# SRM VALLIAMMAI ENGEINEERING COLLEGE

SRM Nagar, Kattankulathur – 603 203.

## (An Autonomous Institution)

## DEPARTMENT OF COMPUTER APPLICATIONS

## QUESTION BANK



## I SEMESTER

**MA4121 – APPLIED PROBABILITY AND STATISTICS**
**Regulation – 2024**
**Academic Year 2025- 2026**
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# SRM VALLIAMMAI ENGEINEERING COLLEGE

## DEPARTMENT OF MATHEMATICS

## QUESTION BANK

**SUBJECT      : MA4121 – APPLIED PROBABILITY AND STATISTICS**
**SEM / YEAR : I / I year MCA**

| Q.No. | Question | Bloom's Taxonomy Level | Competence | Course Outcome |
|---|---|---|---|---|
| **UNIT I  -LINEAR ALGEBRA -** Vector spaces – Norms – Inner Products - QR factorization -  Generalized Eigen vectors - Singular value decomposition and applications – Pseudo inverse -  Least squares method. | | | | |
| **PART – A** | | | | |
| **1**. | Define Vector Space . | BTL -1 | Remembering | CO 1 |
| **2** | Define Subspace of a vector space | BTL -2 | Understanding | CO 1 |
| **3** | Define norm of a vector space. | BTL -2 | Understanding | CO 1 |
| **4** | Define Inner Product of a vector space. | BTL -1 | Remembering | CO 1 |
| **5** | In a Vector Space V (F) if αv=0 then either  α=0 or v=0 prove. | BTL -2 | Understanding | CO 1 |
| **6** | Is $\{(1,4,-6),(1,5,8),(2,1,1),(0,1,0)\}$ is a linearly independent subset of $R^3$ ? Justify your answer | BTL -1 | Remembering | CO 1 |
| **7**. | Find the sum and product of all Eigen values of $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{bmatrix}$ | BTL -2 | Understanding | CO 1 |
| **8**. | Describe Least square method. | BTL -1 | Remembering | CO 1 |
| .**9** | Find the least square solution to the system $x_1 + x_2 = 3$ , $-2x_1 + 3x_2 = 1$ and $2x_1 - x_2 = 2$ | BTL -2 | Understanding | CO 1 |
| **10** | Define Hermitian Matrix. | BTL -1 | Remembering | CO 1 |
| **11** | Write short note on Singular value decomposition of complex matrix A. | BTL -1 | Remembering | CO 1 |
| **12** | State Singular value decomposition theorem. | BTL -1 | Remembering | CO 1 |
| **13** | If A is a nonsingular matrix, then what is $A^+$? | BTL -2 | Understanding | CO 1 |
| **14**. | If the sum of two eigenvalues and trace of a 3x3 matrix A are equal find the value of |A|. | BTL -1 | Remembering | CO 1 |
| **15**. | Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} = A_1$ .Compute A$_2$ using QR algorithm. | BTL -2 | Understanding | CO 1 |
| **16**. | Discuss the nature of the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$. | BTL -2 | Understanding | CO 1 |
| **17**. | Define the generalized Eigen vector, chain of rank m, for a square matrix. | BTL -2 | Understanding | CO 1 |
| **18**. | Find the sum and product of the Eigen values of the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 1 \\ -7 & 2 & -3 \end{bmatrix}$. | BTL -1 | Remembering | CO 1 |

| 19. | Check whether the given matrix is positive definite or not $\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{pmatrix}$ | BTL -1 | Remembering | CO 1 |
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| 20. | Give the nature of quadratic form without reducing into canonical form $x_1{}^2 - 2 x_1 x_2 + x_2{}^2 + x_3{}^2$ | BTL -1 | Remembering | CO 1 |
| 21. | The sum of two Eigen values of a 3x3 matrix is equal to trace of a matrix then find $|A|$ | BTL -2 | Understanding | CO 1 |
| 22. | Determine the canonical basis for $A = \begin{bmatrix} 3 & 5 \\ -2 & -4 \end{bmatrix}$. | BTL -1 | Remembering | CO 1 |
| 23. | Give the nature of the quadratic form without converting whose matrix is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$. | BTL -2 | Understanding | CO 1 |
| 24. | Find the generalized inverse of $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ | BTL -1 | Remembering | CO 1 |
| 25. | Write down the matrix of the quadratic form $2x^2 + 8z^2 + 4xy + 10xz - 2yz$ | BTL -2 | Understanding | CO 1 |
| **PART – B** | | | | |
| 1. | Determine whether the following set is linearly dependent or linearly independent $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 2 & 1 \\ 2 & -2 \end{pmatrix}$ generate $M_{2\times2}(R)$ | BTL -3 | Applying | CO 1 |
| 2. | If $x, y$ and $z$ are vectors in a vector space $V$ such that $x + z = y + z$, then prove that $x = y$ <br> i) The additive identity for any $x \in V$ is unique <br> ii) The additive inverse is unique | BTL -2 | Understanding | CO 1 |
| 3. | Show that the set ,S={(1,3,-4,2),(2,2,-4,0),(1,-3,2,-4),(-1,0,1,0)} is linearly dependent of the other vectors | BTL -3 | Applying | CO 1 |
| 4.(a) | Determine whether the following subset of vector space $R^3(R)$ is a subspace <br> $W_1$={$((a_1, a_2, a_3)$: 2a$_1$-7a$_2$+a$_3$=0} | BTL -3 | Applying | CO 1 |
| 4.(b) | Illustrate that set of all diagonal matrices of order $n \times n$ is a subspace of the vector space $M_{n\times n}(F)$, where $M_{n\times n}$ is the set of all square matrices over the field $F$ | BTL -3 | Applying | CO 1 |
| 5. | Find the QR factorization of A = $\begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$ | BTL -2 | Understanding | CO 1 |
| 6. | Find the QR factorization of A = $\begin{bmatrix} 0 & 0 & 5 \\ 0 & 4 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | BTL -2 | Understanding | CO 1 |
| 7. | Find the QR factorization of A = $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ | BTL -4 | Analyzing | CO 1 |
| 8. | Obtain the singular value decomposition of $\begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}$ | BTL -2 | Understanding | CO 1 |
| 9. | Construct the singular value decomposition for $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \end{pmatrix}$ | BTL -4 | Analyzing | CO 1 |
| 10. | Obtain the singular value decomposition of A = $\begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 4 & -2 \end{bmatrix}$ | BTL -3 | Applying | CO 1 |

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| **11.** | Obtain the singular value decomposition $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ | BTL -3 | Applying | CO 1 |
| **12.** | Solve the following system of equations in the least square sense $x_1 + x_2 + 3x_3 = 1$ ; $x_1 + x_2 + 3 x_3 = 2$ | BTL -3 | Applying | CO 1 |
| **13.** | Solve the following system of equations in the least square sense $x_1 + x_2 + x_3 = 1$ ; $x_1 + x_2 + x_3 = 2$ ; $x_1 + x_2 + x_3 = 3.$ | BTL -4 | Analyzing | CO 1 |
| **14.** | Solve the following system of equations in the least square sense $2x_1 + 2x_2 - 2x_3 = 1$ , $2x_1 + 2x_2 - 2x_3 = 3$, $-2x_1 - 2x_2 + 6x_3 = 2$ | BTL -3 | Applying | CO 1 |
| **15.** | Fit a straight line in the least square sense to the following data <br> X:   -3   -2   -1   0   1   2   3 <br> Y:   8   12   17   25   26   32   40 | BTL -4 | Analyzing | CO 1 |
| **16.** | Fit a straight line in the least square sense to the following data <br> X:   -3   -2   -1   0   1   2   3 <br> Y:   10   15   19   27   28   34   42 | BTL -4 | Analyzing | CO 1 |
| **17.** | Solve the following system of equations in the least square sense $x_1 + x_2 + x_3 = 2$ ; $x_1 + x_2 + x_3 = 4$ ;   $x_1 + x_2 + x_3 = 6.$ | BTL -3 | Applying | CO 1 |
| **18.** | Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ | BTL -4 | Analyzing | CO 1 |

**UNIT – II   PROBABILITY AND RANDOM VARIABLES:** Probability–Axioms of probability–Conditional probability–Baye's theorem-Random variables-Probability function–Moments–Moment generating functions and their properties–Binomial, Poisson, Geometric, Uniform, Exponential, Gamma and Normal distributions–Function of a random variable.

| Q.No. | Question | Bloom's Taxonomy Level | Competence | Course Outcome |
|---|---|---|---|---|
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| **PART – A** | | | | |
| 1. | What is the use of Baye's theorem? | BTL-2 | Understanding | CO 2 |
| 2. | Mention the properties of a discrete probability distribution. | BTL-2 | Understanding | CO 2 |
| 3. | Check whether the function given by $f(x) = \frac{x+2}{25}$ for x=1,2,3,4,5 can serve as the probability distribution of a discrete random variable. | BTL-2 | Understanding | CO 2 |
| 4. | If the random variable X takes the values 1,2,3 and 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$ , find the probability distribution of X | BTL-1 | Remembering | CO 2 |
| 5. | The RV X has the following probability distribution: <br> | x | -2 | -1 | 0 | 1 | <br> | P(x) | 0.4 | k | 0.2 | 0.3 | <br> Find k and the mean value of X | BTL-2 | Understanding | CO 2 |
| 6. | If $f(x) = K(x + x^2)$ in $1 < x < 5$ is a pdf of a continuous | BTL-1 | Remembering | CO 2 |

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| | random variables. Find the value of K. | | | |
|---|---|---|---|---|
| 7. | The p.d.f of a continuous random variable X is $f(x) = k(1+x)$, $2 < x < 5$ Find $k$. | BTL-1 | Remembering | CO 2 |
| 8. | For a continuous distribution $f(x) = k(x - x^2), 0 \leq x \leq 1$, where $k$ is a constant. Find $k$. | BTL-2 | Understanding | CO 2 |
| 9. | If $f(x) = kx^2, 0 < x < 3$, is to be a density function, find the value of $k$. | BTL-2 | Understanding | CO 2 |
| 10. | If the probability that a target is destroyed on any one shot is 0.5, Find the probability that it would be destroyed an $6^{th}$ attempt. | BTL-2 | Understanding | CO 2 |
| 11. | The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution. | BTL-2 | Understanding | CO 2 |
| 12. | The mean and variance of binomial distribution are 5 and 4. Determine the distribution. | BTL-1 | Remembering | CO 2 |
| 13. | If 3% of the electric bulbs manufactured by a company are defective, Find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective. | BTL-2 | Understanding | CO 2 |
| 14. | Messages arrive at a switchboard in a poisson manner at an average rate of six per hour. Find the probability for exactly two messages arrive within one hour. | BTL-1 | Remembering | CO 2 |
| 15. | The number of monthly breakdowns of a computer is a random variable having Poisson distribution with mean 1.8. Find the probability that this computer will function for a month with only one breakdown. | BTL-2 | Understanding | CO 2 |
| 16. | If X is a Poisson variate such that $2P(X = 0) + P(X = 2) = 2P(X = 1)$, find E(X) | BTL-1 | Remembering | CO 2 |
| 17. | The probability that a candidate can pass in an examination is 0.6. What is the probability that he will pass in third trial? | BTL-2 | Understanding | CO 2 |
| 18. | If $f(x) = \dfrac{x^2}{3}, -1 < x < 2$ is the pdf of the random variable X ,then find p(0<x<1) | BTL-1 | Remembering | CO 2 |
| 19. | If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test on the fourth trial | BTL-2 | Understanding | CO 2 |
| 20. | If X has uniform distribution in (-3,3), find $P(|x - 2| < 2)$ | BTL-1 | Remembering | CO 2 |
| 21. | Let X be a random variable with moment generating function $M_X(t) = \dfrac{(2e^t + 1)^4}{81}$ .Find its mean and variance. | BTL-2 | Understanding | CO 2 |
| 22. | A Random variable X is uniformly distributed between 3 and 15. Find the variance of X. | BTL-1 | Remembering | CO 2 |
| 23. | A continuous RV X has the density function $ce^{-\frac{x}{5}}, x > 0$. Find c. Create E(x) and Var(X) | BTL-2 | Understanding | CO 2 |
| 24. | If X is a normal random variable with mean 3 and variance 9, find the probability that X lies between 2 and 5. | BTL-1 | Remembering | CO 2 |
| 25. | A normal distribution has mean $\mu = 20$ and standard deviation $\sigma = 10$. Evaluate $(15 \leq X \leq 40)$. | BTL-1 | Evaluating | CO 2 |

| | **PART –B** | | | |
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| 1(a) | A random variable X has the following probability distribution:<br><br>| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |<br>|---|---|---|---|---|---|---|---|---|<br>| P(X) | 0 | $k$ | $2\,k$ | $2\,k$ | $3\,k$ | $k^2$ | $2k^2$ | $7k^2+$ |<br><br>Find   (i) the value of $k$<br>       (ii) $P(1.5\ <X<\ 4.5\ /\ X>2)$ | BTL-2 | Understanding | CO 2 |
| 1(b) | Find the MGF of Binomial distribution and hence find its mean and variance | BTL-2 | Remembering | CO 2 |
| 2(a) | The probability mass function of a discrete R. V X is given in the following table:<br><br>| X | -2 | -1 | 0 | 1 | 2 | 3 |<br>|---|---|---|---|---|---|---|<br>| P(X=x) | 0.1 | $K$ | 0.2 | $2k$ | 0.3 | $k$ |<br><br>Find (1) Find the value of $k$, (2) P(X<1),(3) P(-1< X ≤ 2) | BTL-2 | Understanding | CO 2 |
| 2(b) | Obtain the MGF of Poisson distribution and hence find its mean and variance | BTL-2 | Remembering | CO 2 |
| 3(a) | The probability mass function of a discrete random variable X is given in the following table<br><br>| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |<br>|---|---|---|---|---|---|---|---|---|---|<br>| P(X) | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 1 |<br><br>Find   (i) the value of a , (ii) $P(\ X<\ 3)$, (iii) Mean of X,  (iv) Variance of X. | BTL-4 | Analyzing | CO 2 |
| 3(b) | Deduce the MGF of a geometric distribution and hence find the mean and variance | BTL-4 | Analyzing | CO 2 |
| 4(a) | If the discrete random variable X has the probability function given by the table.<br><br>| $x$ | 1 | 2 | 3 | 4 |<br>|---|---|---|---|---|<br>| $P(x)$ | $k/3$ | $k/6$ | $k/3$ | $k/6$ |<br><br>Find the value of k and Cumulative distribution of X. | BTL-3 | Applying | CO 2 |
| 4(b) | Derive the MGF of Uniform distribution and hence deduce the mean and variance | BTL-3 | Applying | CO 2 |
| 5(a) | If the probability mass function of a random variable X is given by $P(X=x) = kx^3$, x=1,2,3,4. Find the value of k, mean and variance of X. | BTL-4 | Analyzing | CO 2 |
| 5(b) | Deduce the MGF of Exponential distribution and hence find its mean and variance | BTL-4 | Analyzing | |
| 6(a) | Find the MGF , mean and variance of the random variable X has the pdf<br><br>$$f(x) = \begin{cases} x, 0 < x < 1 \\ 2 - x, 1 < x < 2 \\ 0, otherwise \end{cases}$$ | BTL-4 | Analyzing | CO 2 |
| 6(b) | State and prove the memory less property of exponential distribution | BTL-4 | Analyzing | CO 2 |
| 7(a) | In a large consignment of electric bulbs, 10 percent are defective. A random sample of 20   is taken for inspection. Find the probability that i) all are good bulbs ii) atmost there are 3 defective bulbs iii) exactly there are 3 defective bulbs. | BTL-3 | Applying | CO 2 |

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| **7(b)** | A manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins are defective, what is the probability that a box fail to meet the guaranteed quality. | BTL-3 | Applying | CO 2 |
| **8.** | In a bolt factory machines A, B, C manufacture respectively 25%, 35% and 40% of the total of their output 5, 4, 2 percent are defective bolts. If A bolt is drawn at random from the product and is found to be defective, what are the probabilities that is was manufactured by machines A, B and C? | BTL-4 | Analyzing | CO 2 |
| **9(a)** | If a random variable X has p.d.f $f(x) = \begin{cases} \frac{1}{4}, & \|X\| < 2 \\ 0, & Otherwise \end{cases}$ Find (a) $P(X < 1)$ (b) $P(\|X\| > 1)$ (c) $P(2X + 3 > 5)$. | BTL-4 | Analyzing | CO 2 |
| **9(b)** | Out of 2000 families with 4 children each, Find how many family would you expect to have i) at least 1 boy ii) 2 boys. | BTL-4 | Analyzing | |
| **10.(a)** | Find the MGF of the random variable X having the probability density function f(x) = $\begin{cases} \frac{x}{4}e^{-\frac{x}{2}}, & x > 0 \\ 0 & , otherwise \end{cases}$. Also find the mean and variance | BTL-4 | Analyzing | CO 2 |
| **10(b)** | 4 coins were tossed simultaneously. What is the probability of getting (i) 2 heads, (ii) at least 2 heads, (iii) at most 2 heads. | BTL-4 | Analyzing | CO 2 |
| **11.(a)** | A random variable X has c.d.f $F(x) = \begin{cases} 0, if x < -1 \\ a(1 + x), if -1 < x < 1 \\ 1, if x \geq 1 \end{cases}$. Find the value of a. Also P(X>1/4) and $P(-0.5 \leq X \leq 0)$. | BTL-3 | Applying | CO 2 |
| **11(b)** | The atoms of a radioactive element are randomly disintegrating. If every gram of this element, on average, emits 3.9 alpha particles per second, then what is the probability that during the next second the number of alpha particles emitted from 1 gram is (1) at most 6 (2) at least 2 and (3) at least and at most 5 | BTL-3 | Applying | CO 2 |
| **12.** | If f(x) = $\begin{cases} ax, & 0 \leq x \leq 1 \\ a, & 1 \leq x \leq 2 \\ 3a - ax, & 2 \leq x \leq 3 \\ 0, & elsewhere \end{cases}$ is the p.d.f of X. Calculate (i) The value of a , (ii) The cumulative distribution function of X (iii) If $X_1, X_2$ and $X_3$ are 3 independent observations of X. Find the probability that exactly one of these 3 is greater than 1.5? | BTL-4 | Analyzing | CO 2 |
| **13.(a)** | The Probability distribution function of a R.V. X is given by $f(x) = \frac{4x(9 - x^2)}{81}$, $0 \leq x \leq 3$. Find the mean, variance. | BTL-3 | Applying | CO 2 |
| **13.(b)** | The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month | BTL-3 | Applying | CO 2 |

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| | (1) without breakdown (2) with only one breakdown and (3) with at least one breakdown. | | | |
| 14.(a) | Messages arrive at a switch board in a Poisson manner at an average rate of 6 per hour. Find the probability that exactly 2 messages arrive within one hour, no messages arrives within one hour and at least 3 messages arrive within one hour | BTL-4 | Analyzing | CO 2 |
| 14(b) | An electrical firm manufactures light bulbs that have a life, before burn out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. | BTL-4 | Analyzing | CO 2 |
| 15.(a) | The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. <br> (a) What is the probability that the repair time exceeds 2 hours? <br> (b) What is the conditional probability that a repair time exceeds at least 10 hours that its distribution exceeds 9 hours? | BTL-3 | Applying | CO 2 |
| 15(b) | Let X be a Uniformly distributed R. V. over [-5, 5]. Evaluate (i) $P(X \le 2)$ (ii) $P(|X|>2)$ (iii) Cumulative distribution function of X (iv) Var(X) | BTL-3 | Applying | CO 2 |
| 16.(a) | Buses arrive at a specified stop at 15 minutes interval starting at 7am that is, 7,7:15,7:30,7:45, and so on, If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 am, evaluate the probability that he waits <br> (a) Less than 5 minutes for a bus and <br> (b) At least 12 minutes for a bus | BTL-4 | Analyzing | CO 2 |
| 16(b) | The marks obtained by a number of students for a certain subject is assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are taken at random from this set Find the probability that exactly 2 of them will have marks over 70? | BTL-4 | Analyzing | CO 2 |
| 17. | In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and Standard Deviation of 60 hours. Find the number of bulbs likely to burn for (i) more than 2150 hours (ii) less than 1950 hours and (iii) more than 1920 hours burs less than 2160 hours. | BTL-3 | Applying | CO 2 |
| 18. | The length of time a person speaks over phone follows exponential distribution with mean 6 mins. What is the probability that the person will talk for (1) more than 8 mins (2) between 4 and 8 mins. | BTL-4 | Analyzing | CO 2 |

**UNIT – III TWO DIMENSIONAL RANDOM VARIABLES** Joint distributions – Marginal and conditional distributions – Function of two dimensional random variables – Regression Curve – Correlation.

| Q.No. | Question | Bloom's Taxonomy Level | Competence | Course Outcome |
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| | **PART – A** | | | |

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| 1. | The joint probability distribution of X and Y is given by $$p(x,y) = \frac{x+y}{21},$$ $x = 1,2,3; y = 1, 2$. Find the marginal probability distributions of X | BTL-2 | Understanding | CO 3 |
| 2. | The joint probability function (X,Y) is given by $P(x,y) = k(2x+3y)$, $x = 0,1,2 \; y = 1,2,3$, Find the value of K. | BTL-2 | Understanding | CO 3 |
| 3. | Find the probability distribution of X + Y from the bivariate distribution of (X,Y) given below: <table><tr><td>X \ Y</td><td>1</td><td>2</td></tr><tr><td>1</td><td>0.4</td><td>0.2</td></tr><tr><td>2</td><td>0.3</td><td>0.1</td></tr></table> | BTL-2 | Understanding | CO 3 |
| 4. | If the joint pdf of X and Y is given by f(x,y)=2,in $0 \leq x < y \leq 1$, Find E(X) | BTL-1 | Remembering | CO 3 |
| 5. | Find the marginal distributions of X and Y from the bivariate distribution of (X,Y) given below: <table><tr><td>X \ Y</td><td>1</td><td>2</td></tr><tr><td>1</td><td>0.1</td><td>0.2</td></tr><tr><td>2</td><td>0.3</td><td>0.4</td></tr></table> | BTL-2 | Understanding | CO 3 |
| 6. | Find the value of k,if the joint density function of (X,Y) as $$f(x,y) = \begin{cases} k(1-x)(1-y), 0 < x < 4, 1 < y < 5 \\ 0, otherwise \end{cases}$$ | BTL-1 | Remembering | CO 3 |
| 7. | If the joint probability density function of a random variable X and Y is given by $f(x,y) = \begin{cases} \frac{x^3 y^3}{16}, 0 < x < 2, 0 < y < 2 \\ 0, \qquad\qquad otherwise \end{cases}$. Obtain the marginal density function of X. | BTL-1 | Remembering | CO 3 |
| 8. | The joint pdf of the random variable (X,Y) is given by $f(x,y) = Kxye^{-(x^2+y^2)}, x > 0, y > 0$ Find the value of K. | BTL-1 | Remembering | CO 3 |
| 9. | The joint probability density function of a random variable $(X,Y)$ is $f(x,y) = k\,e^{-(2x+3y)}, x \geq 0, y \geq 0$. Point out the value of $k$. | BTL-2 | Understanding | CO 3 |
| 10. | If the joint pdf of (X, Y) is $f(x,y) = \begin{cases} \frac{1}{4}, 0 < x,y < 2 \\ 0, \; otherwise \end{cases}$. Find $P(X + Y \leq 1)$ | BTL-1 | Remembering | CO 3 |
| 11. | Let X and Y be random variables with joint density function $f(x,y) = \begin{cases} 4xy, \; 0 < x < 1, \; 0 < y < 1 \\ \qquad 0, \; otherwise \end{cases}$ formulate the value of E(XY) | BTL-2 | Understanding | CO 3 |
| 12. | Let the joint density function of a random variable X and Y be given by $f(x,y) = 8xy, 0 < y \leq x \leq 1$.Calculate the marginal probability function of X | BTL-1 | Remembering | CO 3 |
| 13. | What is the condition for two random variables are independent? | BTL-2 | Understanding | CO 3 |
| 14. | If the joint probability density function of X and Y is | BTL-1 | Remembering | CO 3 |

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| | $f(x,y) = e^{-(x+y)}, x,y \geq 0$.Are X and Y independent | | | | |
| 15. | State any tow properties of correlation coefficient | BTL-2 | Understanding | CO 3 |
| 16. | Write the angle between the regression lines | BTL-1 | Remembering | CO 3 |
| 17. | The regression equations are x + 6y = 14 and 2x + 3y = 1. Evaluate the correlation coefficient between X & Y . | BTL-1 | Remembering | CO 3 |
| 18. | If $\bar{X} = 970$ , $\bar{Y} = 18, \sigma_x = 38$ , $\sigma_y = 2$ and r = 0.6 , Find the line of regression of X on Y. | BTL-2 | Understanding | CO 3 |
| 19. | In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible; Variance of X = 9; Regression equations are 8X - 10Y + 66 = 0 and 40X-18Y = 214.  Find the mean values of X and Y? | BTL-1 | Remembering | CO 3 |
| 20. | The regression equations are 3x + 2y = 26 and 6x + y = 31. Find the correlation coefficient. | BTL-2 | Understanding | CO 3 |
| 21. | State the relationship between correlation coefficient and Regression coefficient. | BTL-1 | Remembering | CO 3 |
| 22. | Prove that $-1 \leq r_{xy} \leq 1$ | BTL-2 | Understanding | CO 3 |
| 23. | The equations of two regression lines are 3x+2y=19 and 3y+9x=46. Obtain the mean of X and Y. | BTL-1 | Remembering | CO 3 |
| 24. | The equations of two regression lines are 3x+2y=19 and 3y+9x=46. Derive the correlation coefficient between X and Y. | BTL-1 | Remembering | CO 3 |
| 25. | State the equations of two regression lines. | BTL-2 | Understanding | CO 3 |

**PART –B**

| | | | | |
|---|---|---|---|---|
| 1. | From the following table for bivariate distribution of (X, Y). Find <br> (i) $P(X \leq 1)$ (ii)$P(Y \leq 3)$ (iii) $P(X \leq 1, Y \leq 3)$ (iv)$P(X \leq 1/ Y \leq 3)$ (v) $P(Y \leq 3/ X \leq 1)$ (vi)$P(X + Y \leq 4)$ <br><br> (table below) | BTL-2 | Understanding | CO 3 |

| Y / X | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | $\frac{1}{32}$ | $\frac{2}{32}$ | $\frac{2}{32}$ | $\frac{3}{32}$ |
| 1 | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| 2 | $\frac{1}{32}$ | $\frac{1}{32}$ | $\frac{1}{64}$ | $\frac{1}{64}$ | 0 | $\frac{2}{64}$ |

| | | | | |
|---|---|---|---|---|
| 2.(a) | The two dimensional random variable (X, Y) has the joint probability mass function $f(x,y) = \frac{x+2y}{27}, x = 0,1,2; y = 0,1,2$. Find the marginal distributions of X and Y. Also find the conditional distribution of Y given X = 1 also find the conditional distribution of X given Y = 1. | BTL-3 | Applying | CO 3 |
| 2.(b) | The joint pdf a bivariate R.V(X, Y) is given by $$f(x,y) = \begin{cases} Kxy , & 0 < x < 1, 0 < y < 1 \\ 0 , & otherwise \end{cases}$$ | BTL-3 | Applying | CO 3 |

| | | | | |
|---|---|---|---|---|
| | (1) Find K.   (2)  Find P(X+Y<1).   (3) Are X and Y independent R.V's. | | | |
| **3.(a)** | If the joint pdf of (X, Y) is given by $P(x, y) = $ K(2x+3y), x=0, 1, 2, 3, $y = 1, 2, 3$ Find all the marginal probability distribution. Also find the probability distribution of  X+Y. | BTL-3 | Applying | CO 3 |
| **3.(b)** | The joint pdf of the RV (X,Y) is given by $f(x,y) = kxye^{-(x^2+y^2)}$, $x > 0, y > 0$. Find the value of k. Also prove that X and Y are independent | BTL-4 | Analyzing | CO 3 |
| **4.** | The following table represents the joint probability distribution of the discrete RV (X,Y). i) Find all the marginal distribution ii) Find the conditional distributions of X/Y=2 iii) Find the conditional distribution of Y/X=1. <br><br> | BTL-2 | Understanding | CO 3 |

For Q4:

| X \ Y | 0 | 1 | 2 |
|---|---|---|---|
| -1 | 1/15 | 2/15 | 1/15 |
| 0 | 3/15 | 2/15 | 1/15 |
| 1 | 2/15 | 1/15 | 2/15 |

| | | | | |
|---|---|---|---|---|
| **5.** | Find the marginal distribution of X and Y and also P($P(X \le 1, Y \le 1)$, $P(X \le 1), P(Y \le 1)$. Check whether X and Y are independent. The joint probability mass function of X and Y is | BTL-2 | Understanding | CO 3 |

For Q5:

| X \ Y | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0.10 | 0.04 | 0.02 |
| 1 | 0.08 | 0.20 | 0.06 |
| 2 | 0.06 | 0.14 | .30 |

| | | | | |
|---|---|---|---|---|
| **6.** | The joint pdf of two dimensional random variables (X,Y) is given by $f(x, y) = \begin{cases} 25e^{-5y}, 0 < x < 0.2, y > 0 \\ 0, otherwise \end{cases}$    Find the covariance of x and y. | BTL-4 | Analyzing | CO 3 |
| **7.** | If the joint pdf of a two-dimensional RV(X,Y) is given by $f(x,y) = \begin{cases} x^2 + \frac{xy}{3}; 0 < x < 1, 0 < y < 2 \\ 0, \ elsewhere \end{cases}$. Find (i) $P\left(X > \frac{1}{2}\right)$ (ii) $P(Y < \frac{1}{2}, X < \frac{1}{2})$   (iii)$P\left(Y < \frac{1}{2} / X < \frac{1}{2}\right)$ | BTL-3 | Applying | CO 3 |
| **8.** | The joint pdf of a two dimensional random variable (X, Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1.$ Compute (i) $P\left(X > 1 / Y < \frac{1}{2}\right)$ (ii)$P\left(Y < \frac{1}{2}/X > 1\right)$ (iii) $P(X + Y) \le 1$. | BTL-3 | Applying | CO 3 |

| Q.No. | Question | Bloom's Taxonomy Level | Competence | Course Outcome |
|---|---|---|---|---|
| 9.(a) | (b)The joint pdf of X and Y is given by $f(x,y)=\begin{cases} kx(x-y), 0<x<2, -x<y<x \\ 0, \qquad otherwise \end{cases}$ (i)Find K (ii) Find $f_x(x)$ and $f_y(y)$ | BTL-3 | Applying | CO 3 |
| 10. | Find the Coefficient of Correlation between industrial production and export using the following table <br> | Production (X) | 14 | 17 | 23 | 21 | 25 | <br> | Export (Y) | 10 | 12 | 15 | 20 | 23 | | BTL-3 | Applying | CO 3 |
| 11. | Find the correlation coefficient for the following heights of fathers X,their sons Y and also find the equations of regression lines.  Hence find the height of son when the height of father is 71 <br> | X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 | <br> | Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 | | BTL-3 | Applying | CO 3 |
| 12. | Obtain the lines of regression <br> | X | 50 | 55 | 50 | 60 | 65 | 65 | 65 | 60 | 60 | <br> | Y | 11 | 14 | 13 | 16 | 16 | 15 | 15 | 14 | 13 | | BTL-2 | Understanding | CO 3 |
| 13. | If $f(x,y) = \dfrac{6-x-y}{8}$ , $0 \le x \le 2$, , $2 \le y \le 4$  for a bivariate random variable (X,Y), Evaluate the correlation coefficient $\rho$. | BTL-3 | Applying | CO 3 |
| 14. | Two random variables X and Y have the joint density function $f(x,y) = x + y, 0 \le x \le 1, 0 \le y \le 1$. Evaluate  the Correlation coefficient between X and Y. | BTL-4 | Analyzing | CO 3 |
| 15. | The two regression lines are 4x-5y+33=0 and 20x-9y=107. Find the mean of X and Y. Also find the correlation coefficient between them | BTL-4 | Analyzing | CO 3 |
| 16. | If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of U = X-Y. | BTL-3 | Applying | CO 3 |
| 17. | If X and Y independent Random Variables with pdf $e^{-x}, x \ge 0$ and $e^{-y}, y \ge 0$. Devise the density function of $U = \dfrac{X}{X+Y}$ and $V = X + Y$.  Are they independent? | BTL-3 | Applying | CO 3 |
| 18. | Two random variables X and Y have the following joint probability density function $f(x,y) = \begin{cases} x+y; 0 \le x \le 1, 0 \le y \le 1 \\ 0, \ otherwise \end{cases}$. Find the probability density function of the random variable U = XY. | BTL-4 | Analyzing | CO 3 |

## UNIT – IV TESTING OF HYPOTHESIS

Sampling distributions – Type I and Type II errors – Small and  Large samples - Tests based on Normal ,t and F distributions for testing of  mean, variance and proportion – Test for independence of attributes and goodness of fit.

| Q.No. | Question | Bloom's Taxonomy Level | Competence | Course Outcome |
|---|---|---|---|---|
| | **PART – A** | | | |
| 1. | Define  Statistics | BTL-1 | Remembering | CO 4 |

| 2. | Define Parameter. | BTL-1 | Remembering | CO 4 |
|---|---|---|---|---|
| 3. | Explain null and alternate hypothesis. | BTL-1 | Remembering | CO 4 |
| 4. | Mention the various steps involved in testing of hypothesis. | BTL-1 | Remembering | CO 4 |
| 5. | What is the essential difference between confidence limits and tolerance limits? | BTL-1 | Remembering | CO 4 |
| 6. | Define Standard Error. | BTL-1 | Remembering | CO 4 |
| 7. | Define Type I and Type II error. | BTL-1 | Remembering | CO 4 |
| 8. | What are the parameters and statistics in sampling. | BTL-2 | Understanding | CO 4 |
| 9. | Define level of significance. | BTL-2 | Understanding | CO 4 |
| 10. | What is the test statistic for single proportion test? | BTL-2 | Understanding | CO 4 |
| 11. | A random sample of 25 cups from a certain coffee dispensing machine yields a mean x = 6.9 occurs per cup. Use 0.05 level of significance to test, on the average, the machine dispense $\mu = 7.0$ ounces against the null hypothesis that, on the average, the machine dispenses $\mu < 7.0$ ounces. Assume that the distribution of ounces per cup is normal, and that the variance is the known quantity $\sigma^2 = 0.01$ ounces | BTL-1 | Remembering | CO 4 |
| 12. | Twenty people were attacked by a disease and only 18 were survived. The hypothesis is set in such a way that the survival rate is 85% if attacked by this disease. Will you reject the hypothesis that it is more at 5% level.($Z_{0.05} = 1.645$). | BTL-2 | Understanding | CO 4 |
| 13. | In a large city A, 20 percent of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5 percent of a random sample of 1600 school boys had some defect. Is the difference between the proportions significant? | BTL-2 | Understanding | CO 4 |
| 14. | A standard sample of 200 tins of coconut oil gave an average weight of 4.95 kg with a standard deviation of 0.21 kg. Do we accept that the net weight is 5 kg per tin at 5% level of significance? | BTL-2 | Understanding | CO 4 |
| 15. | Write down the formula of test statistic 't' to test the significance of difference between the population mean and sample mean. | BTL-2 | Understanding | CO 4 |
| 16. | Write down the formula of test statistic 't' to test the significance of difference between two sample means. | BTL-1 | Remembering | CO 4 |
| 17. | What are the applications of t-test? | BTL-1 | Remembering | CO 4 |
| 18. | What is the assumption of t-test? | BTL-1 | Remembering | CO 4 |
| 19. | Write the application of 'F' test. | BTL-1 | Remembering | CO 4 |
| 20. | Define 'F' variate. | BTL-1 | Remembering | CO 4 |
| 21. | What are the properties of "F" test? | BTL-1 | Remembering | CO 4 |
| 22. | Write the formula for the chi- square test of goodness of fit of a random sample to a hypothetical distribution. | BTL-1 | Remembering | CO 4 |
| 23. | State the main use of $\psi^2$-test | BTL1 | Remembering | CO 4 |
| 24. | What are the expected frequencies of 2x2 contingency table? <table><tr><td>a</td><td>b</td></tr><tr><td>c</td><td>d</td></tr></table> | BTL2 | Understanding | CO 4 |
| 25. | State any two applications of $\psi^2$-test. | BTL1 | Remembering | CO 4 |
| **PART –B** | | | | |

| | | | | |
|---|---|---|---|---|
| 1(a) | A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160cms. Can it be reasonably regarded that this sample is from a population of mean 165 cm and standard deviation 10 cm? | BTL-2 | Understanding | CO 4 |
| 1(b) | The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? | BTL-2 | Understanding | CO 4 |
| 2. | Given the following table for hair color and eye color, identify the value of Chi-square. Is there good association between hair color and eye color? <br><br> Hair color table below | BTL-1 | Remembering | CO 4 |
| 3. | Two independent samples of sizes 8 and 7 contained the following values. <br><br> Sample table below <br><br> Test if the two populations have the same mean. | BTL3 | Applying | CO 4 |
| 4. | Two independent samples of 8 and 7 items respectively had the following Values of the variable (weight in kgs.) Use 0.05 LOS to test <br><br> Sample table below <br><br> Whether the variances of the two population's sample are equal. | BTL3 | Applying | CO 4 |
| 5(a) | Two random samples gave the following results: <br><br> Results table below <br><br> Analyze whether the samples have come from the same normal population | BTL3 | Applying | CO 4 |
| 5(b) | The mean braking strength of the cables supplied by manufacture is 1800 with S.D 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cable has increased. To test this claim a sample of 50 cables is tested and is found that the mean braking strength is 1850. Can we support the claim at 1% level of significance | BTL3 | Applying | CO 4 |
| 6. | In a certain factory there are two independent processes manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 Ozs, with a standard deviation of 12 Ozs, while the corresponding | BTL3 | Applying | CO 4 |

**Q2 table (Hair color and Eye color):**

| | | Fair | Brown | Black | Total |
|---|---|---|---|---|---|
| Eye color | Blue | 15 | 5 | 20 | 40 |
| | Grey | 20 | 10 | 20 | 50 |
| | Brown | 25 | 15 | 20 | 60 |
| | Total | 60 | 30 | 60 | 150 |

**Q3 table:**

| Sample I | 19 | 17 | 15 | 21 | 16 | 18 | 16 | 14 |
|---|---|---|---|---|---|---|---|---|
| Sample II | 15 | 14 | 15 | 19 | 15 | 18 | 16 | |

**Q4 table:**

| Sample I | 9 | 11 | 13 | 11 | 15 | 9 | 12 | 14 |
|---|---|---|---|---|---|---|---|---|
| Sample II | 10 | 12 | 10 | 14 | 9 | 8 | 10 | |

**Q5(a) table:**

| Sample | Size | Sample mean | Sum of squares of deviation from the mean |
|---|---|---|---|
| 1 | 10 | 15 | 90 |
| 2 | 12 | 14 | 108 |

| | | | | |
|---|---|---|---|---|
| | figures in a sample of 400 items from the other process are 124 and 14. Is the difference between the two sample means significant? | | | |
| 7. | Records taken of the number of male and female births in 800 families having four Children are as follows : <br> Number of male births       : 0   1   2    3    4 <br> Number of female births      : 4   3   2    1    0 <br> Number of Families        : 32 178 290   236 64 <br> Infer whether the data are consistent with the hypothesis that the binomial law holds the chance of a male birth is equal to female birth, namely $p = \frac{1}{2} = q$. | BTL4 | Analyzing | CO 4 |
| 8. | A survey of 320 families with 5 children each revealed the following distribution <br><br> | Boys | 5 | 4 | 3 | 2 | 1 | 0 | <br> | Girls | 0 | 1 | 2 | 3 | 4 | 5 | <br> | Families | 14 | 56 | 110 | 88 | 40 | 12 | <br><br> Is this result consistent with the hypothesis that male and female births are equally probable? | BTL3 | Applying | CO 4 |
| 9. | The nicotine content in milligram of two samples of toboco where found to be as follows <br> Sample 1    24   27   26   21   25 <br> Sample 2    27   30   28   31   22   36 <br> Can it be said that this samples where from normal population with the same mean. | BTL-1 | Remembering | CO 4 |
| 10. | A group of 10 rats fed on diet A and another group of 8 rats fed on diet B, Recorded the following increase the following increase in weight.(gm) <br><br> | Diet A | 5 | 6 | 8 | 1 | 12 | 4 | 3 | 9 | 6 | 10 | <br> | Diet B | 2 | 3 | 6 | 8 | 10 | 1 | 2 | 8 | - | - | <br><br> Find the variances are significantly different. (Use F-test) | BTL-2 | Understanding | CO 4 |
| 11. | Mechanical engineers testing a new arc welding technique, classified welds both with respect to appearance and an X-ray inspection <br><br> | X-ray/Appearance | Bad | Normal | Good | <br> | Bad | 20 | 7 | 3 | <br> | Normal | 13 | 51 | 16 | <br> | Good | 7 | 12 | 21 | <br><br> Test for independence using 0.05 level of significance. | BTL3 | Applying | CO 4 |
| 12. | A sample of 200 persons with a particular disease was selected. Out of these, 100 were given a drug and the others were not given any drug. The result are as follows: <br><br> | Number of persons | Drug | No drug | Total | <br> | Cured | 65 | 55 | 120 | <br> | Not cured | 35 | 45 | 80 | <br> | Total | 100 | 100 | 200 | <br><br> Test whether the drug is effective or not? | BTL-2 | Understanding | CO 4 |

| 13. | The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week | BTL3 | Applying | CO 4 |
|---|---|---|---|---|

| Days | Sun | Mon | Tues | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|---|
| No. of accidents | 14 | 16 | 08 | 12 | 11 | 9 | 14 |

| 14. | The nicotine content in milligram of two samples of tobacco where found to be as follows, test the significant difference between means of the two samples. | BTL4 | Analyzing | CO 4 |
|---|---|---|---|---|

| Sample I | 21 | 24 | 25 | 26 | 27 | - |
|---|---|---|---|---|---|---|
| Sample II | 22 | 27 | 28 | 30 | 31 | 36 |

| 15. | The marks obtained by a group of 9 regular course students and another group of 11 part time course students in a test are given below : | BTL3 | Applying | CO 4 |
|---|---|---|---|---|

| Sample I | 56 | 62 | 63 | 54 | 60 | 51 | 67 | 69 | 58 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Sample II | 62 | 70 | 71 | 62 | 60 | 56 | 75 | 64 | 72 | 68 | 66 |

Examine whether the marks obtained by regular students and part-time students differ significantly at 5% levels of significance

| 16(a) | A simple sample of heights of 6400 Englishmen has a mean of 170cms and a standard deviation of 6.4cms, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a standard deviation of 6.3cms. Do the data indicate that Americans are, on the average, taller than Englishmen? | BTL4 | Analyzing | CO 4 |
|---|---|---|---|---|
| 16(b) | The theory predicts that the population of beans in the four groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans, the number in the four groups was 882,313,287 and 118. Do the experimental results support the survey? | BTL4 | Analyzing | CO 4 |
| 17(a) | The mean population of a random sample of 4000 villages in Jaipur district was found to be 400 with a standard deviation of 12. The mean population of a random sample of 400 villages in Meerut district was found to be 395 with standard deviation of 15. Is the difference between the two district's means statistically significant? | BTL3 | Applying | CO 4 |
| 17(b) | In a sample of 8 observations, the sum of squares of deviation of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level, given that the 5% point of F for v1=7 and v2=9 defers of freedom is 3,27 | BTL3 | Applying | CO 4 |
| 18. | Test of fidelity and selectivity of 190 radio receivers produced the results shown in the following table | BTL4 | Analyzing | CO 4 |

Fidelity

| Selectivity | Low | Average | High |
|---|---|---|---|
| Low | 6 | 12 | 32 |
| Average | 33 | 61 | 18 |
| High | 13 | 15 | 0 |

| | Use 0.01 level of significance to test whether there is a relationship between fidelity and selectivity. | | | |
|---|---|---|---|---|

**UNIT – V MULTIVARIATE ANALYSIS** Random Vectors and matrices - Mean vectors and covariance matrices – Multivariate normal density and its properties – Principal components – Population principal components – Principal components – Population principal components – Principal components from standardized variables.

| Q.No. | Question | Bloom's Taxonomy Level | Competence | Course Outcome |
|---|---|---|---|---|
| | **PART – A** | | | |
| 1. | Define random vector. | BTL-1 | Remembering | CO 5 |
| 2. | Define covariance matrix | BTL-1 | Remembering | CO 5 |
| 3. | State the properties of multivariate normal density. | BTL-2 | Understanding | CO 5 |
| 4. | Define Principal component analysis. | BTL-1 | Remembering | CO 5 |
| 5. | Define total population variance. | BTL-2 | Understanding | CO 5 |
| 6. | State the general objectives of principal components analysis. | BTL-2 | Understanding | CO 5 |
| 7. | Define the expected value of a random matrix. | BTL-1 | Remembering | CO 5 |
| 8. | $If \sum = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{pmatrix}$ $and$ $V^{1/2} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ find $\rho$ | BTL-2 | Understanding | CO 5 |
| 9. | $If \sum = \begin{pmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{pmatrix}$ Find the standard deviation matrix $V^{1/2}$ | BTL-2 | Understanding | CO 5 |
| 10. | $If$ $X = \begin{pmatrix} 42 & 4 \\ 52 & 5 \\ 48 & 4 \end{pmatrix}$ Find $\bar{x}$. | BTL-2 | Understanding | CO 5 |
| 11. | Define second principle component. | BTL-1 | Remembering | CO 5 |
| 12. | If $X_1$ and $X_2$ are two uncorrelated random variables, then what is the correlation coefficient matrix. | BTL-2 | Understanding | CO 5 |
| 13. | Define multivariate analysis. | BTL-1 | Remembering | CO 5 |
| 14. | State random matrices. | BTL-1 | Remembering | CO 5 |
| 15. | Establish the condition density of bivariate normal distribution. | BTL-2 | Understanding | CO 5 |
| 16. | Explain correlation of variables and components. | BTL-2 | Understanding | CO 5 |
| 17. | Enumerate rescaling the principal components. | BTL-2 | Understanding | CO 5 |
| 18. | Define first principal component. | BTL-1 | Remembering | CO 5 |
| 19. | What is the formula to compute the population variance due to $k^{th}$ principal component. | BTL-2 | Understanding | CO 5 |
| 20. | Explain the principal components obtained from standardized variables. | BTL-2 | Understanding | CO 5 |
| 21. | Define correlation coefficient in terms of variance and covariance | BTL-2 | Understanding | CO 5 |
| 22. | Write the matrix notation for principal component from standardized variables | BTL-2 | Understanding | CO 5 |
| 23. | Write any one theorem about principal component. | BTL-1 | Remembering | CO 5 |
| 24. | Write any two properties of multivariate normal distribution. | BTL-2 | Understanding | CO 5 |
| 25. | Write the multivariate normal density function. | BTL-1 | Remembering | CO 5 |
| | **PART –B** | | | |

| | | | | | |
|---|---|---|---|---|---|
| 1. | Compute the covariance matrix with the following data.<br><br>| X₂ \ X₁ | 0 | 1 | $P_1(x_1)$ |<br>|---|---|---|---|<br>| -1 | 0.24 | 0.06 | 0.3 |<br>| 0 | 0.16 | 0.14 | 0.3 |<br>| 1 | 0.40 | 0 | 0.4 | | BTL-2 | Understanding | CO 5 |
| 2. | Explain partitioning the covariance matrix. | BTL-2 | Remembering | CO 5 |
| 3. | Explain the mean vector and covariance matrix for linear combination of random variables | BTL-3 | Applying | CO 5 |
| 4. | Discuss Bivariate normal density. | BTL-3 | Applying | CO 5 |
| 5. | Prove that the correlation coefficient between the components are the eigen values – eigen vector pairs for sigma. | BTL-4 | Analyzing | CO 5 |
| 6. | Consider the random vector $X' = \{X_1, X_2\}$ The discrete random variable $X_1$ have the following probability function :<br>$X_1$ : -1  0  1<br>$P_1(X_1)$: 0.3  0.3  0.4  and $X_2$ have the following probability function<br>$X_2$ : 0  1<br>$P_1(X_1)$: 0.8  0.2 | BTL-3 | Applying | CO 5 |
| 7. | Let the random variables $X_1, X_2$ and $X_3$ have the covariance matrix $\sum = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ Determine the principal components of $Y_1, Y_2, _3$ | BTL-4 | Analyzing | CO 5 |
| 8. | Let $X_{3x1}$ be $N_3(\mu, \sigma)$ with $\sum = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ Are $X_1, X_2$ independent? What about $(X_1, X_2)$ and $X_3$. | BTL-3 | Applying | CO 5 |
| 9. | Discuss principal components from standardized variables. | BTL-4 | Analyzing | CO 5 |
| 10. | Explain principal component population. | BTL-3 | Applying | CO 5 |
| 11. | Let X be distributed as $N_3(\mu, \Sigma)$ where $\mu' = (1, -1, 2)$ and $\Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ which of the following random variables are independent ? Explain<br>i) $X_1$ and $X_2$<br>ii) $X_1$ and $X_3$<br>iii) $X_2$ and $X_3$<br>iv) $(X_1, X_3)$ and $X_2$ | BTL-4 | Analyzing | CO 5 |

| 12. | For the covariance matrix $\Sigma = \begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix}$ the derived correlation matrix P = $\begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$, Show that the principal components obtained from covariance and correlation matrices are different. | BTL-3 | Applying | CO 5 |
|---|---|---|---|---|
| 13. | Prove that If $\Sigma$ is positive definite so that $\Sigma^{-1}$ exists the $\Sigma$ e $= \lambda e = \Sigma^{-1} e = \left(\frac{1}{\lambda}\right) e$ so $(\lambda, e)$ is an eigen value – eigen vector pair for $\Sigma$ corresponding to the pair $(\frac{1}{\lambda}, e)$ for $\Sigma^{-1}$, also $\Sigma^{-1}$ is positive definite. | BTL-3 | Applying | CO 5 |
| 14. | Prove that the distribution of two linear combination of the components of a random vector. | BTL-4 | Analyzing | CO 5 |
| 15. | Compute the principal components to the following matrix $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ | BTL-3 | Applying | CO 5 |
| 16. | Compute the principal components to the following matrix $A = \begin{pmatrix} 5 & 0 & 3 \\ 4 & 2 & 5 \\ 2 & -2 & -2 \end{pmatrix}$ | BTL-3 | Applying | CO 5 |
| 17. | Compute the principal component to the variance covariance matrix $\Sigma = \begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix}$ | BTL-3 | Applying | CO 5 |
| 18. | If $\Sigma = \begin{pmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{pmatrix}$ then find (i) $V^{1/2}$ and $\rho$ ( ii) Show that $V^{1/}$ $\rho V^{1/2} = \rho$ | BTL-3 | Applying | CO 5 |

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