 1
12.(a)	Let $f: G,*) \to (H,\Delta)$ be group homomorphism then show that $Ker(f)$ is a normal subgroup.	BTL -3	Applying	CO 1
12.(b)	Show that M_2, the set of all $2X2$ nonsingular matrices over R is a group under usual matrix multiplication. Is it abelian?	BTL -3	Applying	CO 1
13.	Prove that in a Ring (R,+,.) The zero element is unique The additive inverse of each ring element is unique If R has a unity then it is unique If R has a unity, x is a unit of R then the multiplicative inverse of x is unique	BTL -4	Analyzing	CO 1
14.	Show that $(Z_6, +_6, \cdot_6)$ is a ring.	BTL -3	Applying	CO 1

15.	Prove that every field is an integral domain and every finite integral domain is a field. Give an example for an integral domain which is nor a field	BTL -4	Analyzing	CO 1
16.	Show that $(Z_{5,} +_5, \cdot_5)$ is a field	BTL -3	Applying	CO 1
17.	Analyze whether $(Z, \oplus, \odot)$ is a ring with the binary operation $x \oplus y = x + y - 1, x \odot y = x + y - xy$ for all $x, y \in Z$	BTL -4	Analyzing	CO 1
18.	Prove that the set $Z_4 = \{0,1,2,3\}$ is a commutative ring with respct ot the binary operation $+_4$ and $\times_4$	BTL -3	Applying	CO 1

	UNIT II - FINITE FIELDS AND POLYNOMIALS Rings – Polynomial rings – Irreducible polynomials over finite fields – Factorization of polynomials over finite fields.			
Q.No.	**Question**	**BT Level**	**Competence**	**COs**
	PART – A			
1.	Define polynomial.	BTL -1	Remembering	CO 2
2.	Define root of a polynomial.	BTL -1	Remembering	CO 2
3.	Find the roots for the function $f(x) = x^2 + 3x + 2 \in \mathbb{Z}_6[x]$.	BTL -1	Remembering	CO 2
4.	What are the roots of $f(x) = x^2 - 6x + 9 \in \mathbb{R}[x]$.	BTL -2	Understanding	CO 2
5.	Find the roots for the function $f(x) = x^2 - 2$ in $R[x] and$ $Q[x]$.	BTL -1	Applying	CO 2
6	How many polynomials in $\mathbb{Z}_5$ has degree 3?	BTL -1	Applying	CO 2
7.	How many polynomials in $\mathbb{Z}_7$ has degree 5?	BTL -2	Understanding	CO 2
8	If $f(x) = 7x^4 + 4x^3 + 3x^2 + x + 4$ & $g(x) = 3x^3 + 5x^2 + 6x + 1$, $f(x), g(x) \in Z_7[x]$, then find $f(x) + g(x)$ & $\deg(f(x) + g(x))$.	BTL -2	Understanding	CO 2
9.	Determine all polynomials of degree 2 in $\mathbb{Z}_2[x]$.	BTL -1	Applying	CO 2
10	Define irreducible polynomial.	BTL -2	Understanding	CO 2
11.	Determine whether $x^2 + 1$ is an irreducible polynomial over the field $\{0,1\}$.	BTL -1	Analyzing	CO 2
12.	Show that $x^2 + x + 1$ is irreducible over Z_5.	BTL -1	Applying	CO 2
13.	Show that $x^2 + 1$ is irreducible in $\mathbb{Z}_3[x]$.	BTL -1	Analyzing	CO 2
14.	Obtain reducible polynomial of degree six with no roots in$\mathbb{Z}_2$.	BTL -2	Understanding	CO 2

15.	Does the set $F = \{0,1,2,3\}$ form a filed with respect to addition modulo 4 and multiplication modulo 4? Why?	BTL -2	Analyzing	CO 2
16.	If $f(x) = x^5 - 2x^2 + 5x - 3$ and $g(x) = x^4 - 5x^3 + 7x$are polynomials in Q[x], determine $q(x)$ and $r(x)$ such that $f(x) = q(x)g(x) + r(x)$	BTL -1	Remembering	CO 2
17.	Find quotient and reminder when g(x) = 2x-1 divides $f(x) = 2x^4 + 5x^3 - 7x^2 + 4x + 8$, where f(x) and g(x) are polynomials over Q[x]	BTL -2	Analyzing	CO 2
18.	What is the remainder when $f(x) = x^7 - 6x^5 + 4x^4 - x^2 + 3x - 7 \in \mathbb{Q}[x]$ is divided by $x - 2$.	BTL -2	Understanding	CO 2
19.	State division algorithm for polynomials.	BTL -1	Analyzing	CO 2
20.	State Remainder theorem.	BTL -1	Applying	CO 2
21.	Define characteristics of a field.	BTL -2	Understanding	CO 2
22.	State Euclidean algorithm.	BTL -2	Analyzing	CO 2
23.	Find two non-zero polynomials $f(x)$ and $g(x)$ in $Z_6[x]$ such that $f(x)g(x) = 0$	BTL -1	Applying	CO 2
24.	Check the reducibility of $f(x) = x^2 + 3x - 1$ in Q[x], R[x] and C[x]	BTL -2	Understanding	CO 2
25.	Check the reducibility of $f(x) = x^3 - 1$ in Q[x], R[x] and C[x]	BTL -2	Analyzing	CO 2
	PART – B			
1.	Let $\mathbb{R}[x]$ be a polynomial ring, then Prove the following (a) If $\mathbb{R}$ is commutative then $\mathbb{R}[x]$ is commutative. (b) If $\mathbb{R}$ is a ring with unity then $\mathbb{R}[x]$ is a ring with unity. (c) $\mathbb{R}[x]$ is an integral domain if and only if $\mathbb{R}$ is an integral domain.	BTL -3	Applying	CO 2
2.(a)	Find $f(x) + g(x)$, $f(x) - g(x)$ and $f(x)\, g(x)$ such that $f(x) = x^4 + x^3 + x + 1$, $g(x) = x^3 + x^2 + x + 1$ over $\mathbb{Z}_2[x]$	BTL -4	Evaluating	CO 2
2.(b)	Find all the roots of $f(x) = x^5 - x$ in $\mathbb{Z}_5[x]$ and then write $f(x)$ as a product of first degree polynomials	BTL -3	Evaluating	CO 2
3.	If $\mathbb{R}$ is a ring then prove that $(\mathbb{R}[x], +, \ .)$ is a ring called a polynomial ring over $\mathbb{R}$.	BTL -3	Applying	CO 2
4.(a)	Let $(\mathbb{R}, +, .)$ be a commutative ring with unity u. Then $\mathbb{R}$ is an integral domain if and only if for all $f(x), g(x) \in \mathbb{R}[x]$, if neither $f(x)$ nor $g(x)$ is the zero polynomial, then prove that degree of $f(x)g(x) = degree f(x) + degree g(x)$.	BTL -3	Applying	CO 2
4.(b)	Find the remainder when $g(x) = 7x^3 - 2x^2 + 5x - 2$ is divided by $f(x) = x - 3$ and $f(x), g(x) \in \mathbb{Z}[x]$	BTL -3	Applying	CO 2

5.	State and Prove (i) Remainder Theorem (ii) Factor theorem	BTL -3	Applying	CO 2
6.(a)	Find all roots of $f(x) = x^2 + 4x$ if $f(x) \in Z_{12}$.	BTL -3	Applying	CO 2
6.(b)	If $g(x) = x^5 - 2x^2 + 5x - 3$ & $f(x) = x^4 - 5x^3 + 7x$ Find $q(x)\,, r(x)\ such\ that\ g(x) = f(x)q(x) + r(x)$.	BTL -3	Applying	CO 2
7.	(i) Check whether Check whether $f(x) = x^4 + x^3 + x^2 + x + 1 \in Z_2[x]$ is irreducible or not? (ii) Discuss whether $x^4 + x^3 + 1$ is reducible over Z_2 .	BTL -3	Remembering	CO 2
8.(a)	If F is a field and $f(x) \in F[x]$ has degree ≥ 1 , then prove that $f(x)$ has at most n roots in F.	BTL -4	Analyzing	CO 2
8.(b)	If $(x) = 3x^5 - 8x^4 + x^3 - x^2 + 4x - 7$, $g(x) = x + 9\ and\ f(x), g(x) \in \mathbb{Z}_{11}[x]$, find the remainder when $f(x)$ is divided by $g(x)$.	BTL -3	Applying	CO 2
9.	(i) Find all the roots of $f(x) = x^3 + 5x^2 + 2x + 6\ in\ Z_7[x]$ And then write f(x) as a product of first degree polynomials (ii)Determine whether the given polynomial is irreducible or not? $f(x) = x^2 + x + 1$ over Z_3, Z_5, Z_7 (iii) Find four distinct linear polynomials $g(x)\,, h(x)\,, s(x)$, $t(x) \in Z_{12}[x]$ so that $f(x) = g(x)h(x) = s(x)t(x)$.	BTL -3	Evaluating	CO 2
10.(a)	Give an example of polynomial $f(x) \in F(x)$, where $f(x)$ Has degree 8 and degree 6, it is reducible but it has no real roots.	BTL -4	Creating	CO 2
10.(b)	Discuss whether $x^4 - 2$ is reducible over $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$.	BTL -3	Applying	CO 2
11.	Let (F ,+ ,.) be a field. If Char (F) > 0, then prove that Char (F) must be Prime.	BTL -4	Analyzing	CO 2
12.(a)	Check whether $f(x) = x^4 + x^3 + x^2 + x + 1 \in Z_2[x]$ is irreducible or not?	BTL -3	Applying	CO 2
12.(b)	Find the gcd of $2x^3 + 2x^2 - x - 1\ and\ 2x^4 - x^2\ in\ \mathbb{Q}\ [x]$	BTL -3	Applying	CO 2
13.	Identify the equivalence classes of $Z_2[x]$ with $S(x) = x^2 + x + 1$	BTL -4	Analyzing	CO 2
14.(a)	Write $f(x) = (2x^2 + 1)(5x^3 - 5x + 3)(4x - 3) \in Z_7[x]$ as a product of the unit and three Monic polynomial.	BTL -3	Applying	CO 2
14.(b)	Determine whether the following polynomial is irreducible or not? $f(x) = x^2 + 3x - 1$ in $\mathbb{R}[x]$, $\mathbb{Q}[x]$, $\mathbb{C}[x]$.	BTL -3	Applying	CO 2
15.	Determine whether the given polynomial is irreducible or not? $f(x) = x^2 + x + 1$ over Z_3, Z_5, Z_7	BTL -4	Analyzing	CO 2
16.(a)	If $f(x) = 4x^2 + 1\,, g(x) = 2x + 3, f(x), g(x) \in Z_8[x]$. Then show that deg $f(x)g(x) = deg f(x) + deg g(x)$.	BTL -4	Analyzing	CO 2
16.(b)	If $f(x) = 2x^4 + 5x^2 + 2\,, g(x) = 6x^2 + 4$, then determine $q(x) and\ r(x) in\ \mathbb{Z}_7[x]$, where $f(x)$ is divided by $g(x)$.	BTL -3	Applying	CO 2

17.	Analyze the GCD of (i) $4x^3 - 2x^2 - 3x + 1$ and $2x^2 - x - 2$ in Q[x] (ii) $x^5 + x^4 + 2x^2 - x - 1$and $x^3 + x^2 - x$ in Q[x]	BTL -4	Analyzing	CO 2
18.(a)	Find the remainder when $g(x) = 6x^4 + 4x^3 + 5x^2 + 3x + 1$ is divided by $f(x) = 3x^2 + 4x + 2$ over polynomials in $\mathbb{Z}_7[x]$	BTL -3	Applying	CO 2
18.(b)	Find the g.c.d of $x^4 + x^3 + 2x^2 + x + 1$ and $x^3 - 1$ over Q.	BTL -3	Applying	CO 2

	UNIT III - ANALYTIC NUMBER THEORY Division algorithm –Prime and composite numbers– GCD – Euclidean algorithm – Fundamental theorem of arithmetic – LCM			
Q.No.	**Question**	**BT Level**	**Competence**	**CO s**
	PART – A			
1.	State divisible algorithm	BTL -1	Remembering	CO 3
2.	State pigeon hole principle	BTL -1	Remembering	CO 3
3.	State principle of inclusion and exclusion	BTL -1	Remembering	CO 3
4.	Find the number of positive integers ≤ 2076 that are divisible by 19	BTL -2	Understanding	CO 3
5.	Find the number of positive integers ≤ 3076 that are not divisible by 17	BTL -2	Applying	CO 3
6	Find the number of positive integers ≤ 3076 that are divisible by 19	BTL -2	Applying	CO 3
7.	Prove that if n is odd then $n^2 - 1$ is divisible by 8	BTL -2	Understanding	CO 3
8	Define union, intersection and complement.	BTL -2	Understanding	CO 3
9.	Give the divisibility relation.	BTL -1	Applying	CO 3
10	Let f(n) denote the number of positive factors of a positive integer n. Evaluate f(12).	BTL -2	Understanding	CO 3
11.	Find the six consecutive integers that are composite	BTL -1	Analyzing	CO 3
12.	Express $(12,15,21)$ as a linear combination of 12,15,and 21	BTL -1	Applying	CO 3
13.	Prove that the product of any two integers of the form 4n+1 is also the same form	BTL -2	Analyzing	CO 3
14.	Use canonical decomposition to Evaluate the GCD of 168 and 180	BTL -2	Understanding	CO 3
15.	Use canonical decomposition to evaluate LCM of 1050 and 2574	BTL -1	Analyzing	CO 3
16.	Find the canonical decomposition of 2520	BTL -1	Remembering	CO 3
17.	Find the prime factorization of 420, 135, 1925	BTL -2	Analyzing	CO 3
18.	Using $(252,360)$ construct $[252,360]$	BTL -2	Understanding	CO 3
19.	Find the twin primes p, q such that $[p, q] = 323$	BTL -1	Analyzing	CO 3
20.	Using recursion evaluate $(18,30,60,75,132)$	BTL -1	Applying	CO 3

21.	Find the GCD (414,662) using Euclidean algorithm	BTL -2	Understanding	CO 3
22.	Find the LCM (120.500)	BTL -2	Analyzing	CO 3
23.	Find the canonical decomposition of 1976	BTL -1	Applying	CO 3
24.	Use canonical decomposition to Evaluate the GCD of 72 and 108	BTL -2	Understanding	CO 3
25.	Use canonical decomposition to Evaluate the LCM of 110 and 210	BTL -1	Analyzing	CO 3
	PART – B			
1.	If $a, b, c \in Z$ then (i) $a/a\,, for\ all\ a \neq 0\ \in Z$ (ii) $a/b\ \ and\ b/c\ \ then\ a/c\ , \forall\ a, b \neq 0, c \neq 0\ \in Z$ (iii) $a/b\ then\ \ a/b\,c, \forall a \neq 0, b \in Z$ $(iv)\ a/b\ \ and\ a/c\ \ then\ a/(xb + yc)\ , \forall\ x, y \in Z, a \neq 0 \in Z$	BTL -3	Applying	CO 3
2.(a)	State and Prove Euclidean algorithm	BTL -4	Evaluating	CO 3
2.(b)	Find the number of positive integers ≤ 3000 divisible by 3, 5 or 7	BTL -4	Evaluating	CO 3
3.	Prove that (i) If p is a prime and p/ab then p/a or p/b (ii) If p is a prime and $p/a_1 a_2 a_3 \cdots a_n$, where $a_{1,} a_{2,} a_{3,} \cdots, a_n$ are positive integers then p/a_i for some i, $1 \leq i \leq n$	BTL -3	Applying	CO 3
4.(a)	Prove that the GCD of two positive integers a and b is a linear combination of a and b	BTL -3	Applying	CO 3
4.(b)	Find the number of positive integers in the range 1976 through 3776 that are divisible by 13 and not divisible by 17	BTL -3	Applying	CO 3
5.	Prove that (i) every integer $n \geq 2$ has a prime factor. (ii) there are infinitely many primes.	BTL -3	Applying	CO 3
6.(a)	Find the number of integers from 1 to 250 that are divisible by any of the integers 2,3,5,7	BTL -3	Applying	CO 3
6.(b)	Prove that $(a, a - b) = 1$ if and only if $(a, b)\ = 1$	BTL -3	Applying	CO 3
7.	State and prove Fundamental Theorem of Arithmetic.	BTL -3	Remembering	CO 3
8.(a)	Prove the following If (i)$(a, m) = 1$and $(a, m) = 1$, then $(ab, m) = 1$ (ii)If a/c and b/c and $(a, b) = 1$, then ab/c	BTL -4	Analyzing	CO 3
8.(b)	Use Euclidean algorithm to find the GCD of $(1819, 3587)$. Also express the GCD as a linear combination of the given numbers	BTL -3	Applying	CO 3
9.	State and Prove Euclid theorem	BTL -4	Evaluating	CO 3
10.(a)	Prove that there are infinitely many primes of the form $4n + 3$	BTL -4	Creating	CO 3

10.(b)	Use Euclidean algorithm to find the GCD of (12345,54321). Also express the GCD as a linear combination of the given numbers	BTL -3	Applying	CO 3
11.	If a and b are positive integers then prove that (i) $[a,b] = \frac{a.b}{(a,b)}$ (ii) Prove that two positive integers a and b are relatively prime iff $[a,b] = ab$	BTL -4	Analyzing	CO 3
12.(a)	Prove that every composite number n has prime factor $\leq [\sqrt{n}]$	BTL -3	Applying	CO 3
12.(b)	Use Euclidean algorithm to find the GCD of (2076, 1776). Also express the GCD as a linear combination of the given numbers	BTL -3	Applying	CO 3
13.	Use Euclidean algorithm to evaluate the GCD of (2024, 1024). Also express the GCD as a linear combination of the given numbers	BTL -4	Analyzing	CO 3
14.(a)	Prove that for every positive integer n there are n consecutive integers that are composite numbers	BTL -3	Applying	CO 3
14.(b)	Use Euclidean algorithm to find the GCD of (4076, 1024). Also express the GCD as a linear combination of the given numbers	BTL -3	Applying	CO 3
15.	(i)If $d = (a,b)$ and d' is any common divisor of a and b then d'/d (ii) For any positive integer m prove that $(ma, mb) = m(a,b)$ (iii) If $d = (a,b)$, then $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$	BTL -4	Analyzing	CO 3
16.(a)	Construct the canonical decomposition of 23!	BTL -4	Analyzing	CO 3
16.(b)	Use Euclidean algorithm to find the GCD of (3076, 1976). Also express the GCD as a linear combination of the given numbers	BTL -3	Applying	CO 3
17.	If $d = (a,b)$ then $(i)(a, a-b) = d$ (ii)For any integer x then $(a,b) = (a, b+ax)$	BTL -4	Analyzing	CO 3
18.(a)	Find [24,28,36,40]	BTL -3	Applying	CO 3
18.(b)	Use Euclidean algorithm to find the GCD of 42823, 6409.Also express the GCD as a linear combination of the given numbers	BTL -3	Applying	CO 3

	UNIT IV - DIOPHANTINE EQUATIONS AND CONGRUENCES Linear Diophantine equations – Congruence‘s – Linear Congruence‘s – Modular exponentiation– Chinese remainder theorem.			
Q.No.	**Question**	**BT Level**	**Competence**	**CO s**
	PART – A			
1.	Define linear Diophantine Equation in two variables.	BTL -1	Remembering	CO 4
2.	Discuss whether $6x + 8y = 25$ is solvable.	BTL -1	Remembering	CO 4
3.	Discuss whether 12x+18y=30 is solvable.	BTL -1	Remembering	CO 4
4.	Is 6x+12y+15z=10 solvable?	BTL -2	Understanding	CO 4
5.	Is 5x+11y+9z=6 solvable?	BTL -2	Applying	CO 4
6	Prove that $a \equiv b(mod\ m)$ if and only if $a = b + km\ for\ some\ integer\ k$.	BTL -1	Applying	CO 4
7.	Find the least residue of 23 modulo 5 , -3 modulo 5.	BTL -2	Understanding	CO 4
8	Define complete sets of residues modulo m.	BTL -2	Understanding	CO 4
9.	Find the Congruence classes modulo 5.	BTL -1	Applying	CO 4
10	Find the Congruence classes modulo 7.	BTL -2	Understanding	CO 4
11.	State Chinese Remainder Theorem.	BTL -2	Analyzing	CO 4
12.	Define Congruence and incongruence solution.	BTL -1	Applying	CO 4
13.	If $a \equiv b(mod\ m)$, then prove that $a^n \equiv b^n(mod\ m)$ for any positive integer n.	BTL -1	Analyzing	CO 4
14.	If $ac \equiv bc(mod\ m)$ and $(c\ , m) = 1\ , then\ \ a \equiv b(mod\ m)$.	BTL -2	Understanding	CO 4
15.	If $ac \equiv bc(mod\ m)$ and $(c\ , m) = d\ , then\ \ a \equiv b\left(mod\ \frac{m}{d}\right)$.	BTL -2	Analyzing	CO 4
16.	Determine whether the congruence $8x \equiv 10(mod\ 6)$ is solvable.	BTL -1	Remembering	CO 4
17.	Determine whether the congruence $2x \equiv 3(mod\ 4)$ is solvable.	BTL -1	Analyzing	CO 4
18.	Determine whether the congruence $4x \equiv 7(mod\ 5)$ is solvable.	BTL -2	Understanding	CO 4
19.	Determine whether the congruence $8x \equiv 10(mod6)$ is solvable.	BTL -2	Analyzing	CO 4
20.	Determine whether the congruence $5x \equiv 2(mod\ 3)$ is solvable.	BTL -1	Applying	CO 4
21.	Using Chinese Remainder theorem, determine whether the linear system is solvable $x \equiv 7(mod\ 9)\ ,\ x \equiv 11(mod\ 12)$.	BTL -2	Understanding	CO 4

22.	Using Chinese Remainder theorem, determine whether the linear system is solvable $x \equiv 3(mod\ 6)$, $x \equiv 5(mod\ 8)$.	BTL -1	Analyzing	CO 4
23.	Using Chinese Remainder theorem, determine whether the linear system is solvable $x \equiv 2(mod\ 10)$, $x \equiv 7(mod\ 15)$.	BTL -2	Applying	CO 4
24.	Using Chinese Remainder theorem, determine whether the linear system is solvable $x \equiv 5(mod\ 7)$, $x \equiv 8(mod\ 8)$.	BTL -2	Understanding	CO 4
25.	Using Chinese Remainder theorem, determine whether the linear system is solvable $x \equiv 9(mod\ 10)$, $x \equiv 10(mod\ 14)$.	BTL -1	Analyzing	CO 4
	PART B			
1.	Prove that the linear Diophantine equation $ax + by = c$ is solvable if and only if d/c, where $d = (a, b)$. If $x_0\ and\ y_0$ is a particular solution of the linear Diophantine equation , then all its solutions are given by $x = x_0 + \frac{dt}{d}, y = y_0 - \frac{at}{d}$ where t is an arbitrary integer	BTL -3	Applying	CO 4
2.	Solve the linear Diophantine equation $28x + 91y = 119$	BTL -4	Evaluating	CO 4
3.	Solve $1776x + 1976y = 4152$	BTL -3	Applying	CO 4
4.	Solve $93x - 81y = 3$	BTL -3	Applying	CO 4
5.	Find the general solution of the linear Diophantine equation $6x + 8y + 12z = 10$	BTL -3	Applying	CO 4
6.	Solve linear Diophantine equation is solvable $(i)1776x + 1976y = 4152$	BTL -3	Applying	CO 4
7.	Solve linear Diophantine equation is solvable $1076x + 2076y = 1155$	BTL -1	Remembering	CO 4
8.	Prove that (i) $a \equiv b(modm)$ if and only if $a = b + km$ for some integer k. (ii) Prove that the relation $' \equiv '$ (congruence) is an equivalence relation.	BTL -3	Applying	CO 4
9.	23 weary travelers entered the outskirts of a lush and beautiful forest. They found 63 equal heaps of plantains put together and seven single fruits are divided then equally. Find the number of fruits in each heap	BTL -4	Evaluating	CO 4
10.	A fruit basket contains apples and oranges. Each apple cost 65 Rs. Each orange cost 45Rs. For a total of 810 Rs. Find the minimum possible numbers of apple in the basket.	BTL -4	Creating	CO 4
11.	Prove that, let $a \equiv b(modm) and\ c \equiv d(modm)$ then $(i) a + c \equiv b + d(modm)$ $(ii) ac \equiv bd(modm)$ (iii) $a^n \equiv b^n(modm)$ for any positive integer n	BTL -4	Analyzing	CO 4
12.	Find the incongruent solutions of $28x \equiv 119(mod\ 91)$	BTL -3	Applying	CO 4
13.	State and prove Chinese remainder theorem.	BTL -4	Analyzing	CO 4
14.(a)	Verify that whether the number of prime of the form $4n + 3$ be expressed as the sum of two squares	BTL -3	Applying	CO 4

14.(b)	If a cock is worth five coins, a hen three coins and three chicks together one coin, how many cocks, hens and chicks, totally 100 can be bought for 100 coins	BTL -3	Applying	CO 4
15.	Find the incongruent solutions of $27x \equiv 117(mod\ 89)$.	BTL -4	Analyzing	CO 4
16.(a)	A child has some marbles in a box. If the marbles are grouped in sevens, there will be five left over; If they are grouped in elevens, there will be six left over; If they are grouped in thirteen , eight will be left over; Determine the latest number of marbles in the box	BTL -4	Analyzing	CO 4
16.(b)	Solve $12x + 16y = 18$.	BTL -3	Applying	CO 4
17.	If n is any integer then show that $(i) n^2 + n \equiv 0(mod2)$ $(ii) n^4 + 2n^3 + n^2 \equiv 0(mod4)$ $(iii) 2n^3 + 3n^2 + n \equiv 0(mod6)$	BTL -4	Analyzing	CO 4
18.(a)	Verify that whether the number of integer of the form $8n + 7$ be expressed as the sum of three squares.	BTL -3	Applying	CO 4
18.(b)	Solve $71x - 50y = 1$.	BTL -3	Applying	CO 4

	UNIT V - CLASSICAL THEOREMS AND MULTIPLICATIVE FUNCTIONS Wilson‘s theorem – Fermat‘s little theorem – Euler‘s theorem – statements – examples - Euler‘s Phi functions – Tau and Sigma functions.			
Q.No.	**Question**	**BT Level**	**Competence**	**CO s**
	PART – A			
1.	State Wilsons Theorem	BTL -1	Remembering	CO 5
2.	State Fermat’s Theorem	BTL -1	Remembering	CO 5
3.	State Euler’s Theorem	BTL -1	Remembering	CO 5
4.	Define Euler Phi Function	BTL -2	Understanding	CO 5
5.	Define Tau Function	BTL -1	Applying	CO 5
6	Define Sigma Function	BTL -1	Applying	CO 5
7.	Show that 11 is self invertible.	BTL -2	Understanding	CO 5
8	Evaluate $\frac{(np)!}{n!\,p^n}$ if n=46, p=5	BTL -2	Understanding	CO 5
9.	How many primes are there of the form m! + 1 when m≤ 100?	BTL -1	Applying	CO 5

10	Find the self-invertible least residue modulo each prime 7 and 19	BTL -2	Understanding	CO 5
11.	Solve $x^2 \equiv 1\ mod(6)$	BTL -2	Analyzing	CO 5
12.	Find the least residues of $1,2,\dots,p-1\ modulo\ 7$	BTL -1	Applying	CO 5
13.	Find $\emptyset\ (11)$.	BTL -1	Analyzing	CO 5
14.	Evaluate the inverse of 12 modulo 7	BTL -2	Understanding	CO 5
15.	Solve the linear congruence of 12x ≡ 6 (mod 7)	BTL -2	Analyzing	CO 5
16.	Solve the linear congruence of 24x ≡ 11 (mod 17)	BTL -1	Remembering	CO 5
17.	Find $\emptyset(18)$.	BTL -1	Analyzing	CO 5
18.	Solve the linear congruence of 35x ≡ 47 (mod 24)	BTL -2	Understanding	CO 5
19.	Define Multiplication Theorem.	BTL -2	Analyzing	CO 5
20.	Compute $\emptyset\ (7919)$	BTL -1	Applying	CO 5
21.	Compute $\emptyset\ (15{,}625)$	BTL -2	Understanding	CO 5
22.	Find the twin primes p and q if $\emptyset\ (pq) = 288$	BTL -2	Analyzing	CO 5
23.	Compute $\emptyset(47)$.	BTL -2	Applying	CO 5
24.	Compute $\sigma(97)$.	BTL -2	Understanding	CO 5
25.	Compute $\sigma(97)$.	BTL -1	Remembering	CO 5
	PART – B			
1.	Apply Wilson's theorem to find the reminder of (i) 51! When divided by 91 (ii) 67! When divided by 71.	BTL -3	Applying	CO 5
2.(a)	Find the reminder of 13! When divided by 19	BTL -4	Evaluating	CO 5
2.(b)	Find the remainder when 7^{1001} is divided by 17	BTL -4	Evaluating	CO 5
3.	Verify $(p-1)! \equiv -1(modp)$, when p=13 (i)Without using Wilson's theorem (ii)Using Wilson's theorem	BTL -3	Applying	CO 5
4.(a)	Find the reminder of 17! When divided by 23	BTL -3	Applying	CO 5
4.(b)	Find the remainder when 24^{1947} is divided by 17	BTL -3	Applying	CO 5
5.	Find $\emptyset(1976)$ and $\sigma\ (496)$	BTL -3	Applying	CO 5

6.(a)	If n is a positive integer such that$(n-1)! \equiv -1(modp)$	BTL -3	Applying	CO 5
6.(b)	Find the remainder when 15^{1976} is divided by 23	BTL -3	Applying	CO 5
7.	Create the reminder of (i)55^{1876} when divided by 12 (ii)25^{2550} when divided by 18	BTL -1	Remembering	CO 5
8.(a)	Evaluate the linear congruence equations $12x \equiv 6(mod\ 7)$ using Fermat's little theorem	BTL -4	Analyzing	CO 5
8.(b)	Find the remainder when 31^{1706} is divided by 23	BTL -3	Applying	CO 5
9.(a)	Evaluate the linear congruence equations $8x \equiv 3(mod\ 11)$ using Fermat's little theorem.	BTL -4	Evaluating	CO 5
9.(b)	Compute (i) $\emptyset(7919), \emptyset(666)$ using Euler phi function	BTL -4	Evaluating	CO 5
10.(a)	Solve the linear congruence $5x \equiv 3(mod\ 24)$	BTL -4	Creating	CO 5
10.(b)	Compute $\tau(6491), \tau(2187), \tau(44982)$ using Euler phi function	BTL -3	Applying	CO 5
11.	Evaluate $\tau\ (n) and\ \sigma\ (n)$ for each $n = 43, 1560, 44982$	BTL -4	Analyzing	CO 5
12.	Find the remainder when 245^{1040} is divided by 18 and find the remainder when 7^{1020} is divided by 15.	BTL -3	Creating	CO 5
13.(a)	Compute $\tau(6120)\ and\ \sigma\ (6120)$.	BTL -4	Analyzing	CO 5
13.(b)	Apply Wilson's theorem to find the reminder of (i) 52! When divided by 71 (ii) 76! When divided by 51 .	BTL -3	Applying	CO 5
14.	Compute $\sigma(331), \sigma(1024), \sigma(2187)$ using Euler phi function.	BTL -3	Applying	CO 5
15.	Evaluate the remainder when 199^{2020} is divided by 28 and the remainder when 79^{1776} is divided by 24	BTL -4	Analyzing	CO 5
16.(a)	Find the remainder when $35^{32} + 51^{24}$ is divisible by 1785	BTL -4	Analyzing	CO 5
16.(b)	Compute $\tau(36)\ and\ \sigma\ (36)$	BTL -3	Applying	CO 5
17.(a)	Compute $\tau(1560), \tau(6120)$	BTL -4	Analyzing	CO 5
17.(b)	Using Euler's Theorem, evaluate the ones digit in the decimal value of each (i) 17^{666} $(ii)23^{7777}$	BTL -4	Analyzing	CO 5
18.(a)	Compute □(81), □(2187)	BTL -3	Applying	CO 5
18.(b)	Solve the linear congruence $15x \equiv 7(mod13)$	BTL -3	Applying	CO 5