SRM VALLIAMMAI ENGINEERING COLLEGE
(An Autonomous Institution)

S.R.M. Nagar, Kattankulathur - 603203

DEPARTMENT OF MATHEMATICS

QUESTION BANK

IV SEMESTER
(Common to MDE)

MA3427 - APPLIED MATHEMATICS FOR BIO-MEDICAL ENGINEERING
Regulation – 2023

Academic Year – 2024 – 2025

Prepared by

Dr. Bhooma S, Assistant Professor / Mathematics

SRM VALLIAMMAI ENGNIEERING COLLEGE
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DEPARTMENT OF MATHEMATICS

SUBJECT: MA3427 – APPLIED MATHEMATICS FOR BIOMEDICAL ENGINEERING

SEM / YEAR: IV / II year B.E. (Common to MDE)

<table>
<tr><td colspan="5">UNIT I - PROBABILITY AND RANDOM VARIABLES
Axioms of probability – Conditional Probability-Discrete and continuous random variables – Moments – Moment generating functions</td></tr>
<tr><th>Q.No.</th><th>Question</th><th>BT Level</th><th>Competence</th><th>Course Outcome</th></tr>
<tr><td colspan="5">PART – A</td></tr>
<tr><td>1.</td><td>Define Probability.</td><td>BTL-1</td><td>Remembering</td><td>CO 1</td></tr>
<tr><td>2.</td><td>Write the axioms of Probability.</td><td>BTL-1</td><td>Remembering</td><td>CO 1</td></tr>
<tr><td>3.</td><td>What is the probability that a non-leap year selected at random will contain 53 Tuesdays?</td><td>BTL-1</td><td>Remembering</td><td>CO 1</td></tr>
<tr><td>4.</td><td>If A and B are events in S such that $P(A) = 1/3$, $P(B) = 1/4$ and $P(A \cup B) = 1/2$. Find $P(A \cap \overline{B})$ and $P(A|\overline{B})$.</td><td>BTL-2</td><td>Understanding</td><td>CO 1</td></tr>
<tr><td>5.</td><td>Define Moment Generating function of a random variable.</td><td>BTL-1</td><td>Remembering</td><td>CO 1</td></tr>
<tr><td>6.</td><td>If a random variable X has the MGF $M_X(t) = \frac{2}{2-t}$. Find the mean of X.</td><td>BTL-2</td><td>Understanding</td><td>CO 1</td></tr>
<tr><td>7.</td><td>Show that the function $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ is a probability density function of a continuous random variable X.</td><td>BTL-2</td><td>Understanding</td><td>CO 1</td></tr>
<tr><td>8.</td><td>Find the Moment generating function of a continuous random variable X whose pdf is $f(x) = \begin{cases} xe^{-x/2}, & x > 0 \\ 0, & x \leq 0 \end{cases}$</td><td>BTL-2</td><td>Understanding</td><td>CO 1</td></tr>
<tr><td>9.</td><td>Define discrete and continuous random variables with examples</td><td>BTL-1</td><td>Remembering</td><td>CO 1</td></tr>
<tr><td>10.</td><td>The number of hardware failures of a computer system in a week of operations has the following p.d.f, Calculate the value of K.<table><tr><td>No. of failures</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Probability</td><td>K</td><td>2 K</td><td>2 K</td><td>K</td><td>3 K</td><td>K</td><td>4 K</td></tr></table></td><td>BTL-2</td><td>Understanding</td><td>CO 1</td></tr>
<tr><td>11.</td><td>A random variable X has following probability distribution.<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr></table></td><td>BTL-2</td><td>Understanding</td><td>CO 1</td></tr>
</table>

	P(x) | 0.4K | 0.3K | 0.2K | 0.1K Find K.			
12.	The pdf of a continuous random variable X is $f(x) = k(1+x), 2 < x < 5$, Find *k*.	BTL-2	Understanding	CO 1
13.	For a continuous distribution $f(x) = k(x - x^2), 0 \leq x \leq 1$, where k is a constant. Find k.	BTL-2	Understanding	CO 1
14.	If $f(x) = kx^2, 0 < x < 3$, is to be a density function, find the value of k.	BTL-2	Understanding	CO 1
15.	If the pdf of a RV is $f(x) = \frac{x}{2}, 0 \leq x \leq 2$, find $P(X > 1.5)$.	BTL-2	Understanding	CO 1
16.	If X is a CRV with p.d.f. $f(x) = 2x, 0 < x < 1$, then find the pdf of the RV $Y = 8X^3$.	BTL-2	Understanding	CO 1
17.	If X and Y are independent RVs with variances 2 and 3 .Find the variance of 3X + 4Y.	BTL-2	Understanding	CO 1
18.	If the RV X takes the values 1, 2, 3, 4 such that 2P(X =1)=3P(X =2) =P (X =3) = 5 P (X =4), find the probability distribution.	BTL-2	Understanding	CO 1
19.	The first four moments of a distribution about 4 are 1, 4, 10 and 45 respectively. Show that the mean is 5 and variance is 3.	BTL-2	Understanding	CO 1
20.	If X is a normal random variable with mean 3 and variance 9, find the probability that X lies between 2 and 5.	BTL-2	Understanding	CO 1
21.	If a RV has the pdf $f(x) = \begin{cases} 2e^{-2x} & \text{for } x > 0 \\ 0, & \text{for } x \leq 0 \end{cases}$, find the mean of X	BTL-1	Remembering	CO 1
22.	A continuous random variable X has p.d.f $f(x) = 2x$, $0 \leq x \leq 1$. Find $(X > 0.5)$.	BTL-2	Understanding	CO 1
23.	Check whether the function given by $f(x) = \frac{x+2}{25}$ for x=1, 2, 3, 4, 5 can serve as the probability distribution of a discrete random variable.	BTL-1	Remembering	CO 1
24.	If a RV has the probability density $f(x) = \begin{cases} 2e^{-2x} & \text{for } x > 0 \\ 0, & \text{for } x \leq 0 \end{cases}$, find the probabilities that will take a value between 1 and 3.	BTL-2	Understanding	CO 1
25.	If the MGF of a continuous RV is $\frac{1}{t}(e^{5t} - e^{4t})$ what is the distribution of X? What are the mean and variance of X?	BTL-2	Understanding	CO 1
	PART – B			
1.	The Probability distribution function of a R.V. X is given by $f(x) = \frac{4x(9-x^2)}{81}$, $0 \leq x \leq 3$. Find the mean, variance	BTL-3	Applying	CO 1
2.	Find the MGF, mean and variance of the random variable X which has the pdf	BTL-3	Applying	CO 1

<table>
<tr><td></td><td>$$f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & otherwise \end{cases}$$</td><td></td><td></td><td></td></tr>
<tr><td>3.</td><td>A random variable X has the following probability distribution:
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>P(X)</td><td>0</td><td>k</td><td>$2k$</td><td>$2k$</td><td>$3k$</td><td>k^2</td><td>$2k^2$</td><td>$7k^2$+k</td></tr>
</table>
Find (i) the value of k
(ii) $P(1.5 < X < 4.5 / X > 2)$</td><td>BTL-3</td><td>Applying</td><td>CO 1</td></tr>
<tr><td>4.</td><td>The probability mass function of a discrete R. V X is given in the following table:
<table>
<tr><td>X</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P(X=x)</td><td>0.1</td><td>k</td><td>0.2</td><td>$2k$</td><td>0.3</td><td>k</td></tr>
</table>
(1) Find the value of k, (2) P(X<1), (3) P(-1< X ≤ 2), (4) E(X)</td><td>BTL-3</td><td>Applying</td><td>CO 1</td></tr>
<tr><td>5.</td><td>A test engineer discovered that the CDF of the lifetime of an equipment in years is given by $F_X(x)=\begin{cases} 0, & x<0 \\ 1-e^{-\frac{x}{5}}, & 0\le x<\infty \end{cases}$.
(i) What is the expected lifetime of the equipment? (ii) What is the variance of the lifetime of the equipment?</td><td>BTL-3</td><td>Applying</td><td>CO 1</td></tr>
<tr><td>6.</td><td>A student doing a summer internship in a company was asked to model the lifetime of certain equipment that the company makes. After a series of tests, the student proposed that the lifetime of the equipment can be modeled by a random variable X that has the PDF $f(x)=\begin{cases} \frac{xe^{-x/10}}{100}, & x\ge 0 \\ 0, & otherwise \end{cases}$. (i) Show that $f(x)$ is a valid PDF. (ii) What is the probability that the lifetime of the equipment exceeds 20? (iii) What is the expected value of X?</td><td>BTL-3</td><td>Applying</td><td>CO 1</td></tr>
<tr><td>7.</td><td>The probability mass function of a discrete R. V X is given in the following table
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>P(X)</td><td>a</td><td>3a</td><td>5a</td><td>7a</td><td>9a</td><td>11a</td><td>13a</td><td>15a</td><td>17a</td></tr>
</table>
Find (i) the value of a , (ii) $P(X < 3)$, (iii) Mean of X, (iv) Variance of X.</td><td>BTL-3</td><td>Applying</td><td>CO 1</td></tr>
<tr><td>8.</td><td>The probability mass function of a RV X is given by $P(X=r)=kr^3, r=1,2,3,4$. Find (1) the value of k, (2) $P(\frac{1}{2}<X<\frac{5}{2}/X>1)$, (3) Mean and (4) Variance.</td><td>BTL-3</td><td>Applying</td><td>CO 1</td></tr>
<tr><td>9.</td><td>If f(x) = $\begin{cases} ax, & 0\le x\le 1 \\ a, & 1\le x\le 2 \\ 3a-ax, & 2\le x\le 3 \\ 0, & elsewhere \end{cases}$ is the pdf of X.
Calculate (i) the value of a , (ii) the cumulative distribution function of X</td><td>BTL -4</td><td>Analyzing</td><td>CO 1</td></tr>
<tr><td>10(a).</td><td>Two events A and B are such that $P[A\cap B]$ = 0.15, $P[A\cup B]$ = 0.65, and $P[A|B]$ = 0.5. Find $P[B|A]$.</td><td>BTL-3</td><td>Applying</td><td>CO 1</td></tr>
<tr><td>10(b).</td><td>If the discrete random variable X has the probability function given by the table.</td><td>BTL-3</td><td>Applying</td><td>CO 1</td></tr>
</table>

	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(x)$</td><td>$k/3$</td><td>$k/6$</td><td>$k/3$</td><td>$k/6$</td></tr></table> Find the value of k and Cumulative distribution of X.			
11.	Let X be a continuous R.V with probability density function $f(x)=\begin{cases} xe^{-x}, & x>0 \\ 0, & otherwise \end{cases}$, Find (1) The cumulative distribution of X, (2) Moment Generating Function $M_X(t)$ of X, (3) P(X<2) and (4) E(X)	BTL-4	Analyzing	CO 1
12.	The probability distribution of an infinite discrete distribution is given by P[X = j] = $\frac{1}{2^j}$ (j = 1,2,3…) Find (1) Mean of X, (2) P [X is even], (3) P(X is odd).	BTL-4	Analyzing	CO 1
13.	The probability density function of a random variable X is given by $f(x)=\begin{cases} x, & 0<x<1 \\ k(2-x), & 1\le x\le 2 \\ 0, & otherwise \end{cases}$ (i) Find the value of k (ii) P (0.2<x<1.2) (iii) What is P[0.5 < x < 1.5 | x ≥1] (iv) Find the distribution function of f(x).	BTL -4	Analyzing	CO 1
14.	If X is a discrete random variable with probability function p(x) = $\frac{1}{K^x}$, x =1,2 ... (K constant) then find the moment generating function ,mean and variance.	BTL -3	Applying	CO 1
15.	Let the random variable X has the p.d.f. f(x) = $\begin{cases} \frac{1}{2}e^{-\frac{x}{2}}, & x>0 \\ 0. & otherwise \end{cases}$ Find the mean and variance.	BTL -3	Applying	CO 1
16.	Find the mean & variance of the probability distribution <table><tr><td>X_i</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>P_i</td><td>0.08</td><td>0.12</td><td>0.19</td><td>0.24</td><td>0.16</td><td>0.10</td><td>0.07</td><td>0.04</td></tr></table>	BTL -3	Applying	CO 1
17(a).	Two events A and B have the following probabilities: $P[A]$ = 1/4, $P[B\|A]$ = 1/2, and $P[A\|B]$ = 1/3. Compute (a) $P[A \cap B]$, (b) $P[B]$, and (c) $P[A \cup B]$.	BTL -3	Applying	CO 1
17(b).	Two events A and B have the following probabilities: $P[A]$ = 0.6, $P[B]$ = 0.7, and $P[A \cap B] = p$. Find the range of values that p can take.	BTL -3	Applying	CO 1
18(a).	If a random variable X has p.d.f $f(x)=\begin{cases} \frac{1}{4}, & \|X\|<2 \\ 0, & Otherwise \end{cases}$ Find (a) $P(X<1)$ (b) $P(\|X\|>1)$ (c) $P(2X+3>5)$.	BTL -3	Applying	CO 1
18(b).	If X is a continuous r.v. with p.d.f $f(x)=\begin{cases} A(2x-x^2), & 0<x<2 \\ 0, & Otherwise \end{cases}$ Find A and P(X>1).	BTL -3	Applying	CO 1

UNIT-II TWO - DIMENSIONAL RANDOM VARIABLES

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and linear regression

<table>
<tr><th>Q.No.</th><th>Question</th><th>BT Level</th><th>Competence</th><th>Course Outcome</th></tr>
<tr><td colspan="4">PART – A</td><td></td></tr>
<tr><td>1.</td><td>The joint probability distribution of X and Y is given by
$p(x,y)=\frac{x+y}{21}$,
x = 1,2,3; y = 1, 2. Find the marginal probability distributions of X and Y.</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>2.</td><td>The joint probability function (X,Y) is given by $P(x,y) = k(2x+3y)$,
$x = 0,1,2 \;\; y = 1,2,3,$ Find the value of K.</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>3.</td><td>Find the probability distribution of X + Y from the bivariate distribution of (X,Y) given below:
<table>
<tr><th>X \ Y</th><th>1</th><th>2</th></tr>
<tr><td>1</td><td>0.4</td><td>0.2</td></tr>
<tr><td>2</td><td>0.3</td><td>0.1</td></tr>
</table></td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>4.</td><td>If the joint pdf of X and Y is given by f(x,y)=2,in $0 \le x < y \le 1$, Find E(XY)</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>5.</td><td>Find the marginal distributions of X and Y from the bivariate distribution of (X,Y) given below:
<table>
<tr><th>X \ Y</th><th>1</th><th>2</th></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td></tr>
<tr><td>2</td><td>0.3</td><td>0.4</td></tr>
</table></td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>6.</td><td>Find the value of k,if the joint density function of (X,Y) is
$f(x,y)=\begin{cases} k(1-x)(1-y), 0<x<4, 1<y<5 \\ 0, otherwise \end{cases}$</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>7.</td><td>If the joint probability density function of a random variable X and Y is given by $f(x,y) = \begin{cases} \frac{x^3y^3}{16}, 0<x<2, 0<y<2 \\ 0, \quad otherwise \end{cases}$.
Obtain the marginal density function of X.</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>8.</td><td>The joint pdf of the random variable (X,Y) is given by $f(x,y)=Kxye^{-(x^2+y^2)}, x>0, y>0$ Find the value of K.</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>9.</td><td>The joint probability density function of a random variable (X,Y) is $f(x,y)=k\,e^{-(2x+3y)}, x \ge 0, \; y \ge 0$. Point out the value of k.</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>10.</td><td>If the joint pdf of (X, Y) is $f(x,y) = \begin{cases} \frac{1}{4}, 0<x,y<2 \\ 0, \;\; otherwise \end{cases}$.
Find $P(X+Y \le 1)$</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>11.</td><td>Let X and Y be random variables with joint density function
f(x,y) $=\begin{cases} 4xy, \;\; 0<x<1, \; 0<y<1 \\ 0, \; otherwise \end{cases}$ formulate the value of E(XY)</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
</table>

<table>
<tr><td>12.</td><td>Let the joint density function of a random variable X and Y be given by $f(x,y) = 8xy,\ 0 < y \le x \le 1$.Calculate the marginal probability function of X</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>13.</td><td>What is the condition for two random variables are independent?</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>14.</td><td>If the joint probability density function of X and Y is $f(x,y) = e^{-(x+y)}, x, y \ge 0$.Are X and Y independent</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>15.</td><td>State any two properties of correlation coefficient</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>16.</td><td>Write the angle between the regression lines</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>17.</td><td>The regression equations are x + 6y = 14 and 2x + 3y = 1. Evaluate the correlation coefficient between X & Y .</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>18.</td><td>If $\bar{X} = 970$, $\bar{Y} = 18, \sigma_x = 38$, $\sigma_y = 2$ and r = 0.6 , Find the line of regression of X on Y.</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>19.</td><td>In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible; Variance of X = 9; Regression equations are 8X - 10Y + 66 = 0 and 40X-18Y = 214. Find the mean values of X and Y?</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>20.</td><td>The regression equations are 3x + 2y = 26 and 6x + y = 31. Find the correlation coefficient.</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td>21.</td><td>Let X and Y have the joint p.m.f
<table>
<tr><td>Y \ X</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>0</td><td>0.1</td><td>0.4</td><td>0.1</td></tr>
<tr><td>1</td><td>0.2</td><td>0.2</td><td>0</td></tr>
</table>
Find P(X+Y > 1) .</td><td>BTL -1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>22.</td><td>Define the conditional distribution function of two dimensional discrete and continuous random variables.</td><td>BTL -1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>23.</td><td>The equations of two regression lines are 3x+2y=19 and 3y+9x=46. Obtain the mean of X and Y.</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>24.</td><td>The equations of two regression lines are 3x+2y=19 and 3y+9x=46. Derive the correlation coefficient between X and Y.</td><td>BTL-1</td><td>Remembering</td><td>CO 2</td></tr>
<tr><td>25.</td><td>State the equations of two regression lines.</td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
<tr><td colspan="4">PART – B</td><td></td></tr>
<tr><td>1.</td><td>From the following table for bivariate distribution of (X, Y). Find (i) $P(X \le 1)$ (ii)$P(Y \le 3)$ (iii) $P(X \le 1, Y \le 3)$ (iv)$P(X \le 1/ Y \le 3)$ (v) $P(Y \le 3/ X \le 1)$ (vi)$P(X + Y \le 4)$
<table>
<tr><td>X \ Y</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>$\frac{1}{32}$</td><td>$\frac{2}{32}$</td><td>$\frac{2}{32}$</td></tr>
<tr><td>1</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{8}$</td><td>$\frac{1}{8}$</td><td>$\frac{1}{8}$</td></tr>
</table></td><td>BTL-2</td><td>Understanding</td><td>CO 2</td></tr>
</table>

	<table><tr><td>2</td><td>$\frac{1}{32}$</td><td>$\frac{1}{32}$</td><td>$\frac{1}{64}$</td><td>$\frac{1}{64}$</td><td>0</td><td>$\frac{2}{64}$</td></tr></table>				
2.(a)	The two dimensional random variable (X, Y) has the joint probability mass function $f(x,y) = \frac{x+2y}{27}, x = 0,1,2; y = 0,1,2$. Find the marginal distributions of X and Y. Also find the conditional distribution of Y given X = 1 also find the conditional distribution of X given Y = 1.	BTL-3	Applying	CO 2
2.(b)	The joint pdf a bivariate R.V(X, Y) is given by $f(x,y) = \begin{cases} Kxy, & 0 < x < 1, 0 < y < 1 \\ 0, & otherwise \end{cases}$ (1) Find K. (2) Find P(X+Y<1). (3) Are X and Y independent R.V's.	BTL-3	Applying	CO 2
3.(a)	If the joint pdf of (X, Y) is given by P(x, y) = K(2x+3y), x=0, 1, 2, 3, y = 1, 2, 3 Find all the marginal probability distribution. Also find the probability distribution of X+Y.	BTL-3	Applying	CO 2
3.(b)	The joint pdf of the RV (X,Y) is given by $f(x,y) = kxye^{-(x^2+y^2)}$, $x > 0, y > 0$. Find the value of k. Also prove that X and Y are independent	BTL-4	Analyzing	CO 2
4.	The following table represents the joint probability distribution of the discrete RV (X,Y). Find all the marginal and conditional distributions.<table><tr><td rowspan="2">Y</td><td colspan="3">X</td></tr><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>1</td><td>1/2</td><td>1/6</td><td>0</td></tr><tr><td>2</td><td>0</td><td>1/9</td><td>1/5</td></tr><tr><td>3</td><td>1/18</td><td>1/4</td><td>2/15</td></tr></table>	BTL-2	Understanding	CO 2
5.	Find the marginal distribution of X and Y and also P($P(X \leq 1, Y \leq 1)$, $P(X \leq 1), P(Y \leq 1)$. Check whether X and Y are independent. The joint probability mass function of X and Y is<table><tr><td>Y / X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>0</td><td>0.10</td><td>0.04</td><td>0.02</td></tr><tr><td>1</td><td>0.08</td><td>0.20</td><td>0.06</td></tr><tr><td>2</td><td>0.06</td><td>0.14</td><td>.030</td></tr></table>	BTL-2	Understanding	CO 2
6.	The joint pdf of two dimensional random variables (X,Y) is given by $f(x,y) = \begin{cases} 25e^{-5y}, 0 < x < 0.2, y > 0 \\ 0, otherwise \end{cases}$ Find the covariance of x and y.	BTL-4	Analyzing	CO 2

7.	If the joint pdf of a two-dimensional RV(X,Y) is given by $f(x,y)=\begin{cases} x^2+\frac{xy}{3}; 0<x<1, 0<y<2 \\ 0, \; elsewhere \end{cases}$. Find (i) $P\left(X>\frac{1}{2}\right)$ (ii) $P(Y<\frac{1}{2}, X<\frac{1}{2})$ (iii) $P\left(Y<\frac{1}{2} / X<\frac{1}{2}\right)$	BTL-3	Applying	CO 2
8.	The joint pdf of a two dimensional random variable (X, Y) is given by $f(x,y)=xy^2+\frac{x^2}{8}, 0\le x\le 2, 0\le y\le 1$. Compute (i) $P\left(X>1 / Y<\frac{1}{2}\right)$ (ii) $P\left(Y<\frac{1}{2}/X>1\right)$ (iii) $P(X+Y)\le 1$.	BTL-3	Applying	CO 2
9.	(b)The joint pdf of X and Y is given by f(x,y)= $\begin{cases} kx(x-y), 0<x<2, -x<y<x \\ 0, \quad otherwise \end{cases}$ (i)Find K (ii) Find $f_x(x)$ and $f_y(y)$	BTL-3	Applying	CO 2
10.	Find the Coefficient of Correlation between industrial production and export using the following table Production (X): 14, 17, 23, 21, 25 Export (Y): 10, 12, 15, 20, 23	BTL-2	Understanding	CO 2
11.	Find the correlation coefficient for the following heights of fathers X,their sons Y and also find the equations of regression lines. Hence find the height of son when the height of father is 71 X: 65, 66, 67, 67, 68, 69, 70, 72 Y: 67, 68, 65, 68, 72, 72, 69, 71	BTL-2	Understanding	CO 2
12.	Obtain the lines of regression X: 50, 55, 50, 60, 65, 65, 65, 60, 60 Y: 11, 14, 13, 16, 16, 15, 15, 14, 13	BTL-2	Understanding	CO 2
13.	If $f(x,y)=\frac{6-x-y}{8}$, $0\le x\le 2$, , $2\le y\le 4$ for a bivariate random variable (X,Y), Evaluate the correlation coefficient ρ.	BTL-3	Applying	CO 2
14.	Two random variables X and Y have the joint density function $f(x,y)=x+y, 0\le x\le 1, 0\le y\le 1$. Evaluate the Correlation coefficient between X and Y.	BTL-4	Analyzing	CO 2
15	The two regression lines are 4x-5y+33=0 and 20x-9y=107. Find the mean of X and Y. Also find the correlation coefficient between them	BTL-3	Applying	CO 2
16.	From the following data , Find (i)The two regression equations (ii) The coefficient of correlation between the marks in Mathematics and Statistics (iii) The most likely marks in Statistics when marks in Mathematics are 30 Marks in Maths :25 28 35 32 31 36 29 38 34 32 Marks in Statistics: 43 46 49 41 36 32 31 30 33 39	BTL-4	Analyzing	CO 2

17.(a)	If X and Y independent Random Variables with pdf $e^{-x}, x \ge 0$ and $e^{-y}, y \ge 0$. Devise the density function of $U = \frac{X}{X+Y}$ and $V = X+Y$. Are they independent?	BTL-4	Analyzing	CO 2
17.(b)	Two random variables X and Y have the following joint probability density function $f(x,y) = \begin{cases} x+y; 0 \le x \le 1, 0 \le y \le 1 \\ 0, \ otherwise \end{cases}$. Find the probability density function of the random variable U = XY.	BTL-3	Applying	CO 2
18	Out of the two lines of regression given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$, which one is the regression line of X on Y? Analyze the equations to find the means of X and Y. If the variance of X is 12, find the variance of Y.	BTL-4	Analyzing	CO 2

UNIT-III: RANDOM PROCESSES

Classification – Stationary process – Markov chain- Poisson process

Q.No.	Question	BT Level	Competence	Course Outcome
	PART – A			
1.	What are the four types of a stochastic process?	BTL-1	Remembering	CO3
2.	Define Discrete Random sequence with example.	BTL-1	Remembering	CO3
3.	Define Discrete Random Process with example.	BTL-1	Remembering	CO3
4.	Define Continuous Random sequence with example.	BTL-1	Remembering	CO3
5.	Define Continuous Random Process with example.	BTL-1	Remembering	CO3
6.	Define wide sense stationary process. .	BTL-1	Remembering	CO3
7.	Define Strict Sense Stationary Process.	BTL-1	Remembering	CO3
8.	Show that the random process $X(t) = A\cos(\omega_c t + \theta)$ is not stationary if it is assumed that A and ω_c are constants and θ is a uniformly distributed variable on the interval (0,π).	BTL-2	Understanding	CO3
9.	A random process X (t) = A sin *t* + B cos *t* where A and B are independent random variables with zero means and equal standard deviations. Find the mean of the process.	BTL-1	Remembering	CO3
10.	Consider the random process X (*t*) = *cos* (t + ϕ), where ϕ is uniform random variable in $\left(-\pi/2, \pi/2\right)$. Check whether the process is stationary.	BTL-2	Understanding	CO3
11.	Consider the random process X (*t*) = *cos* (ω_0t + θ), where θ is uniform random variable in $(-\pi, \pi)$. Check whether the process is stationary or not	BTL-1	Remembering	CO3

12.	Find the mean of a stationary random process whose auto correlation function is given by $R_{(Z)} = \frac{25Z^2 + 36}{6.25Z^2 + 4}$.	BTL-2	Understanding	CO3
13.	Find the mean of a stationary random process whose auto correlation function is given by $R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2}$.	BTL-2	Understanding	CO3
14.	A random process has the autocorrelation function $R_{xx}(\tau) = \frac{4\tau^2+6}{\tau^2+1}$, find the mean square value of the problem.	BTL-2	Understanding	CO3
15.	Compute the mean value of the random process whose auto correlation function is given by $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$.	BTL-2	Understanding	CO3
16.	Define Poisson process.	BTL-1	Remembering	CO3
17.	State and two properties of Poisson process.	BTL-1	Remembering	CO3
18.	Check whether the Poisson process $X(t)$ given by the probability law $P\{X(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, n = 0,1,2,\cdots$ is stationary or not.	BTL-1	Remembering	CO3
19.	A hospital receives on an average of 3 emergency calls in 10 minutes interval. What is the probability that there are 3 emergency calls in a 10 minute interval	BTL-2	Understanding	CO3
20.	Define Markov chain	BTL-1	Remembering	CO3
21.	A random process is defined by $X(t) = K\cos\omega t, t \geq 0$ where ω is a constant and K is uniformly distributed between 0 and 2. Determine $E[X(t)]$.	BTL-1	Remembering	CO3
22.	Consider the Markov chain with 2 states and transition probability matrix $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Find the stationary probabilities of the chain.	BTL-1	Remembering	CO3
23.	The one-step transition probability matrix of a Markov chain with states (0,1) is given by $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Evaluate whether it is irreducible Markov chain?	BTL-2	Understanding	CO3
24.	Compute the variance of the random process $X(t)$ whose autocorrelation function is given by $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$.	BTL-1	Remembering	CO3
25.	Check whether the Markov chain with transition probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ is irreducible or not?	BTL-2	Understanding	CO3
	PART – B			
1.	The process {X(t)} whose probability distribution under certain conditions is given by	BTL-3	Applying	CO 3

	$P\{X(t)=n\}=\begin{cases}\frac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2\\ \frac{at}{(1+at)}, & n=0\end{cases}$. Show that it is not stationary.			
2.(a)	A radioactive source emits particles at a rate of 5 per minute in accordance with poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 partcles are recorded in 4 minute period	BTL-3	Applying	CO 3
2.(b)	Find the mean and autocorrelation of the Poisson processes	BTL-3	Applying	CO 3
3.(a)	If the random process {X(t)} takes the value -1 with probability 1/3 and takes the value +1 with probability 2/3, find whether {X(t)} is a stationary process or not.	BTL-3	Applying	CO 3
3.(b)	Prove that the sum of two independent Poisson process is a Poisson process.	BTL-3	Applying	CO 3
4.(a)	Consider a random process $X(t) = B\ cos\ (50\ t + \Phi)$ where B and Φ are independent random variables. B is a random variable with mean 0 and variance 1.Φ is uniformly distributed in the interval [-π,π]. Determine the mean and auto correlation of the process.	BTL-3	Applying	CO 3
4.(b)	Prove that the difference of two independent Poisson process is not a Poisson process.	BTL-3	Applying	CO 3
5.(a)	Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide sense stationary, if A and ω are constant and θ is a uniformly distributed random variable in (0, 2π).	BTL-3	Applying	CO 3
5.(b)	A fisherman catches a fish at a poisson rate of 2 per hour from a large lake with lots of fish. If he starts fishing at 10 am. What is the probability that he catches one fish by 10.30 am and three fishes by noon.	BTL-4	Analyzing	CO 3
6.(a)	Suppose that customers arrive at a bank according to poisson process with mean rate of 3 per minute. Find the probability that during a time of two minutes (1) Exactly 4 customers arrive (2) Greater than 4 customers arrive (3) Fewer than 4 customers arrive	BTL-3	Applying	CO 3
6.(b)	Prove that the inter arrival time of the Poisson process follows exponential distribution	BTL-3	Applying	CO 3
7.	Show that the random process $X(t) = Acos\omega t + Bsin\omega t$ is wide sense stationary process if A and B are random variables such that $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$and $E(AB) = 0$	BTL-3	Applying	CO 3
8.	A salesman's territory consists of three regions A, B, C. He never sells in the same region on successive days. If he sells in region A, then the next day he sells in B. However, if he sells either B or C, then the next day he is twice as likely to sell in A as in the other region. Explain how often does he sell in each of the regions in the steady state?	BTL-3	Applying	CO 3

9.	There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. The state of the related markov chain is the number of red marbles in urn A after the interchange. What is the probability that there are 2 red marbles in urn A after the interchange? What is the probability that there are 2 red marbles in urn A after 3 steps? In the long run, what is the probability that there are 2 red marbles in urn A	BTL-3	Applying	CO 3
10.(a)	A hard disk fails in a computer system and it follows a poisson distribution with mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If we have extra hard disks and the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in next 10 weeks.	BTL-4	Analyzing	CO 3
10.(b)	The probability of a dry day following a rainy day is 1/3 and that the probability of a rainy day following a dry day is ½. Given that May 1st is a dry day. Obtain the probability that May 3rd is a dry day also May 5th is a dry day	BTL-3	Applying	CO 3
11.	A fair die is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, Evaluate the transition probability matrix P of the Markov chain $\{X_n\}$. Find also $P\{X_2 = 6\}$ and P^2.	BTL-4	Analyzing	CO 3
12.	The transition probability matrix of a Markov chain $\{X_n\}$, n = 1, 2, 3, … having 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is P(0) = (0.7, 0.2, 0.1). Evaluate i) $P(X_2 = 3)$ ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$	BTL-3	Applying	CO 3
13.	Consider the Markov chain $\{X_n, n= 0, 1, 2, 3, \ldots\}$ having 3 states space S={1,2,3} and one step TPM $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ and initial probability distribution $P(X_0=i)=1/3$, i= 1, 2, 3. Compute (1) $P(X_3=2.\ X_2=1, X_1=2/X_0=1)$ (2) $P(X_3=2, X_2=1/X_1=2, X_0=1)$ (3) $P(X_2=2/X_0=2)$ (4) Invariant Probabilities of the Markov Chain.	BTL-3	Applying	CO 3
14.(a)	Let $\{X_n : n = 1, 2, 3, \ldots\}$ be a Markov chain on the space S = {1,2,3} with one step t.p.m $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$ 1. Sketch the transition diagram, 2. Is the chain irreducible? Explain.	BTL-3	Applying	CO 3

14.(b)	If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, Evaluate the probability that the interval between 2 consecutive arrivals is (a) more than 1 minute, (b) between 1 minute and 2 minutes and (c) 4 minutes or less	BTL-4	Analyzing	CO 3
15.	Three boys A, B and C are throwing a ball to each other. *A* always throws the ball to *B* and *B* always throws the ball to *C* but *C* is just as likely to throw the ball to *B* as to *A*. Show that the process is Markovian. Find the transition probability matrix and classify the states	BTL-4	Analyzing	CO 3
16.	Consider a Markov chain chain $\{X_n, n= 0, 1, 2, \ldots\}$ having states space S={ 1,2} and one step TPM $P = \begin{bmatrix} \frac{4}{10} & \frac{6}{10} \\ \frac{8}{10} & \frac{2}{10} \end{bmatrix}$. (1) Draw a transition diagram, (2) Is the chain irreducible?	BTL-4	Analyzing	CO 3
17.	Classify the states of the Markov chain for the one-step transition probability matrix $P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$ with state space S = {1,2,3}	BTL-4	Analyzing	CO 3
18.	On a given day, a retired English professor, Dr. Charles Fish amuses himself with only one of the following activities reading (i), gardening (ii) or working on his book about a river valley (iii) for $1 \le i \le 3$, let $X_n = i$, if Dr. Fish devotes day *n* to activity *i*. Suppose that $\{X_n : n=1, 2, \ldots\}$ is a Markov chain, and depending on which of these activities on the next day is given by the t. p. m $P = \begin{bmatrix} 0.30 & 0.25 & 0.45 \\ 0.40 & 0.10 & 0.50 \\ 0.25 & 0.40 & 0.35 \end{bmatrix}$ Find the proportion of days Dr. Fish devotes to each activity.	BTL-3	Applying	CO 3

UNIT-IV: VECTOR SPACES

Vector spaces – Subspaces – Linear combinations– Linear independence and linear dependence – Bases and dimensions (definition only)

Q.No.	Question	BT Level	Competence	Course Outcome
	PART – A			
1.	Define Vector Space	BTL-1	Remembering	CO 4
2.	Define Subspace of a vector space	BTL-1	Remembering	CO 4
3.	What are the possible subspace of R^2	BTL-1	Remembering	CO 4

4.	In a Vector Space V (F) if αv=0 then either α=0 or v=0 prove.	BTL-2	Understanding	CO 4
5.	Is $\{(1,4,-6),(1,5,8),(2,1,1),(0,1,0)\}$ is a linearly independent subset of R^3 ? Justify your answer	BTL-2	Understanding	CO 4
6.	State Replacement Theorem	BTL-1	Remembering	CO 4
7.	In a vector Space V(F), prove that 0v=0 , for all $v \in V$	BTL-2	Understanding	CO 4
8	Write the vectors $v = (1,-2,5)$ as a linear combination of the vectors $x = (1,1,1), y = (1,2,3)$ and $z = (2,-1,1)$	BTL-2	Understanding	CO 4
9.	What is the Dimension of $M_{2x2}(R)$?	BTL-2	Understanding	CO 4
10.	Determine whether the set W=$\{(a_1,a_2,a_3)\epsilon R^3$:$a_1$+2$a_2-3a_3$=1}is a subspace of R^3under the operations of addition and scalar multiplication.	BTL-2	Understanding	CO 4
11.	Determine whether $w = (4,-7,3)$ can be written as a linear combination of $v_1 = (1,2,0)$ and $v_2 = (3,1,1)$ in R^3	BTL-2	Understanding	CO 4
12.	For which value of k will the vector $u = (1,-2,k)$ in R^3 be a linear combination of the vectors $v = (3,0,-2)$ and $w = (2,-1,5)$?	BTL-2	Understanding	CO 4
13.	Define Infinite dimensional vector Space	BTL-1	Remembering	CO 4
14.	Point out whether the set $W_1 = \{(a_1,a_2,a_3) \in R^3 : a_1 - 4a_2 - a_3 = 0\}$ is a subspace of R^3 under the operations of addition and scalar multiplication defined on R^3	BTL-2	Understanding	CO 4
15.	If W is a Subspace of the Vector Space V(F) prove that W must contain 0 vector in V	BTL-2	Understanding	CO 4
16.	Point out whether $w = (3,4,1)$ can be written as a linear combination of $v_1 = (1,-2,1)$ and $v_2 = (-2,-1,1)$ in R^3	BTL-2	Understanding	CO 4
17.	What are the possible subspaces of R^3	BTL-2	Understanding	CO 4
18.	Show that the vectors $\{(1,1,0),(1,0,1)$ and $(0,1,1)\}$ genarate R^3	BTL-2	Understanding	CO 4
19.	If $v_1,v_2 \in V(F)$ and $\alpha_1, \alpha_2 \in F$.Show that the set $\{v_1, v_2, \alpha_1 v_1 + \alpha_2 v_2\}$ is linearly dependent	BTL-2	Understanding	CO 4
20.	Test whether S={(2,1,0) , (1,1,0),(4,2,0)} in R^3 is a basis of R^3 over R	BTL-2	Understanding	CO 4
21.	Define finite dimensional Vector Space	BTL-1	Remembering	CO 4
22.	Is $v = (2,-5,4)$ a linear combination of (1,-3,2) and (2,-1,1) in R^3(R) ?	BTL-1	Remembering	CO 4
23.	Define linear span	BTL-1	Remembering	CO 4
24.	Show that the set S= $\{(0,1,0),(1,0,1)$ and $(1,1,0)\}$ in $R^3$is a basis over R	BTL-2	Understanding	CO 4
25.	Define linear combination of vectors	BTL-1	Remembering	CO 4
	PART – B			

1.	Determine whether the following set is linearly dependent or linearly independent $\begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix}, \begin{pmatrix}0 & -1\\ 1 & 1\end{pmatrix}, \begin{pmatrix}-1 & 2\\ 1 & 0\end{pmatrix}$ and $\begin{pmatrix}2 & 1\\ 2 & -2\end{pmatrix}$ generate $M_{2\times 2}(R)$	BTL-3	Applying	CO 4
2.	If x, y and z are vectors in a vector space V such that $x+z=y+z$, then prove that $x=y$ i) The vector 0 (identity) is unique ii) The additive identity for any $x \in V$ is unique	BTL-4	Analyzing	CO 4
3.	Show that the set ,S={(1,3,-4,2), (2,2,-4,0), (1,-3,2,-4), (-1,0,1,0)} is linearly dependent of the other vectors	BTL-4	Analyzing	CO 4
4.	Determine whether the following subset of vector space R^3(R) is a subspace $W_1=\{((a_1, a_2, a_3)$: $2a_1-7a_2+a_3=0\}$	BTL-3	Applying	CO 4
5.	Illustrate that set of all diagonal matrices of order $n \times n$ is a subspace of the vector space $M_{n\times n}(F)$, where $M_{n\times n}$ is the set of all square matrices over the field F.	BTL-4	Analyzing	CO 4
6.	Evaluate that $W_1=\{(a_1, a_2, \ldots a_n) \in F^n; a_1+a_2+\cdots+a_n=0\}$ is a subspace of F^n and $W_2=\{(a_1, a_2, \ldots a_n) \in F^n; a_1+a_2+\cdots+a_n=1\}$ is not a subspace.	BTL-3	Applying	CO 4
7.	Illustrate that the vectors {(1,1,0), (1,0,1), (0,1,1)} generate R^3	BTL-4	Analyzing	CO 4
8.	Determine the following sets $\{1-2x-2x^2, -2+3x-x^2, 1-x+6x^2\}$ are bases for P_2(R).	BTL-3	Applying	CO 4
9.	Analyze that the matrices $\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}, \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$ and $\begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}$ generate $M_{2\times 2}(R)$.	BTL-4	Analyzing	CO 4
10.	Identify whether the set $\{x^3+2x^2, -x^2+3x+1, x^3-x^2+2x-1\}$in $P_3(R)$ is linearly independent or not.	BTL-3	Applying	CO 4
11.	Determine the following sets $\{1+2x-x^2, 4-2x+x^2, -1+18x-9x^2\}$ are bases for P_2(R).	BTL-3	Applying	CO 4
12.	Illustrate that the set $\{1, x, x^2, \ldots . x^n\}$ is a basis for $P_n(F)$.	BTL-3	Applying	CO 4
13.	Determine whether the set of vectors {(1,0,0,-1), (0,1,0,-1), (0,0,1,-1), (0,0,0,1)} is a basis for R^4	BTL-3	Applying	CO 4
14.	Determine the basis and dimension of the solution space of the linear homogeneous system x+y-z=0, -2x-y+2z=0, -x+z =0.	BTL-3	Applying	CO 4
15.	Determine x so that the vectors (1,-1,x-1),(2,x,-4),(0,x+2,-8) are linearly dependent over R	BTL-3	Applying	CO 4
16.	Decide whether or not the set $S=\{x^3+3x-2, 2x^2+5x-3, -x^2-4x+4\}$ is a basis for $P_2(R)$	BTL-4	Analyzing	CO 4
17.	Determine whether the set of vectors $X_1=(1,0,-1)$, $X_2=(2,5,1)$, and $X_3=(0,-4,3)$ is a basis for R^3	BTL-3	Applying	CO 4
18.	The set of solutions to the system of linear equations $x_1-2x_2+x_3=0$, $2x_1-3x_2+x_3=0$ is a subspace of R^3. Find a basis for this subspace.	BTL-4	Analyzing	CO 4

UNIT-V: LINEAR TRANSFORMATION Linear transformation – Null spaces and ranges – Dimension theorem – Matrix representation of linear transformations				
Q.No.	**Question**	**BT Level**	**Competence**	**Course Outcome**
PART – A				
1.	Define linear transformation of a function	BTL-1	Remembering	CO 5
2.	If $T: V \rightarrow W$ be a linear transformation then prove that $T(-v) = -v$ for $v \in V$	BTL-2	Understanding	CO 5
3.	If $T: V \rightarrow W$ be a linear transformation then prove that $T(x - y) = x - y$ for all $x, y \in V$	BTL-2	Understanding	CO 5
4.	Prove that the transformation T is linear if and only if $T(cx + y) = cT(x) + T(y)$	BTL-2	Understanding	CO 5
5.	Illustrate that the transformation $T: R^2 \rightarrow R^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_2)$ is linear	BTL-2	Understanding	CO 5
6.	Evaluate that the transformation $T: R^3 \rightarrow R^2$ defined by $T(x, y, z) = (x, 0,0)$ a linear transformation.	BTL-2	Understanding	CO 5
7.	Describe explicitly the linear transformation $T: R^2 \rightarrow R^2$such that $T(2,3) = (4,5) and\ T(1,0) = (0,0)$	BTL-1	Remembering	CO 5
8.	Illustrate that the transformation $T: R^2 \rightarrow R^3$defined by $T(x, y) = (x + 1, 2y, x + y)$ is not linear	BTL-2	Understanding	CO 5
9.	Is there a linear transformation $T: R^3 \rightarrow R^3$such that $T(1,0,3) = (1,1)$ and $T(-2,0,-6) = (2,1)$?	BTL-2	Understanding	CO 5
10.	Define null space.	BTL-1	Remembering	CO 5
11.	Define matrix representation of T relative to usual basis $\{e_i\}$	BTL-1	Remembering	CO 5
12.	Find the matrix $[T]_e$ whose linear operator $is\ T(x, y) = (5x + y,\ 3x - 2y)$	BTL-2	Understanding	CO 5
13.	Find a basis for the null space of the matrix $A = \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix}$	BTL-2	Understanding	CO 5
14.	Let V = R^4 and consider the following subset of V: W = $\{(x_1, x_2, x_3, x_4) \in R^4 \mid 2x_1-3x_2+x_3-7x_4=0\}$. Is W a subspace of V?	BTL-1	Remembering	CO 5
15.	Find the matrix representation of usual basis $\{e_i\}$ to the linear operator $T(x, y, z) = (2y + z, x - 4y, 3x)$	BTL-2	Understanding	CO 5
16.	State the dimension theorem for matrices.	BTL-1	Remembering	CO 5
17.	Verify the dimension theorem for $T_\theta((a_1, a_2)) = (0, a_2)$	BTL-2	Understanding	CO 5
18.	Verify the dimension theorem for $T_\theta((a_1, a_2)) = (a_1 \cos\theta - a_2 \sin\theta,\ a_1 \sin\theta + a_2 \cos\theta)$	BTL-2	Understanding	CO 5
19.	Show that $T: E^2 \rightarrow E^2$, defined by $T\big((x_1, x_2)\big) = (x_1 + x_2, x_1 - x_2 + 1)$ is not linear.	BTL-2	Understanding	CO 5
20.	Show that $T: E^2 \rightarrow E^1$, defined by $T\big((x_1, x_2)\big) = x_1^2 + x_2^2$ is not linear.	BTL -2	Understanding	CO 5
21.	Define Range	BTL-1	Remembering	CO 5

22.	Show that $T: C^2 \to C^2$, defined by $T((x_1, x_2)) = (z_1 + z_2, z_1 - 2z_2)$ is linear.	BTL -2	Understanding	CO 5
23.	Find ker T, where $T: E^3 \to E^2$ is defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 - x_3)$.	BTL -2	Understanding	CO 5
24.	Let $T: (E^2, S) \to (E^2, \tau)$ be defined by $T((x_1, x_2)) = (x_1 + 2x_2, x_1 - x_2)$. Find the matrix of T when $S = \tau = \{\boldsymbol{e_1}, \boldsymbol{e_2}\}$.	BTL -2	Understanding	CO 5
25.	Let $T: (E^2, S) \to (E^2, \tau)$ be defined by $T((x_1, x_2)) = (x_1 + 2x_2, x_1 - x_2)$. Find the matrix of T when $S = \tau = \{(1,2), (3,-1)\}$.	BTL -2	Understanding	CO 5
	PART – B			
1.	For each of the following linear operators T on a vector space V and ordered basis β,compute $[T]_\beta$, V=R^2, $T\binom{a}{b}=\binom{10a-6b}{17a-10b}$, β=$\{\binom{1}{2},\binom{2}{3}\}$	BTL-3	Applying	CO 5
2.	Let $T: P_2(R) \to P_3(R)$ be defied by $T[f(x)] = 2f'(x) + \int_0^x 3f(t)dt$. Prove that T is linear, find the bases for $N(T)$ and $R(T)$. Compute the nullity and rank of T. Determine whether T is one-to-one or onto.	BTL-3	Applying	CO 5
3.	Let $T: P_2(R) \to P_3(R)$ be defined by $T[f(x)] = xf(x) + f'(x)$ is linear. Find the bases for both $N(T)$, $R(T)$, nullity of T, rank of T and determine whether T is one –to-one or onto.	BTL-3	Applying	CO 5
4.	Let $T: R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Evaluate a basis and dimension of null space N(T) and range space R(T) and range space R(T). Also verify dimension theorem	BTL-3	Applying	CO 5
5.	Find a linear map $T: R^3 \to R^4$ whose image is generated by (1,2,0,-4) and (2,0,-1,-3)	BTL-3	Applying	CO 5
6.	Point out that T is a linear transformation and find bases for both N(T) and R(T). Compute nullity rank T. Verify dimension theorem also verify whether T is one –to-one or onto where $T: P_2(R) \to P_3(R)$ defined by $T[f(x)] = xf(x) + f'(x)$	BTL-4	Analyzing	CO 5
7.	For each of the following linear operators T on a vector space V and ordered basis β,compute $[T]_\beta$,V=P_1(R), T(a+bx)=(6a-6b)+(12a-11b)x and β={3+4x,2+3x}	BTL-3	Applying	CO 5
8.	Let $T: R^2 \to R^3$ be defined by $T(x, y) = (x + 3y, 0, 2x - 4y)$. Compute the matrix of the transformation with respect to the standard bases of R^2 and R^3. Find N(T) and R(T) . Is T one –to-one? Is T onto. Justify your answer.	BTL-4	Analyzing	CO 5
9.	Let T be the linear operator on R^3 defined by $T(x, y, z) = (2x - 7y - 4z, 3x + y + 4z, 6x - 8y + z)$ (i) Find the matrix of T in the basis { f_1=(1,1,1), f_2=(1,1,0) f_3=(1,0,0) and (ii) Verify $[T]_f$ $[T]_v$ = $[T(v)]_f$ for any vector vϵR^3	BTL-4	Analyzing	CO 5
10.	Let α=$\{\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}0&1\\0&0\end{pmatrix},\begin{pmatrix}0&0\\1&0\end{pmatrix},\begin{pmatrix}0&0\\0&1\end{pmatrix}\}$, β={1,x,$x^2$} and γ={1},Define T:$M_{2x2}$(F) → M_{2x2}(F) by T(A)=A^T. Compute $[T]_\alpha$.	BTL -3	Applying	CO 5

11.	Let V be the space of 2x2 matrices over R and let M=$\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}$. Let T be linear operator defined by T(A)=MA. Find the trace of T.	BTL -4	Analyzing	CO 5
12.	Let V and W be vector spaces over F, and suppose that $\{v_1, v_2 \dots \dots \dots . v_n\}$ is a basis for V, For $w_1, w_2 \dots \dots \dots . w_n$ in W. Prove that there exists exactly one linear transformation $T: V \to W$ such that T(v_i) $= w_i$ for i=1,2,…n	BTL-4	Analyzing	CO 5
13.	Consider the basis S={v_1,v_2,v_3} for R^3 where v_1=(1,1,1), v_2=(1,1,0) and v_3=(1,0,0). Let T:$R^3 \to R^2$ be the linear transformation such that T(v_1)=(1,0), T(v_2)=(2,-1) and T(v_3) = (4,3). Find the formula for T(x_1,x_2,x_3), then use this formula to compute T(2,-3,5)	BTL-4	Analyzing	CO 5
14.	Let T be a linear operator defined by $T(a,b,c) = (-4a + 3b - 6c, 6a - 7b + 12c, 6a - 6b + 11c), \beta$ be the ordered basis then find $[T]_\beta$.	BTL -3	Applying	CO 5
15.	Let $T: R^3 \to R^2$ be defined by $T(x,y,z) = (2x - y, 3z)$ verify whether T is linear or not. Find N(T) and R(T) and hence verify the dimension theorem.	BTL-4	Analyzing	CO 5
16.	Let $T: M_{22} \to M_{22}$ be defined by $T(A) = A - A^T$. Given the standard basis $S = \left\{\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\right\} = \{\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}, \boldsymbol{e_4}\}$, find the matrix for T w.r.t. S.	BTL-4	Analyzing	CO 5
17.	Let $T: P_1 \to P_2$ be defined by $T(a + bx) = ax + \left(\frac{b}{2}\right)x^2$. Given P_1 and P_2 the standard bases $S = \{1, x\}$ and $\tau = \{1, x, x^2\}$, respectively. Find the matrix of T w.r.t. theses bases.	BTL -3	Applying	CO 5
18.	Let $T: (E^2, S) \to (E^2, \tau)$ be defined by $T\big((x_1, x_2)\big) = (x_1 + 2x_2, x_1 - x_2)$. Find the matrix of T when (i) $S = \tau = \{(3,-1), (1,2)\}$. (ii) $S = \{(1,-1), (1,1)\}, \tau = \{(1,0), (0,-1)\}$. (iii) $S = \{(1,-1), (1,1)\}$, $\tau = \{(0,-1), (1,0)\}$.	BTL -3	Applying	CO 5